

An introduction to statistical physics with simple examples

Hugo Touchette
School of Mathematical Sciences
Queen Mary University of London
London, United Kingdom
ht@maths.qmul.ac.uk

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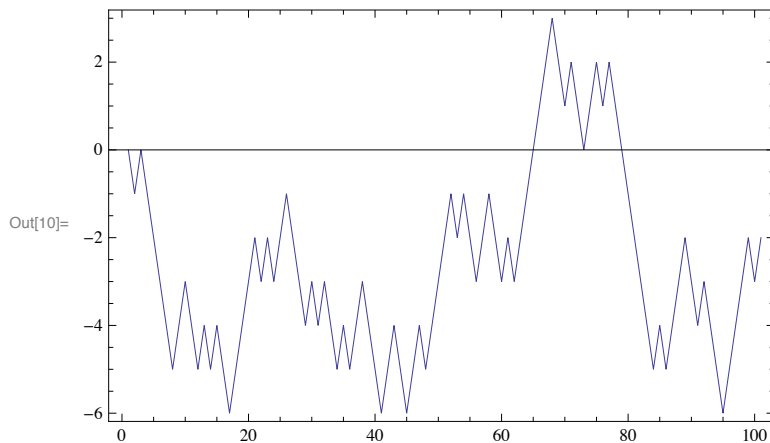
Presentation at <http://www.maths.qmul.ac.uk/~ht/talks/>

Random walks

■ 1D random walk

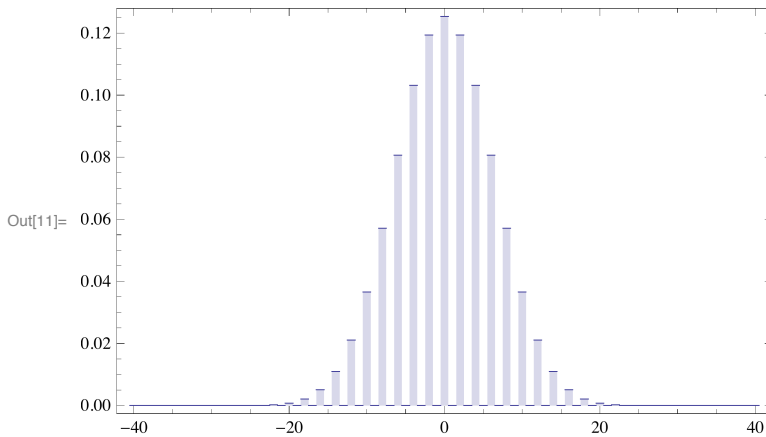
```
In[9]:= BiasedStep[a_] := If[RandomReal[] < a, -1, 1]
```

```
In[10]:= With[{Nrw = 1, Nsteps = 100, a = 0.5},  
  rwlist = {};  
  Do[  
    steplist = Table[BiasedStep[a], {i, Nsteps}];  
    rw = FoldList[Plus, 0, steplist];  
    AppendTo[rwlist, rw];  
    , {j, Nrw};  
  ListPlot[Evaluate@rwlist, Joined -> True]  
]
```



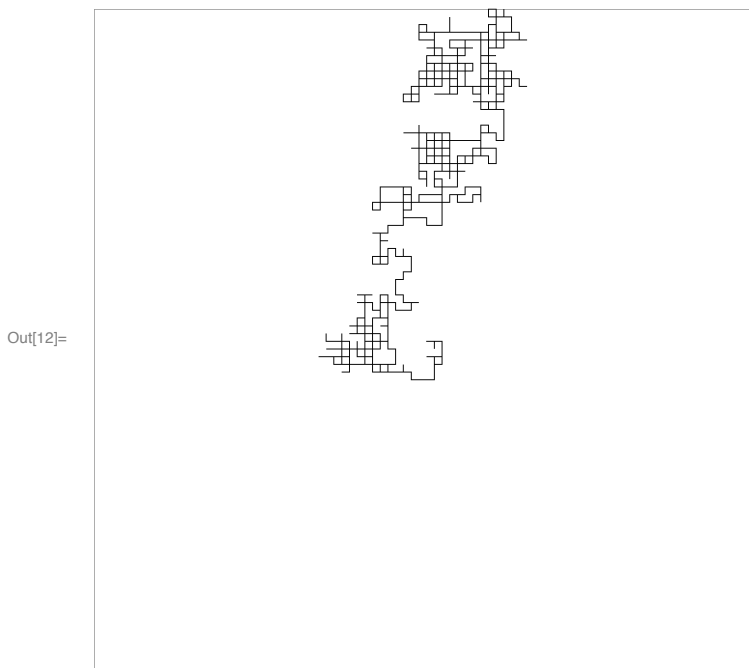
■ Binomial distribution

```
In[11]:= With[{a = 0.5, Nsteps = 40},
  DiscretePlot[PDF[BinomialDistribution[Nsteps, a], (Nsteps - s) / 2],
    {s, -Nsteps, Nsteps, 1}, PlotRange -> All, ExtentSize -> 1, Frame -> True, Joined -> False]
]
```



■ 2D random walk

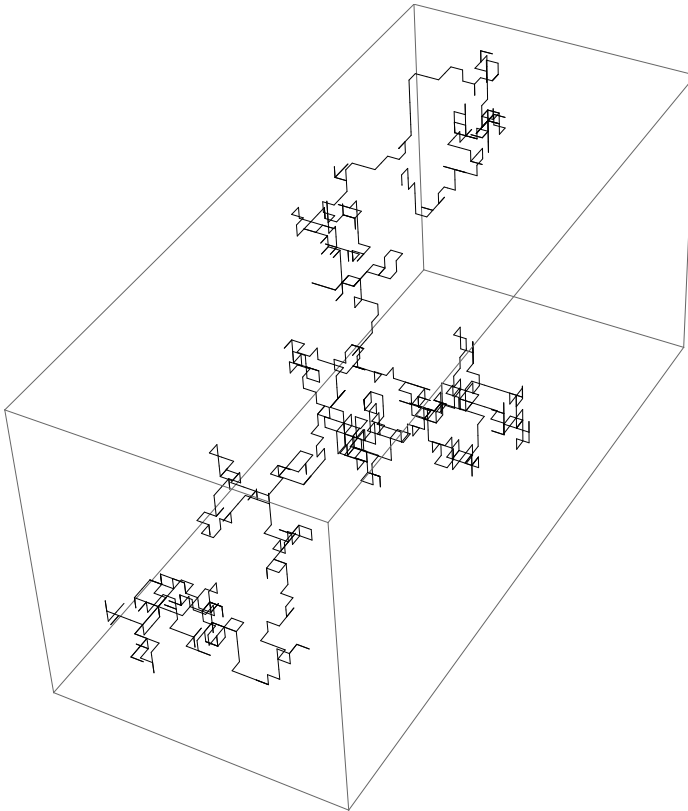
```
In[12]:= With[{Nsteps = 1000},
  moves = {{0, -1}, {-1, 0}, {0, 1}, {1, 0}};
  randommoves = Table[RandomChoice[moves], {i, Nsteps}];
  traj = FoldList[Plus, {0, 0}, randommoves];
  maxv = Max[Abs[traj]];
  box = {EdgeForm[Thin], Transparent, Rectangle[{-maxv, -maxv}, {maxv, maxv}]};
  Graphics[{box, Line[traj]}]
]
```



■ 3D random walk

```
In[13]:= With[{Nsteps = 1000},
  moves = {{-1, 0, 0}, {1, 0, 0}, {0, -1, 0}, {0, 1, 0}, {0, 0, -1}, {0, 0, 1}};
  randommoves = Table[RandomChoice[moves], {i, Nsteps}];
  traj = FoldList[Plus, {0, 0, 0}, randommoves];
  maxv = Max[Abs[traj]];
  box = {EdgeForm[Thin], Transparent, Rectangle[{-maxv, -maxv}, {maxv, maxv}]}];
  Graphics3D[Line[traj]]
]
```

Out[13]=

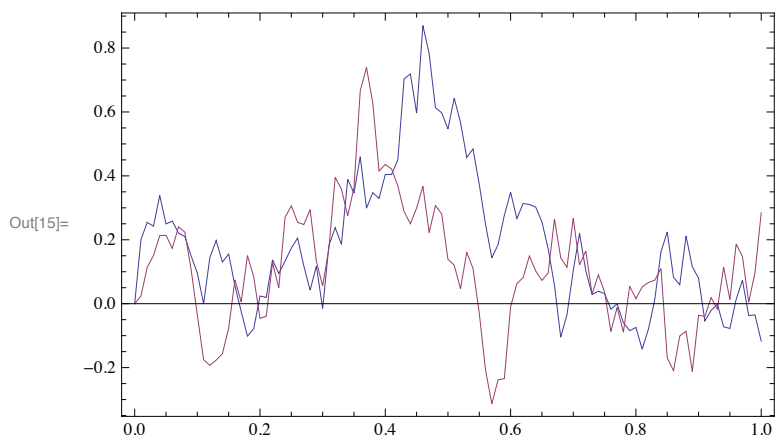


Brownian motion

■ 1D

```
In[14]:= BrownianMotion1[σ_, T_, dt_] := Module[{TimeList, dB, BMT},
  TimeList = Range[0, T, dt];
  dB = Sqrt[dt] RandomReal[NormalDistribution[0, σ], T / dt];
  BMT = Transpose[{TimeList, FoldList[Plus, 0, dB]}]
]
```

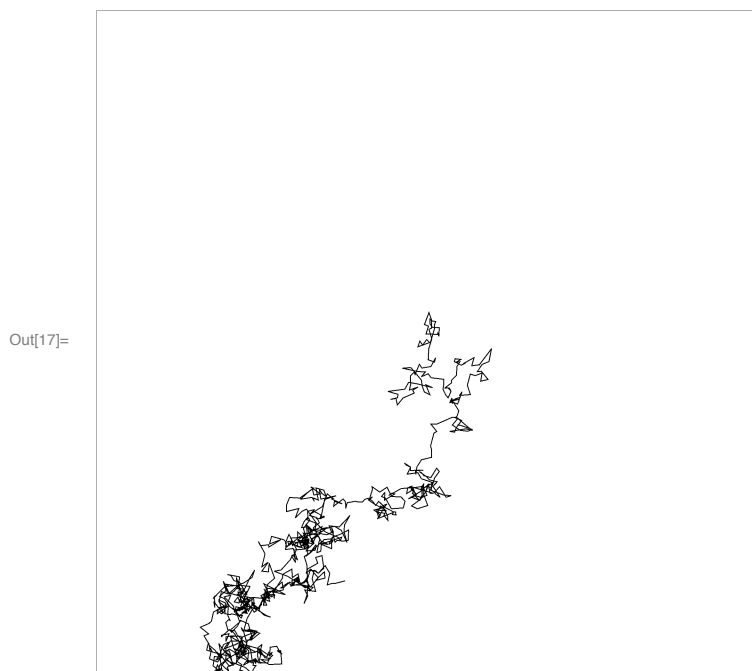
```
In[15]:= ListPlot[Table[BrownianMotion1[1, 1, 0.01], {2}],
  Joined → True, PlotRange → All, Frame → True]
```



■ 2D trace

```
In[16]:= BMDynamics[{x_, y_}, σ_] := {x, y} + RandomVariate[NormalDistribution[0, σ], {2}]
```

```
In[17]:= With[{dt = 0.01, σ = 1, N = 1000, v = 5},
  traj = NestList[BMDynamics[#, σ √dt] &, {0, 0}, N];
  maxv = Max[Abs[traj]];
  box = {EdgeForm[Thin], Transparent, Rectangle[{-maxv, -maxv}, {maxv, maxv}]};
  Graphics[{box, Line[traj]}]
]
```



■ 2D animation

```
In[18]:= With[{dt = 0.01, σ = 1, v = 5},
  box = {EdgeForm[Thin], Transparent, Rectangle[{-v, -v}, {v, v}]};
  pos = {0, 0};
  Graphics[{box, Disk[Dynamic[pos = BMDynamics[pos, σ √dt]], 0.2]}]
]
```

■ 2D animation with trace

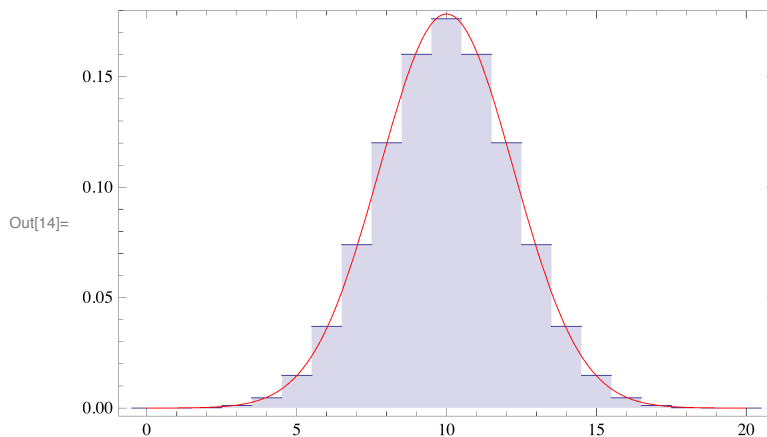
```
In[19]:= With[{dt = 0.01, σ = 1, v = 5},
  box = {EdgeForm[Thin], Transparent, Rectangle[{-v, -v}, {v, v}]};
  {x, y} = {0, 0};
  traj = {{x, y}};
  Graphics[{box, Line[Dynamic[AppendTo[traj, BMDynamics[Last[traj], σ √dt]]]]]}]
]
```

■ Many particles

```
In[22]:= With[{dt = 0.01, σ = 0.1, v = 100, Npart = 2},
  box = {EdgeForm[Thin], Transparent, Rectangle[{-v, -v}, {v, v}]};
  pos = Table[{0, 0}, {i, Npart}];
  Graphics[{box, Point[Dynamic[pos = (BMDynamics[#, 1] & /@ pos)]]]}]
]
```

Galton board (and the binomial distribution)

```
In[14]:= With[{nmax = 20, p = 0.5},
  σ = StandardDeviation[BinomialDistribution[nmax, p]];
  p1 = DiscretePlot[PDF[BinomialDistribution[nmax, p], i],
    {i, 0, nmax}, Frame → True, PlotRange → All, ExtentSize → 1];
  p2 = Plot[PDF[NormalDistribution[nmax / 2, σ], x], {x, 0, nmax},
    PlotRange → All, PlotStyle → Red];
  Show[{p1, p2}]
]
```



Maxwell's distribution

Boltzmann's constant:

```
In[32]:= kB = 1.380648 × 10-23;
```

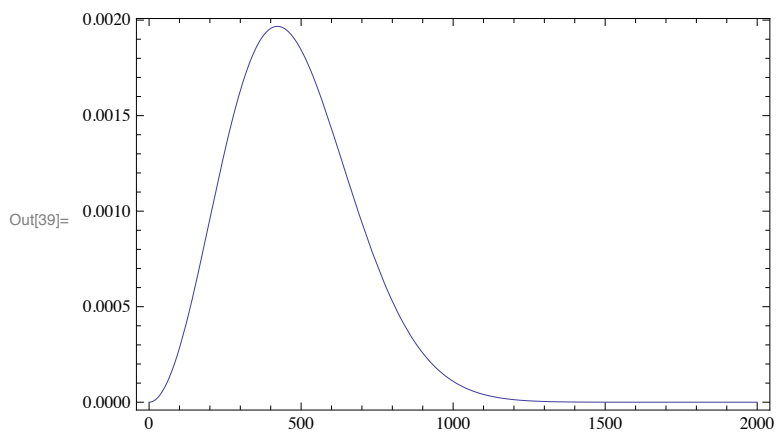
Mass of diatomic nitrogen N₂:

```
In[38]:= mN2 = 2 × 14.0067 × 1.660538 × 10-27;
```

```
In[25]:= pmaxwell[v_, T_, m_] := 
$$\sqrt{\frac{2}{\pi}} \left(\frac{m}{kB T}\right)^3 v^2 \text{Exp}\left[-\frac{m v^2}{2 kB T}\right]$$

```

```
In[39]:= Plot[pmaxwell[v, 300, mN2], {v, 0, 2000}]
```



```
In[40]:=  $\sqrt{\frac{2 \text{ kB } 300}{\text{mN2}}}$ 
```

Out[40]= 421.997

```
In[41]:= Plot[Evaluate@Table[pmaxwell[v, T, mN2], {T, 100, 300, 100}], {v, 0, 2000}]
```

