Granular Brownian motion with solid friction

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A. Gnoli, A. Puglisi, HT
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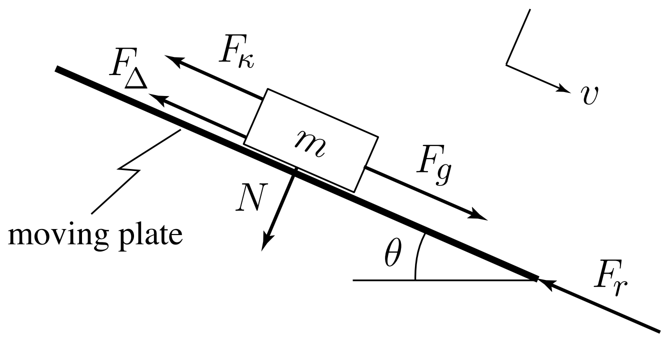
Solid friction



- Newton's equation:

$$m\dot{v} = -\underbrace{\alpha v(t)}_{\text{viscous}} - \underbrace{\Delta_F\,\sigma(v)}_{\text{dry}} + \underbrace{F}_{\text{ext}} \qquad \sigma(v) = \begin{cases} -1 & v<0 \\ 0 & v=0 \\ +1 & v>0 \end{cases}$$

- Solid friction = dry friction = Coulomb friction
- Critical angle:

$$\theta_c = \arctan(\Delta/g), \qquad \Delta = \Delta_F/m$$

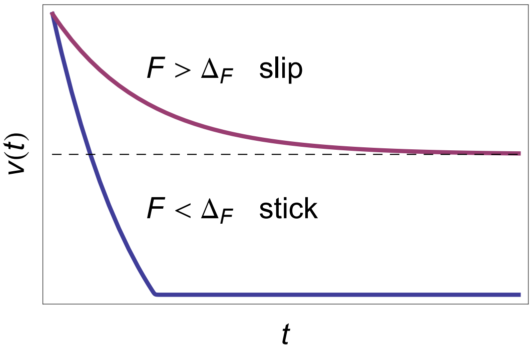
Solid friction (cont'd)

$$m\dot{v} = -\alpha v(t) - \Delta_F\,\sigma(v) + F$$

- $|F| < \Delta_F$
  - $v^* = 0$ stable
  - Reached in finite time

$$\tau = \frac{m}{\alpha}\ln\left(1 + \frac{v_0\alpha}{\Delta_F}\right)$$

- $|F| > \Delta_F$
  - $v^* = (F - \Delta_F)/\alpha$
  - Reached for $t = \infty$



Motion from $v = 0$ iff $|F| > \Delta_F$

Brownian motion with solid friction

Caughey 60s, de Gennes 2005

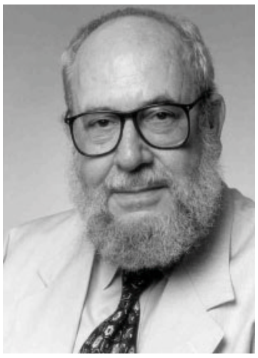
- Langevin equation:

$$m\,\dot{v}(t) = -\alpha v(t) - \Delta_F\,\sigma(v) + F + \xi(t)$$

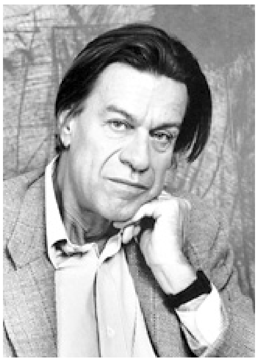
- Gaussian white noise:

$$\langle \xi(t)\rangle = 0$$
$$\langle \xi(t)\xi(0)\rangle = m^2\Gamma\delta(t)$$

- Diffusion with solid friction
- No fluctuation-dissipation relation
- Noise and friction have different sources



Thomas K. Caughey 1927-2004



Pierre-Gilles de Gennes 1932-2007

Solution

- Propagator:

$$P = p(v, t|v_0, 0) = \mathrm{Prob}\{v(t) = v|v(0) = v_0\}$$

- Fokker-Planck equation:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial v}\left(\Phi'(v)P + \frac{\partial P}{\partial v}\right) = -\frac{\partial j}{\partial v}$$

- Potential:

$$\Phi(v) = \frac{(|v| + \Delta)^2}{2} - av$$

- $\Delta = \Delta_F/m$, $a = F/m$
- Correlation function:

$$\langle v(\tau)v(0)\rangle = \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dv'\, v\, v'\, p(v, \tau|v', 0)\rho_*(v')$$

Piecewise linear $\Rightarrow$ exactly solvable

Predictions

- Stationary distribution

$$f(v) = C\,\mathrm{e}^{-2U(v)/\Gamma}, \quad U(v) = \frac{\gamma v^2}{2} + \Delta|v| - \frac{F}{m}v, \qquad \gamma = \frac{\alpha}{m}$$

  - Cusp at $v = 0$

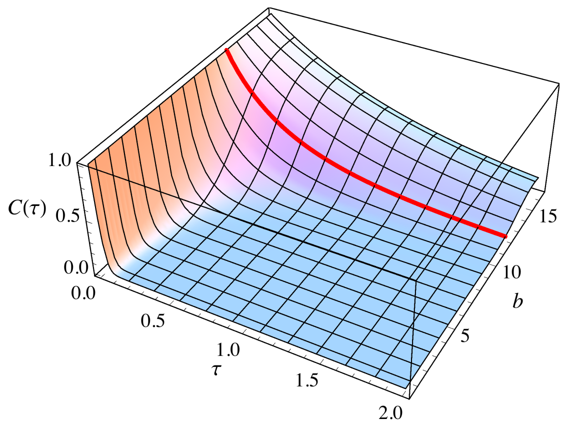
- Drift velocity

$$\begin{aligned} v_* &= \alpha^{-1}F && \text{(Brownian)} \\ v_* &\sim \Gamma F/{\Delta_F}^2 && \text{(Dry friction)} \quad |F| < \Delta_F \end{aligned}$$

- Correlation function

$$\langle v(t)v(0)\rangle = De^{-C_\Delta t}$$

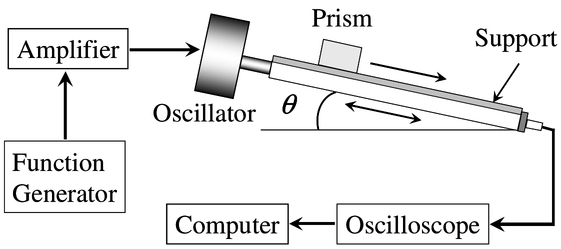
  - de Gennes JSP 2005 ($\gamma = F = 0$)
  - HT, Straeten, Just JPA 2010
  - Stick-slip crossover

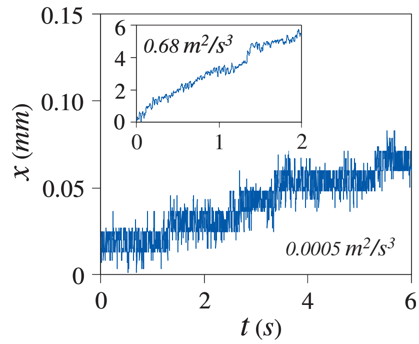


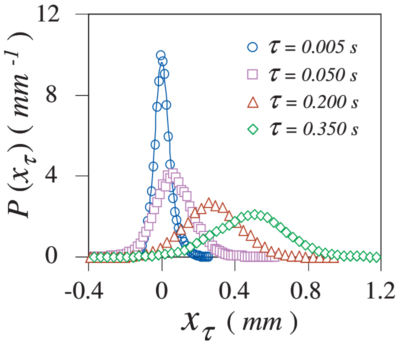
- Power spectrum

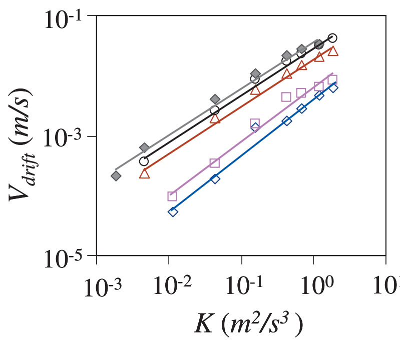
Previous experiment

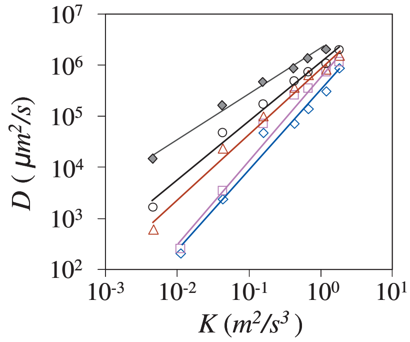
Goohpattader & Chaudhury, J Chem Phys 133, 024702, 2010







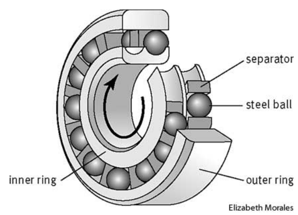


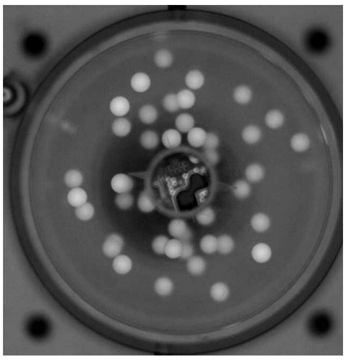


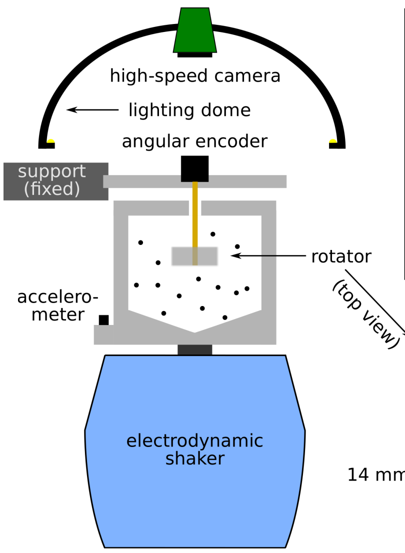
Experiment

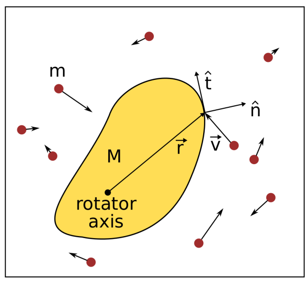
With Andrea Puglisi, Andrea Gnoli (La Sapienza, Rome)

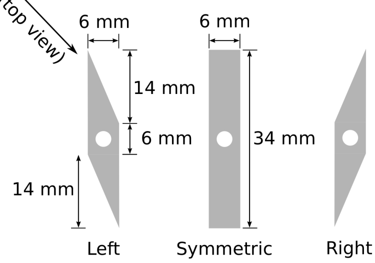
- Rotating pawl in granular gas
- Granular gas
  - $\sim$ 50 spheres, 6mm diameter
  - Gaussian velocity pdf (by camera tracking)
- Angle encoder (rate: 1 kHz)
- Solid friction: Ball bearing
- Frequency regimes:
  - High: Gaussian noise
  - Low: Poisson noise



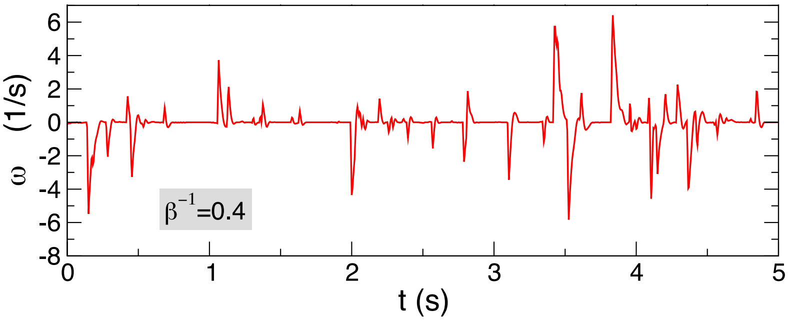


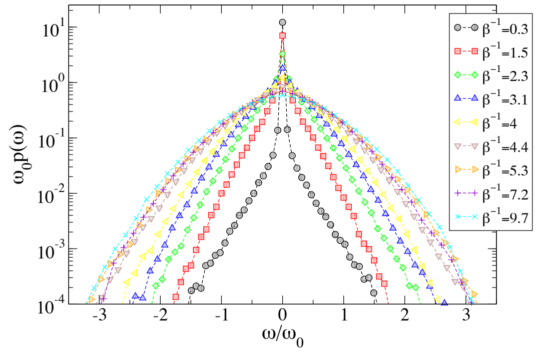






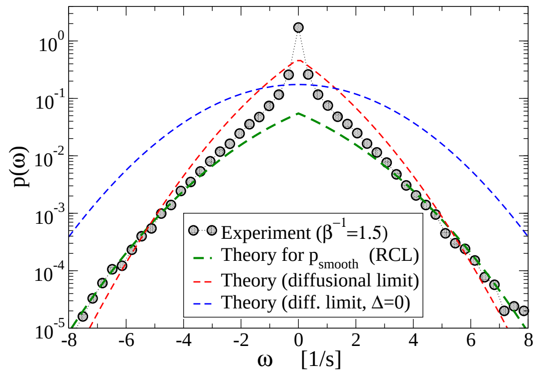
Low frequency regime (Poisson)





- Effective granular temperature:

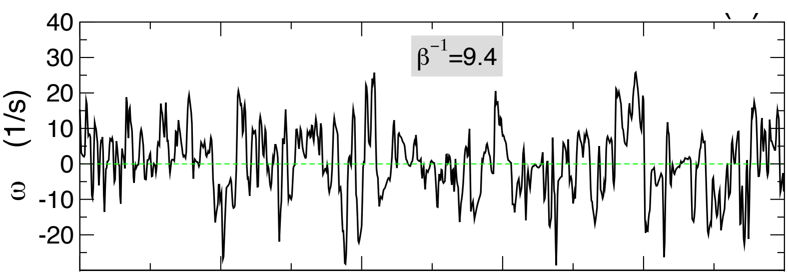
$$\beta^{-1} \sim \frac{\tau_\Delta}{\tau_c} \ll 1$$

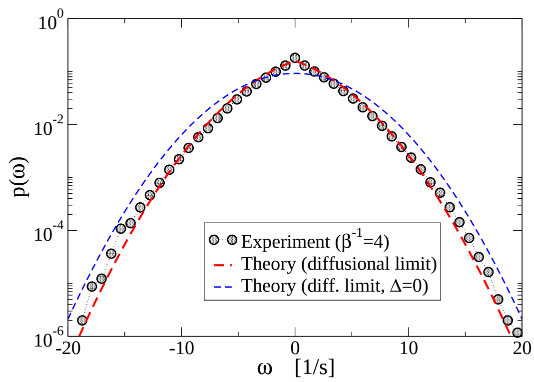


- Collisions followed by relaxation
- Poisson collision model
- Stationary distribution:

$$p(\omega) = a\delta(\omega) + (1-a)p_{smooth}(\omega)$$

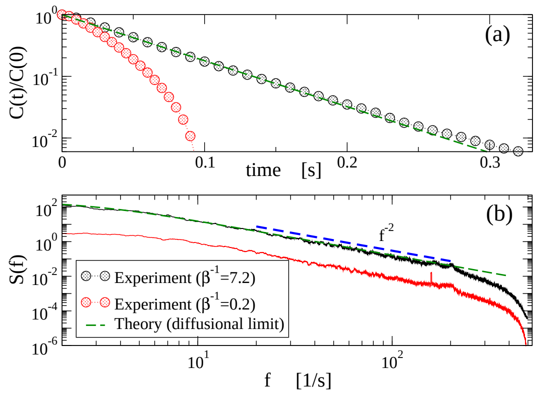
High frequency regime (Gaussian)





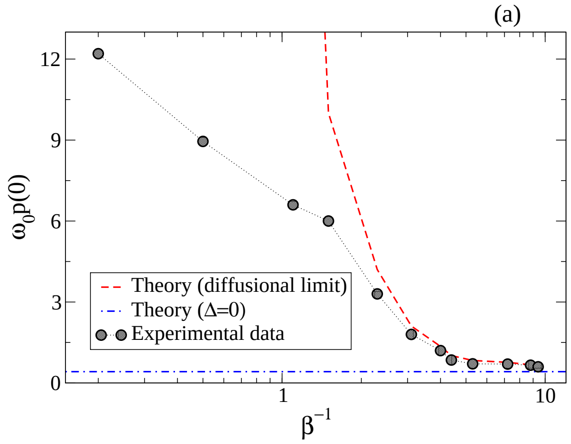
- $\beta^{-1} \sim \dfrac{\tau_\Delta}{\tau_c} \gg 1$
- Caughey–de Gennes Brownian model
- Stationary distribution:

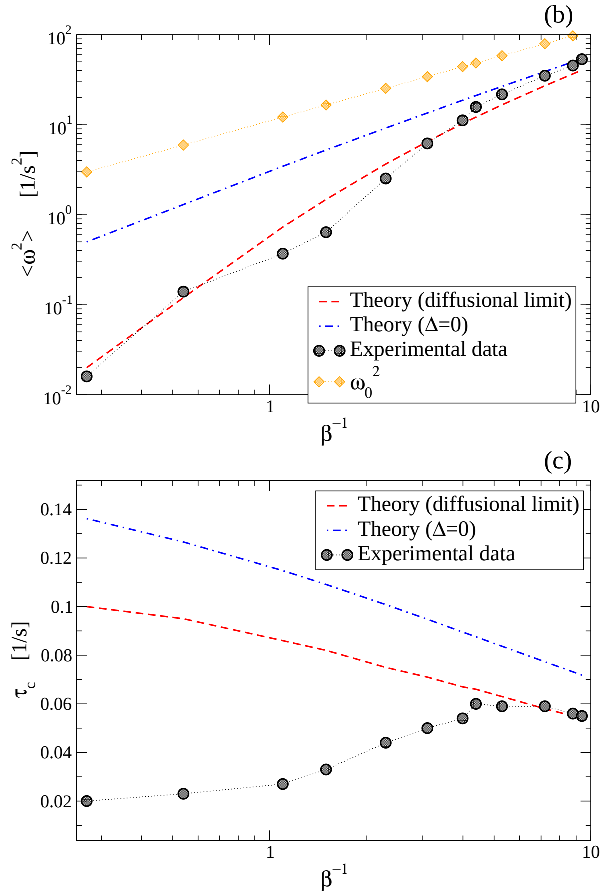
$$p(\omega) = C\,\exp\left[-\frac{(|\omega| + \Delta/\gamma)^2}{\Gamma_g/\gamma}\right]$$



- Cusp at $\omega = 0$
- $C(t) = \langle \omega(t)\omega(0) \rangle$
- No fitting parameter

Crossover





- Good fit for $\beta^{-1} \gg 1$
- Gaussian limit works
- Solid friction important
- No theory for Poisson limit

Conclusions

Main results

- Stationary distribution $p(\omega)$
- Correlation function $C(t) = \langle \omega(t)\omega(0)\rangle$
- Power spectrum $S(f)$
- Good fit with Gaussian (Brownian) theory
- No fitting parameter

Missing theory

- Poisson theory for $C(t)$ and $S(f)$
- Crossover between Poisson and Gaussian

Future experiments

- Effect of external force
- Stick-slip crossover with noise
- Predictions available

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