Integral conceptions: Transition from single to multivariable integral calculus in Engineering

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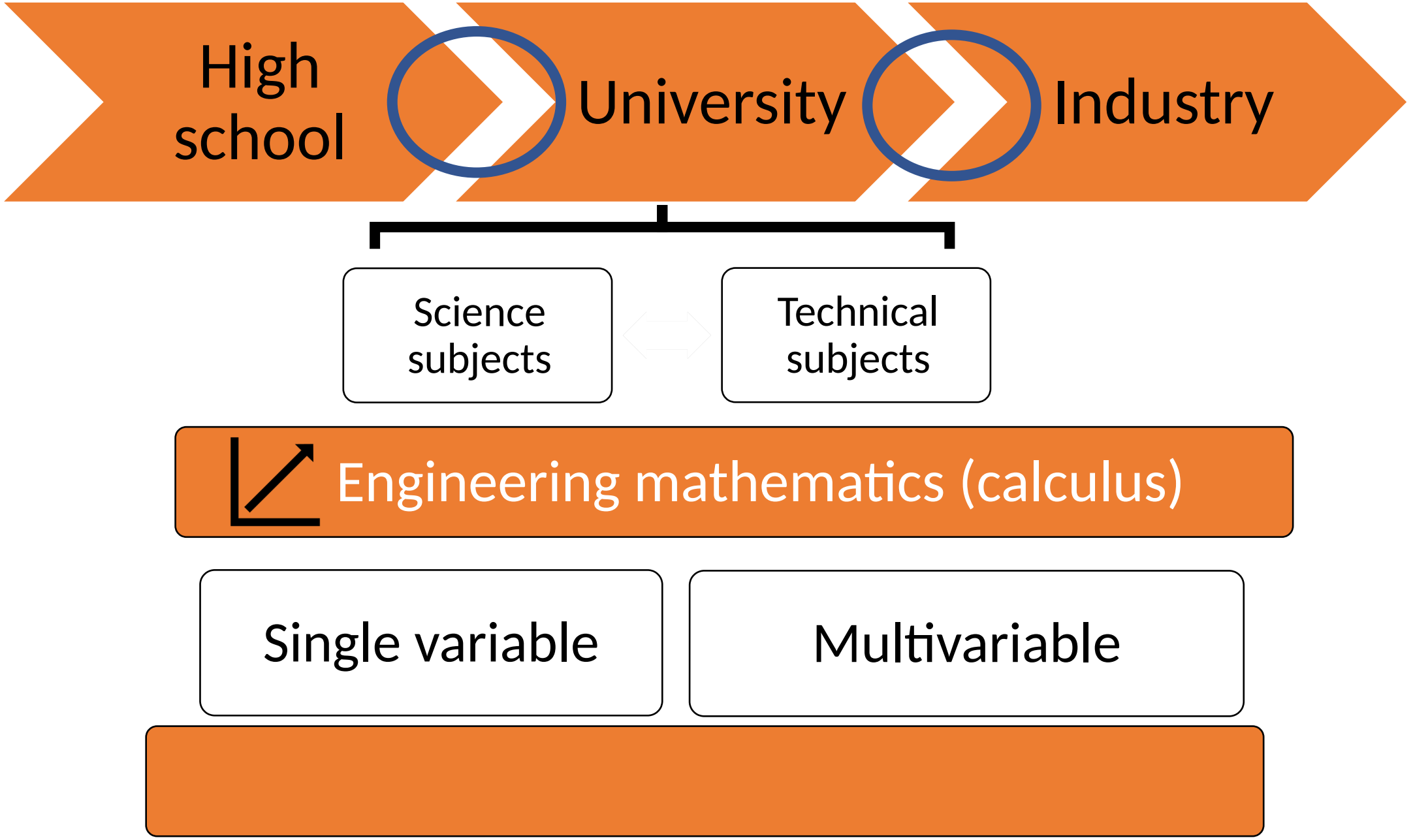
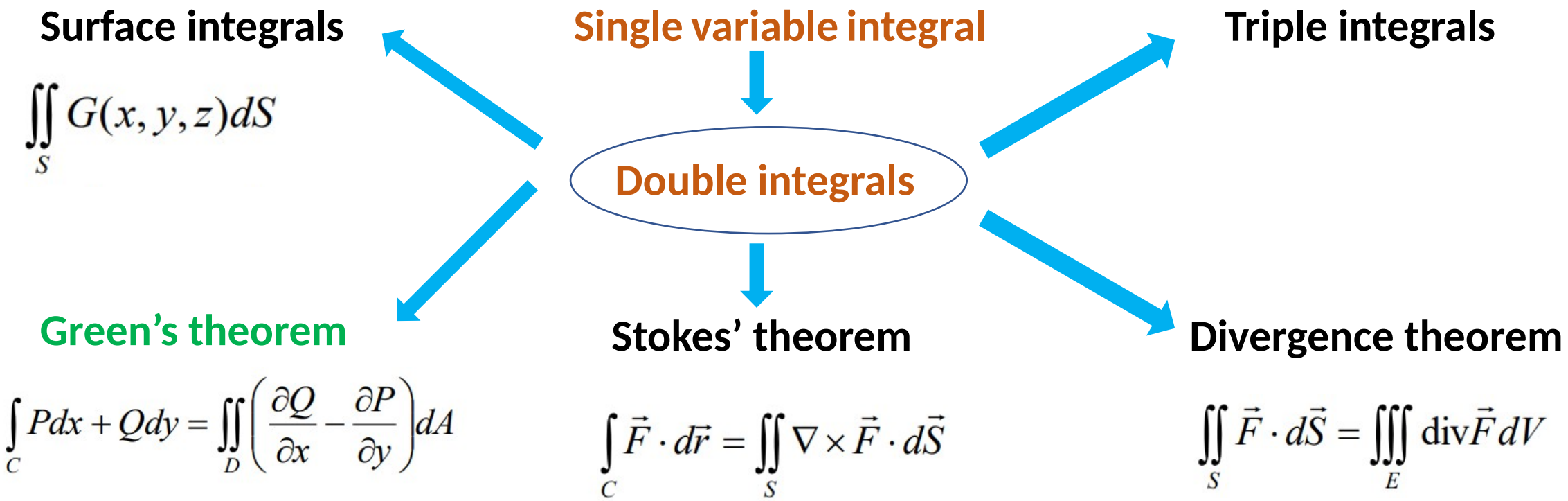
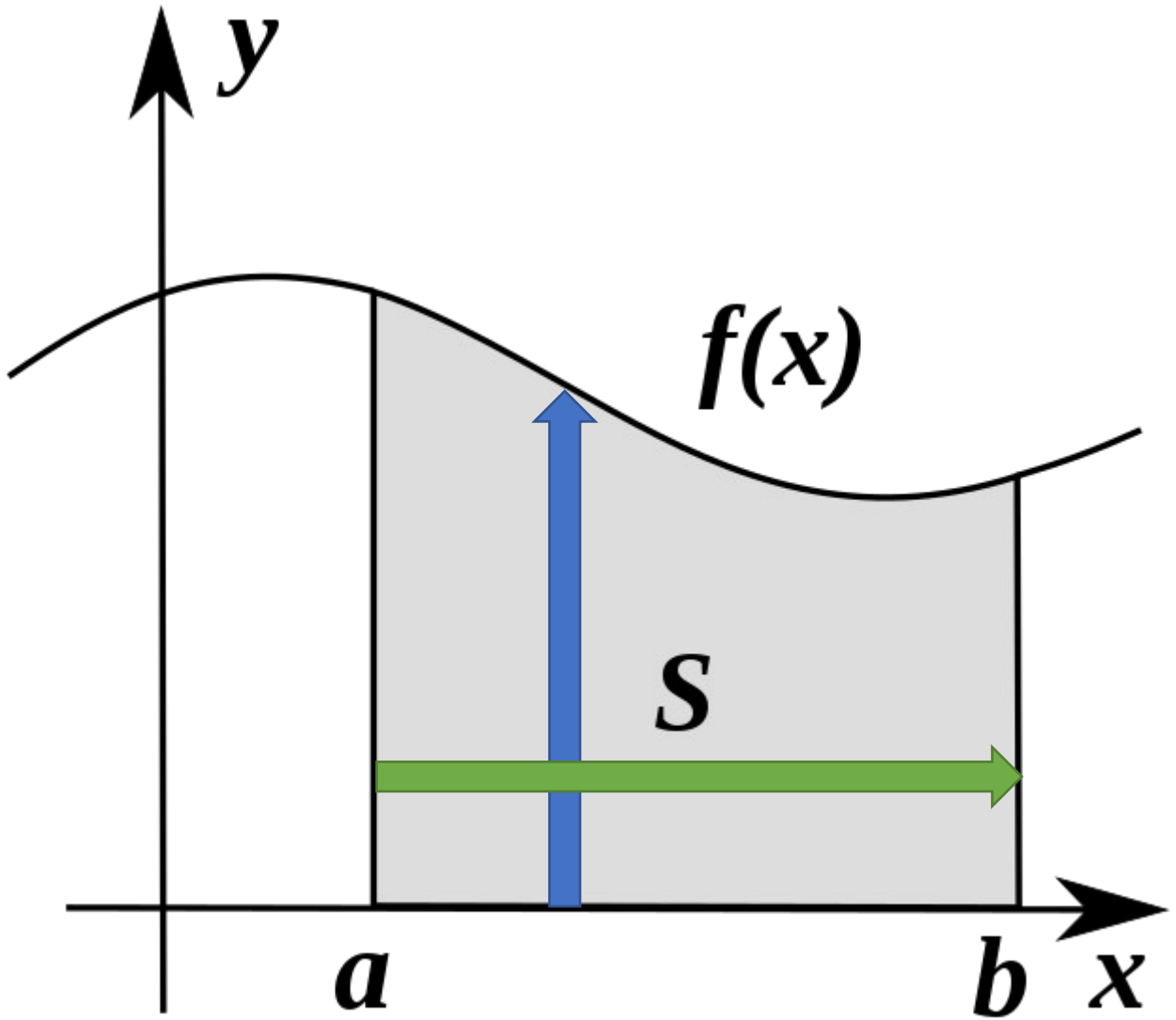

High school
University
Industry
Science subjects
Technical subjects
Engineering mathematics (calculus)
Single variable
Multivariable

Double integrals in multivariable integration



Single VS Double integral

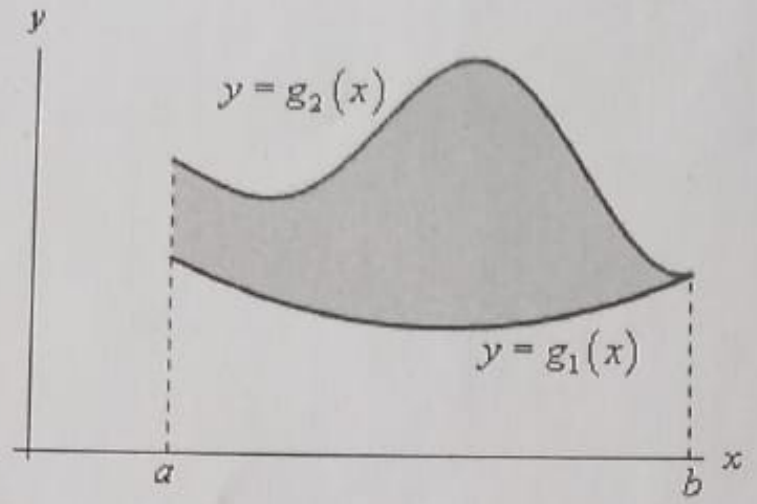


Single Integral

Double Integral

How do single variable conceptions affect students' understanding of double integrals?

Setup an integral that can be used to evaluate the shaded area



a) As a single integral

$$\int_a^b \left(g_2(x) - g_1(x)\right) dx$$

b) As a double integral

$$\int_a^b \int_{g_1(x)}^{g_2(x)} dy\,dx$$

1. What is the relationship between the double Riemann sum and the double integral? Explain

Riemann sum is ~~what~~ basically finding the area using a limit. it is similar to integration, They all lead to area, just that different methods.

4. What do you understand by $I = \int_1^2 \int_0^5 dxdy$?

I is the area located between $0 \leq x \leq 5$ and $1 \leq y \leq 2$

How do single variable conceptions affect students' understanding of double integrals?

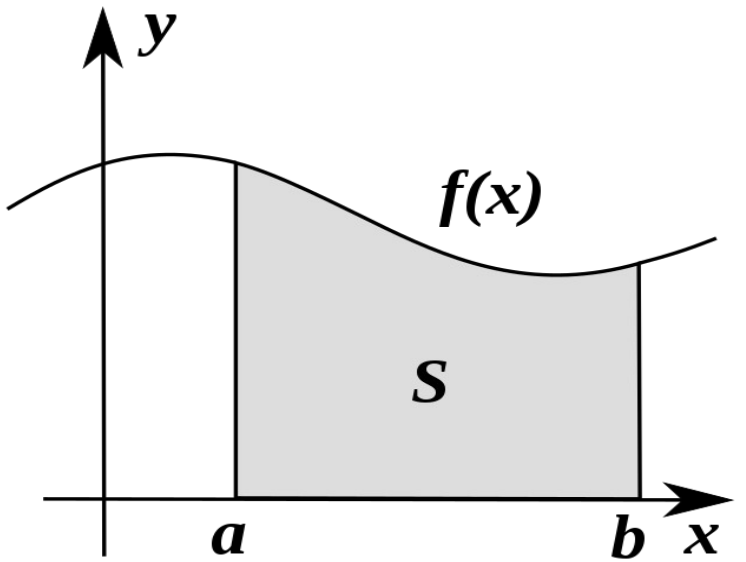
- **Area under the curve**

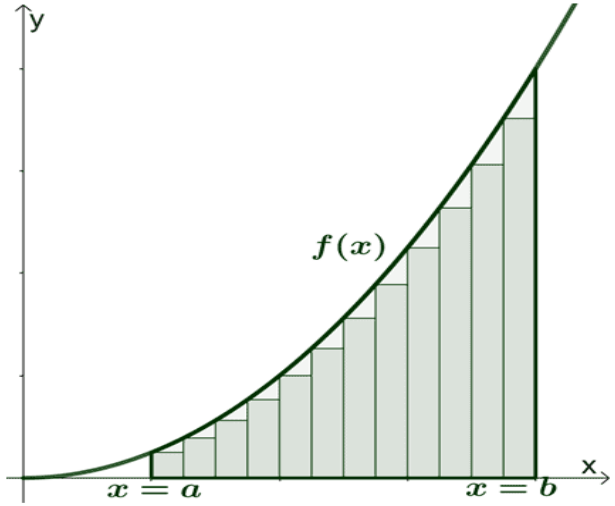
  Volume below surface

  Sum of Vertical slices

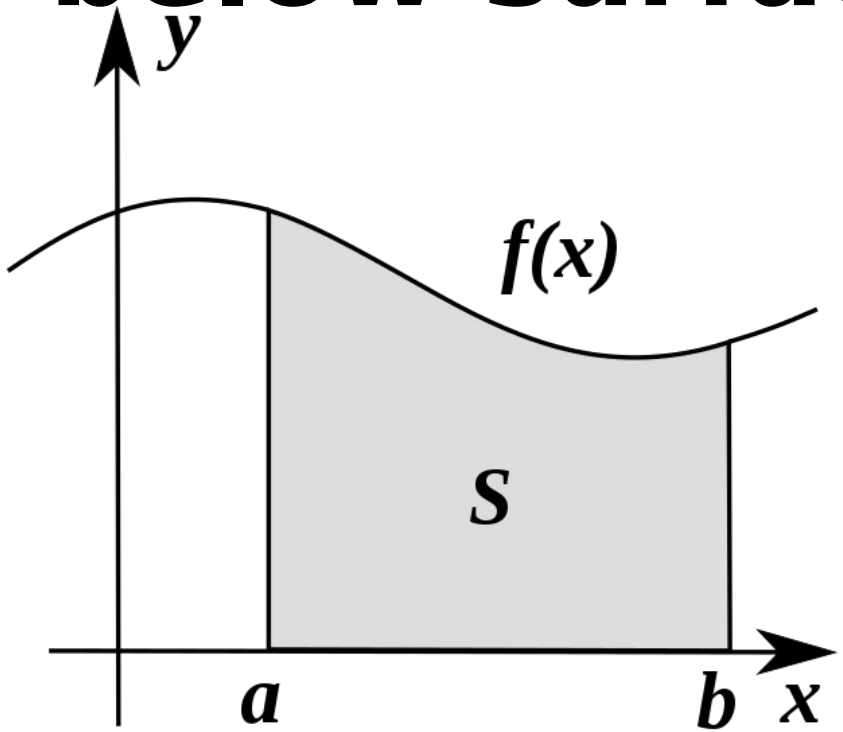
- **Riemann approach**

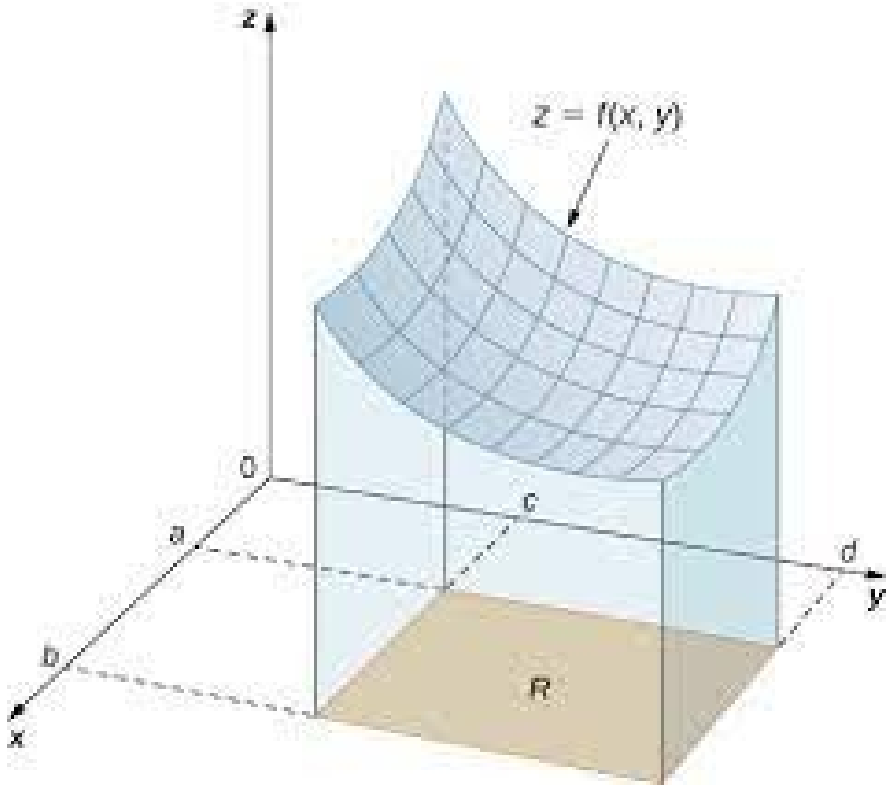
  Multiple representations





Area under the curve → Volume below surface



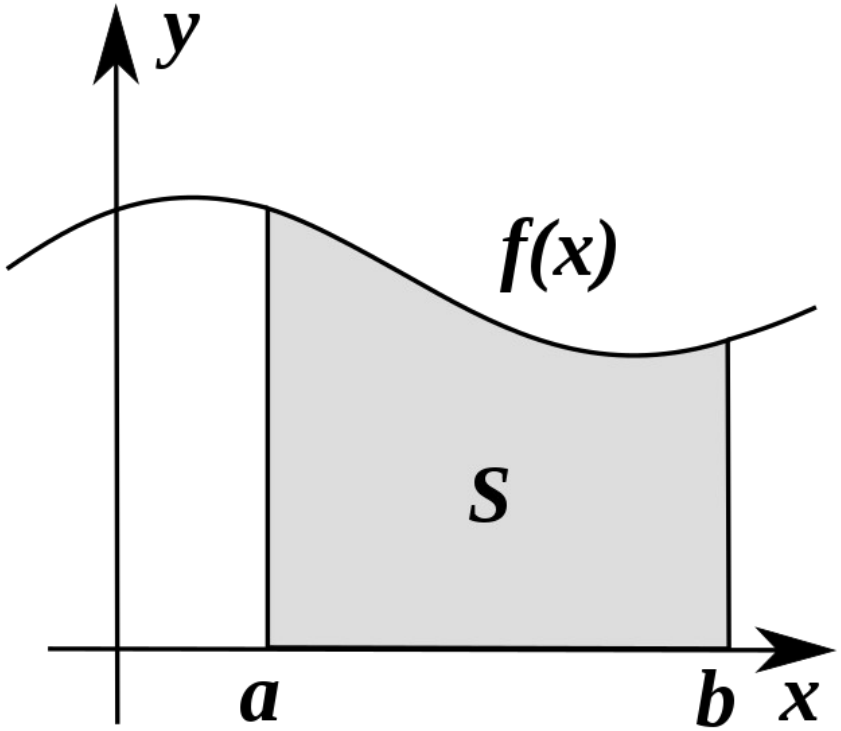


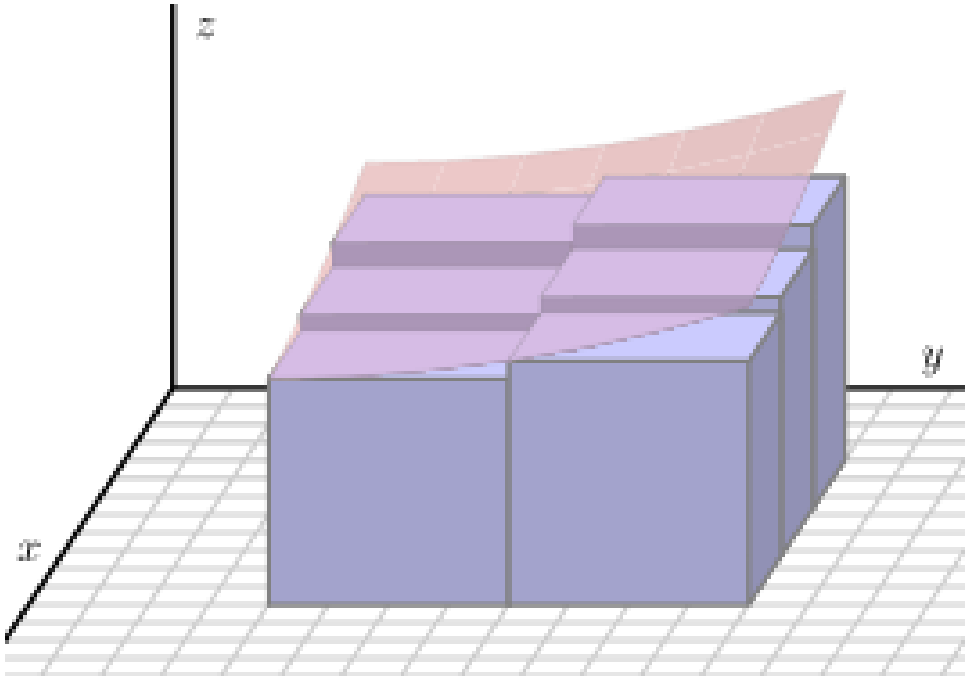
The integrand is the top boundary of the area in single variable integrals as is for a double integral

$$= \int_a^b f(x)\,dx$$

$$V = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

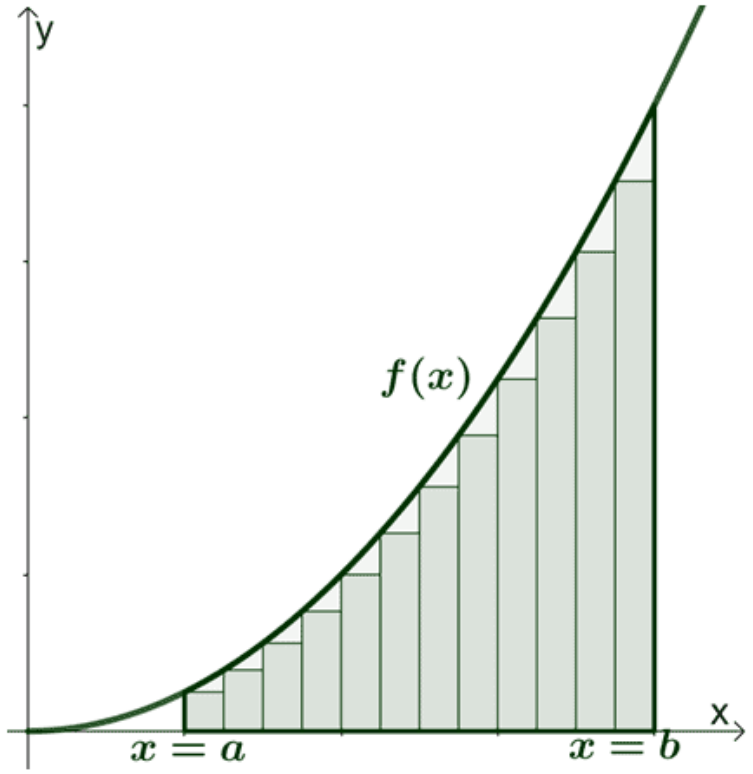
Area under the curve → Sum of vertical slices

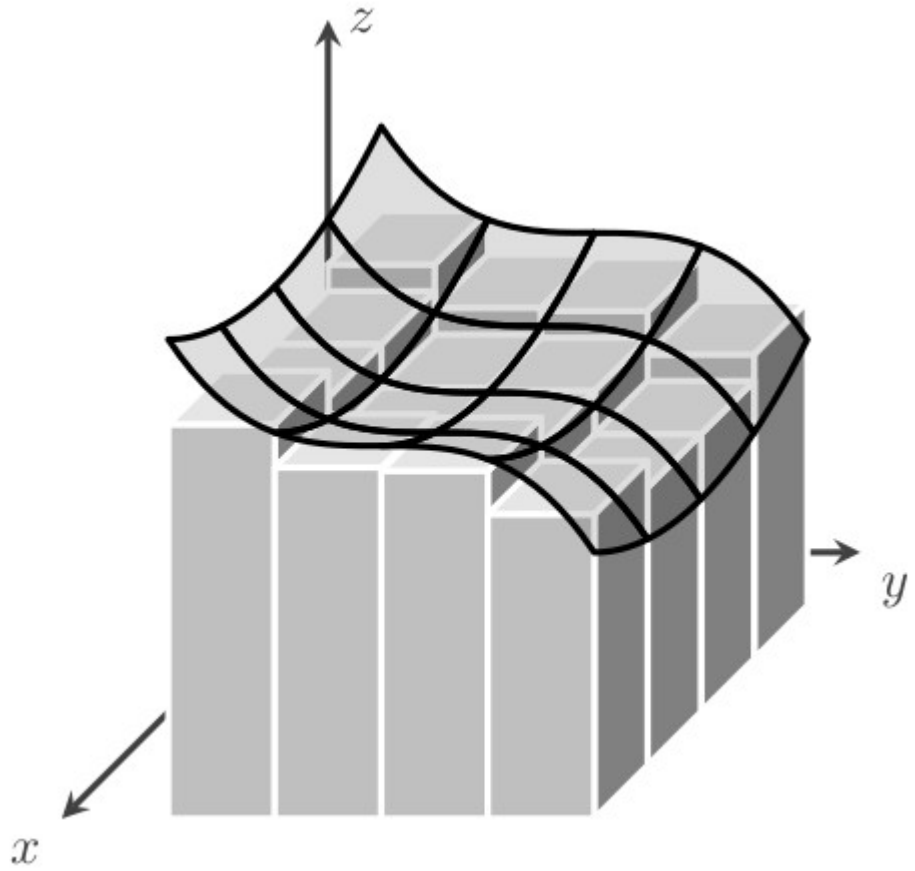




Layers of Area S are stuck together to give the volume of the solid

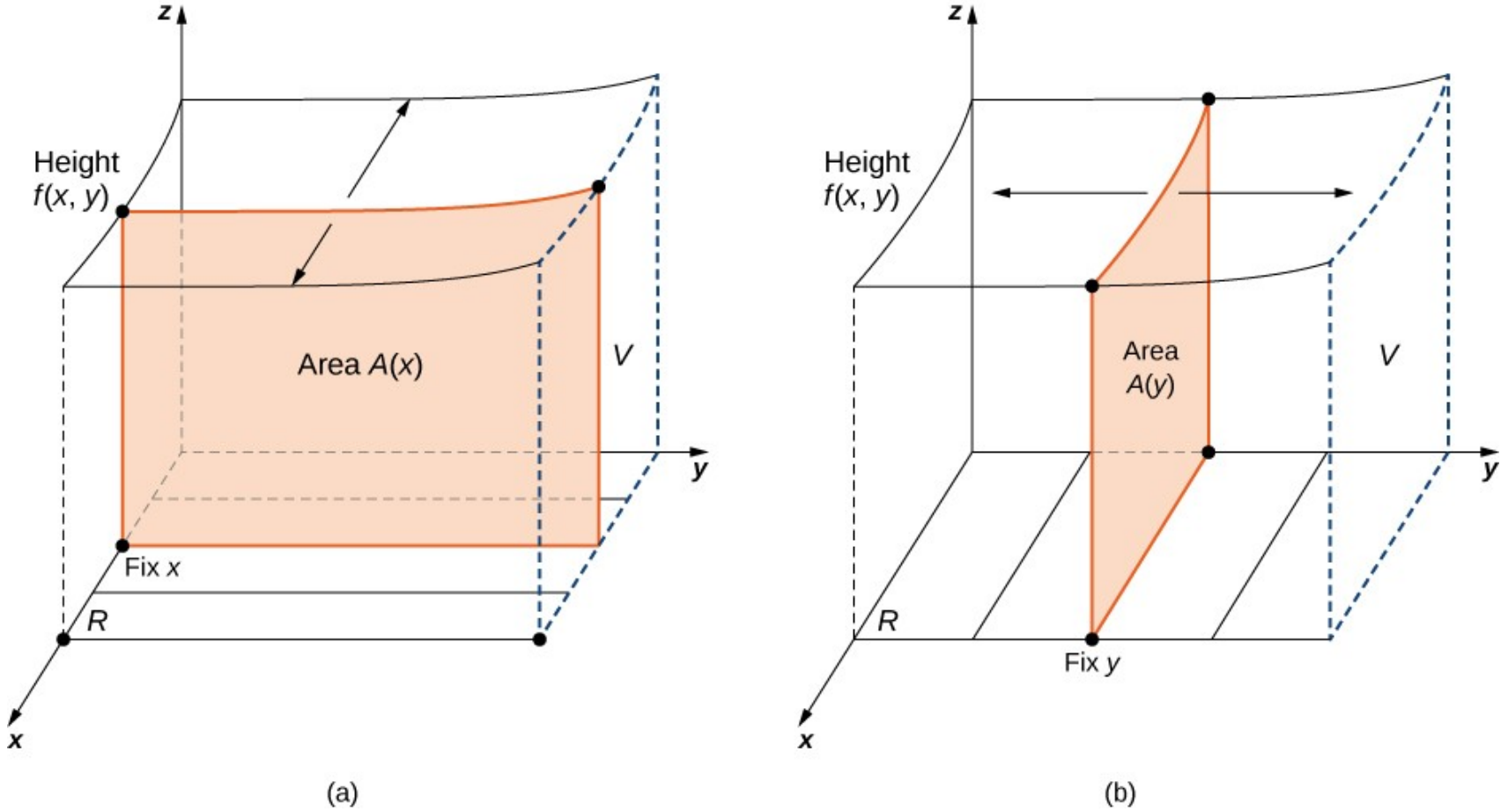
Riemann approach → Multiple representations





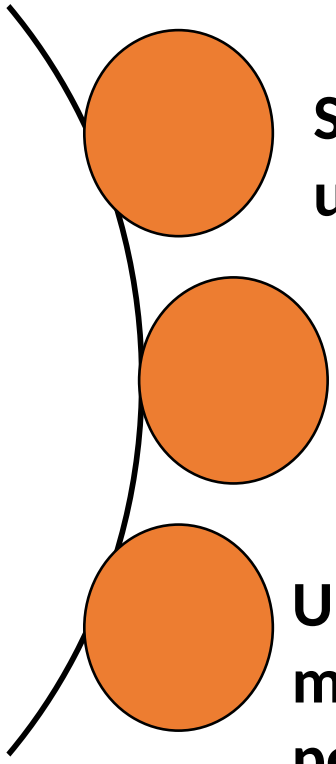
Students with a Riemann understanding were able to distinguish a volume calculation of a double integral and a triple integral

Other conceptions derived from a Riemann understanding



Horizontal slices made up from the region of integration in the -plane were also related to the Riemann approach

Conclusion



Students with an area under the curve conception have difficulties in understanding double integrals.

Students with a Riemann understanding were able to sketch images that represented both single and multivariable integrals.

Understanding this transition can help instructors design instructional materials that will help students understand double integrals and improve performance.

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References:

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3. https://math.libretexts.org/Bookshelves/Calculus/Map%3A_Calculus__Early_Transcendentals_%28Stewart%29/15%3A_Multiple_Integrals/15.01%3A_Double_Integrals_over_Rectangles