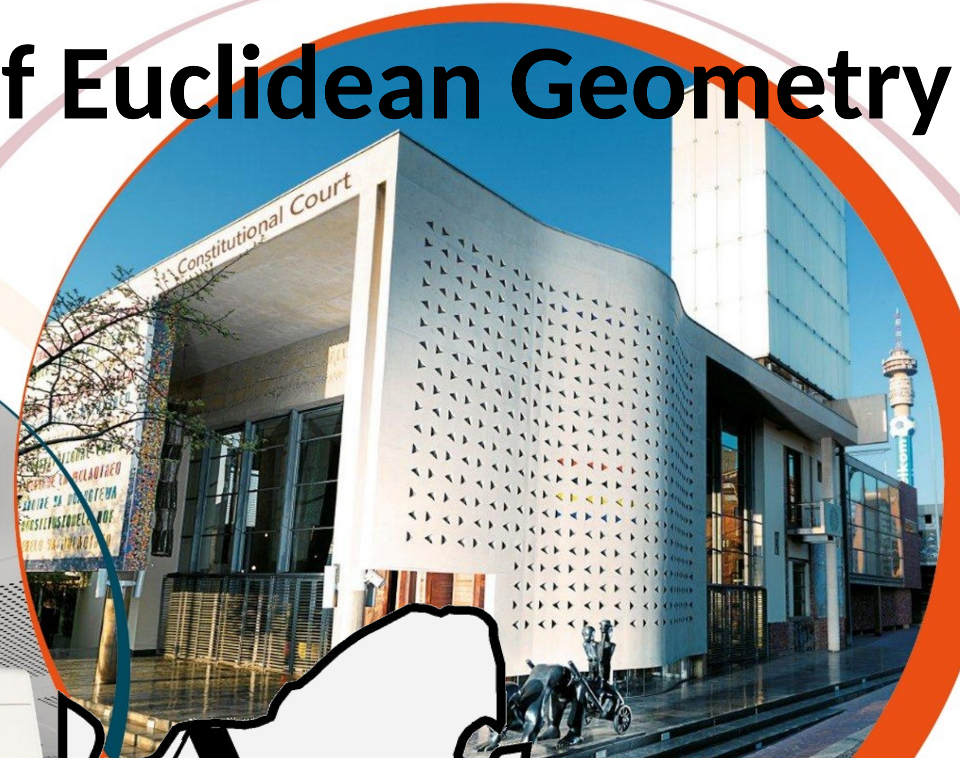
Pedagogical Practices in Integration of IKS in Teaching and Learning of Euclidean Geometry

South African Mathematical Society
SAMS



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Learn without limits.

UNISA college of law

Presentation Overview

- Brief Introduction & Background
- Cognitive understanding in mathematics
- Cognitive Artifacts
- Visualization
- **Examples of sound pedagogical practices**
- Active Learning
- Relevance of Geometry
- Geometry Teaching and Learning Methods
- Indigenous Knowledge System (IKS)
- Theoretical Underpinnings
- Pattern variation and Higher order Learning
- Methodology
- Examples of IKS integration
- Conclusion & Recommendations

- Pedagogy refers to the methodology and process of how instructors approach teaching and learning using a specific curriculum with specific goals in mind.
- Regardless of the approach and learning space, effective pedagogical practices must be designed with the learner in mind in order to maximize engagement and ultimately, impact mastery of mathematics understanding.
- The development of new knowledge needs to take place in more realistic situations to improve understanding.
- Everyday concepts are imbued with personal meaning but are tied to concrete experiences that resist systematicity.
- Thus a teaching process that builds connections between the everyday and scientific concepts is required when teaching maths.

Examples of sound pedagogical practices

Active Learning	A learner-centered approach in which Teachers facilitate, students and learners interact, engage and reflect.
Authentic Assessment	Measurement of student learning through real-world tasks or meaningful application of knowledge or skills. These could be projects that apply (i) Higher-order thinking and application of knowledge (Analyze, Evaluate, and Create), (ii) Emulation of real-world situations in a particular field or industry, and (iii) Opportunities to justify, challenge, and revise.
Experiential and Case-Based Learning	Teaching strategies that use real-life examples to offer a shared learning experience, solve problems, make decisions, and generally think critically together
Flipping the classroom	is an instructional strategy that leverages a blended learning model to achieve what the name implies
Mind mapping	a strategy that helps students visualize and analyze difficult concepts. Starting with a prompt or topic, students identify supporting themes or subtopics and illustrate connections using Cognitive artefacts or media.

Active learning

Active Learning

- Reflection
- Analysis
- Real world application
- Small group discussions
- Student videos and presentations
- Demonstrations

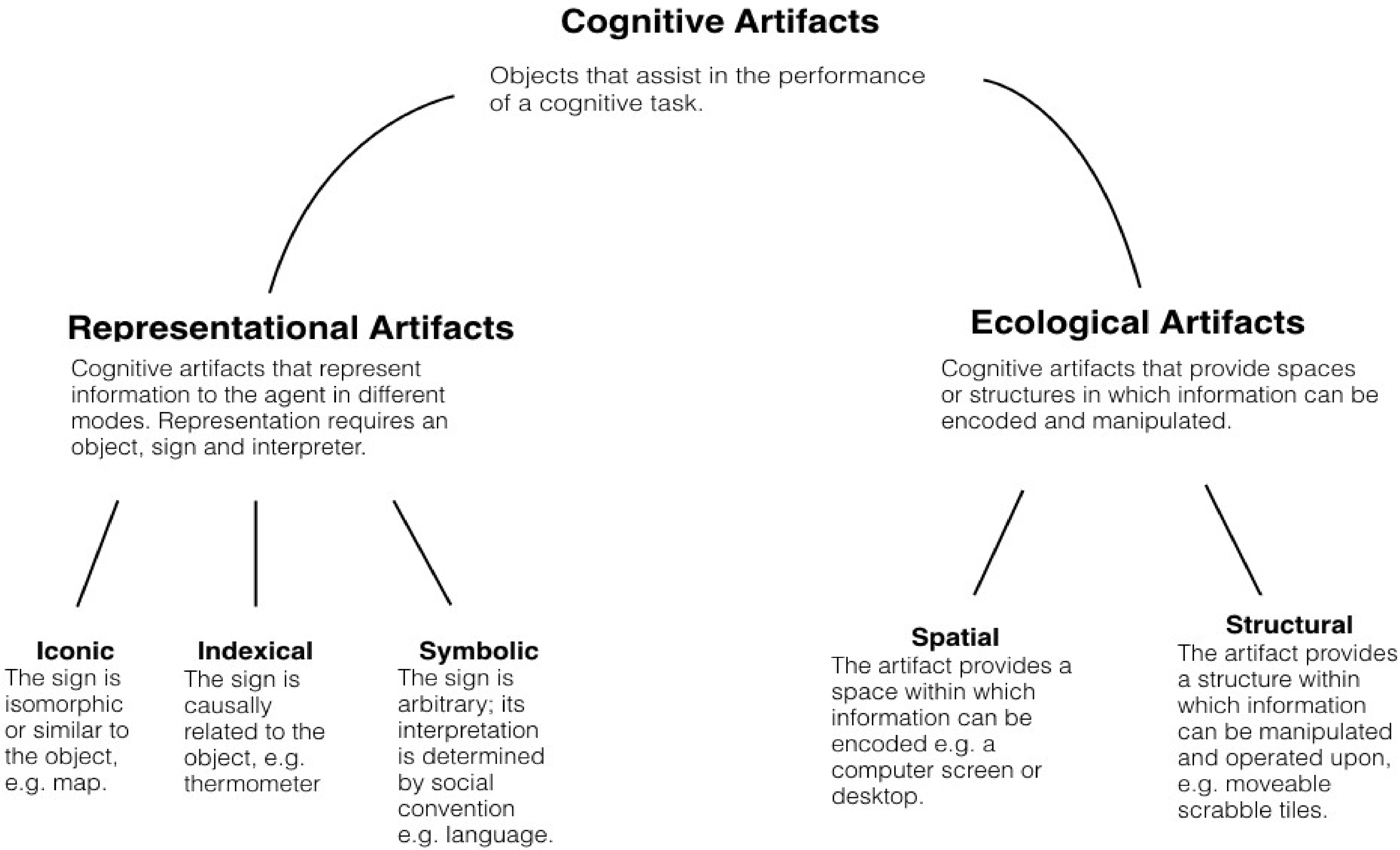
Virtual Flipped Classroom like discussion, debate focused, group based, conventional, micro, and in-class

Authentic Assessment

- require creativity, ingenuity, and resourcefulness
- similar to tests that show up in real life
- complex enough that learners can apply a variety of skills and information-seeking methods in order to reach viable solutions.
- action-oriented and re-iterative

Experiential and case based learning

- Dice Probability
- Growing a garden
- Jewelry making and beadwork-creation of patterns
- Exponent card games



From Richard Heersmink 'A Taxonomy of Cognitive Artifacts' (2013) *Review of Philosophical Psychology* DOI 10.1007/S13164-013-0148-1

Subtypes of mathematics difficulties

Semantic Memory

what enables us to remember words, meanings and concepts, as opposed to specific events.

Procedural Memory

- relates to the use of strategies and procedures to solve math problems, including identification of patterns, discrimination of similarities and differences between objects,

Visual Spatial Memory

Reversing or putting numbers out of sequence and misunderstanding spatially represented numbers (e.g., exponents). Interpreting charts and graphs and **developing an understanding of depicted geometric figures**

Relevance of geometry

- It develops critical thinking and problem solving, and geometrical shapes are part of our lives, appear almost everywhere, and are utilized in science and art,
- It is used to understand other concepts like fraction and decimal numbers; rectangles, squares, areas and circles as techniques of the operations.
- architects and engineers use geometric shapes and characteristics in explanations of physics and chemistry phenomena
- geometry needs a strong pedagogical approach besides deep knowledge to be able to provide an enjoyable and intellectual atmosphere for students

Geometry Teaching and Learning Methods

- Teaching the conceptual and graphical categories of Geometry require different approaches.
- Duval (1998) suggests that the conceptual parts must be transformed into perception by visualization by the graphical parts using (a) immediate perpetual approach - refers to the interpretation of the diagrams; (b) Operative approach -used to determine the sub-configurations for problem solving; and © discursive approach- refers to the description of the problems given.
- Fischbein (1993) refers to geometry in the form of figural and conceptual parts and cautions the teachers to focus on both and describe the geometrical objects and their relations to each other since the figural part refers to abstract objects.
- The challenge is students are expected to read any diagram or graphic only by looking and transform the data given by the diagram in their minds;

If you are on the ridge to the left, can you jump to the one on the right?

- Netz (2003) defines a diagram, as a figure marked out by lines curves or circles, a visual symbol, usually two-dimensional, commonly used in geometry.

- Hohol & Milkowski (2019) conclude that the immense role of cognitive artifacts in the mathematical practice, from lettered diagrams, technical languages, and notations to computational devices, suggests that the epistemology of mathematics, which is the pinnacle of human intellectual activity, must also account for social and cultural factors.

- It is against that background that this presentation focuses on Pedagogical Practices in Integration of IKS in Teaching and Learning Geometry.

Indigenous Knowledge System (IKS)

The Curriculum and Assessment Policy Statement (CAPS) for mathematics for the different phases identifies a number of Principles which are intended to guide and direct interaction and learning in mathematics classrooms. One of these Principles is:

'Valuing Indigenous Knowledge Systems [through] acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution' (DBE, 2011: 5).

Indigenous knowledge is defined as knowledge which is spatially and/or culturally context specific, collective, holistic, and adaptive. An indigenous perspective of mathematics, that mathematics is intrinsically connected to culture and has many different cultural expressions

The art or technique of understanding, explaining, learning about, coping with, and managing the natural, social, and political environment through processes like counting, measuring, sorting, ordering and inferring - processes that result from well-identified cultural groups.

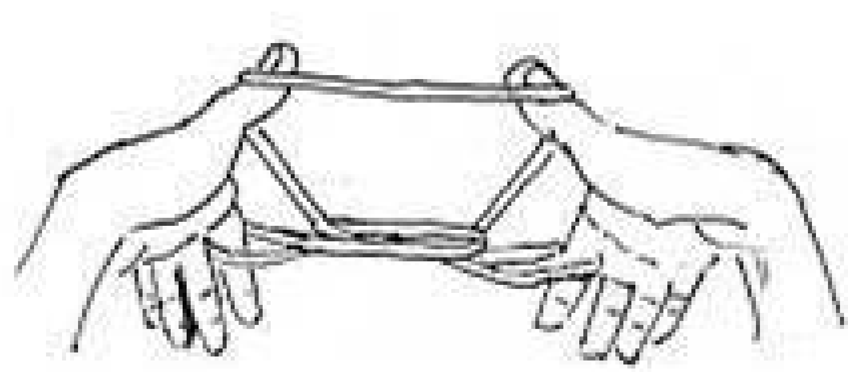
D'Ambrosio, 1989

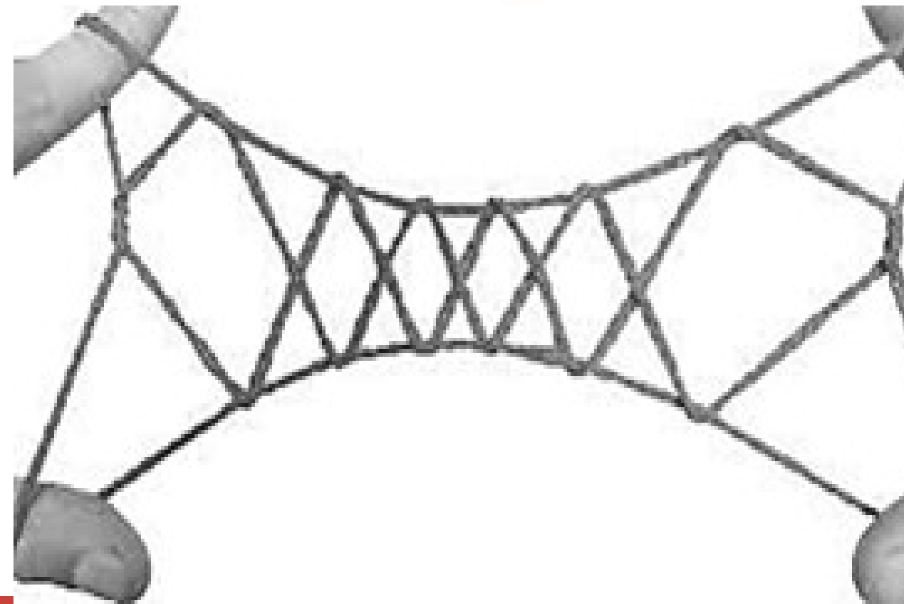
Enthonomathematics refers to the six fundamental activities that are characteristic of every culture. The six activities are counting, locating, measuring, designing, playing and explaining.

Bishop, 1988

The field of research that tries to study mathematics (or mathematical ideas) in its (their) relationship to the whole of cultural and social life.

Gerdes, 1994





Van Hiele Levels of Geometric Thinking and Constructivist Based Teaching Practices

I think that teachers' content knowledge is related to the quality of teaching geometry positively together with the quality of their designing and implementing their lessons. Teachers having sufficient geometry content knowledge can instruct by using appropriate models, representations and materials effectively.

In addition, they are more likely to use real-life examples and make connection with other disciplines consistent with the subject, and teachers encourage students to ask questions and discuss the subject with them. They guide and encourage their students investigate and construct their knowledge by providing appropriate atmosphere and interactions.

van Hieles' levels of Geometric Thinking

Visualization: In the first level, students recognize geometric shapes on the basis of their complex visual perception, while the orientation of the shapes is dominant.

Analysis: Students already know the properties of geometric shapes, but they do not yet perceive the relationships between individual properties. They define geometric shapes by listing all of their properties, even those that are unnecessary.

Informal Deduction: Students are aware of the relationships between the properties of shapes, they know that the individual properties are arranged and interconnected. They formulate the correct abstract definitions and begin to use implication, deduction and abstraction regarding statements in their thinking.

Formal Deduction: Students are aware of the need for a logical system of geometry and the meanings of deduction, position and tasks of axioms, as well as sentences and definitions. They are aware of the need to prove claims and can provide simple evidence at the secondary school level.

Rigor: Students can compare axiomatic systems and describe the effect of adding or removing axioms in a given geometric system. They understand the formal aspects of deduction and they are able to use all types of proofs.

Phases of Learning

Students' geometry thinking level depends on neither their age or their maturity but rather on the lessons received.

There are five phases of learning (Usiskin, 1982):

Phase 1: Information: a two-way teacher-student interaction is essential in understanding certain geometrical shapes such as making observations, asking questions and understanding the vocabulary for that particular geometrical shape.

Phase 2: Orientation: Students explore the topic about geometry as arranged by the teacher. The activities involved should enable students to identify the geometrical shape that is to be learned.

Phase 3: Explanation: Based on previous experiences, students are to express their opinions and discuss about the geometrical shapes that had been observed.

Phase 4: Free Orientation: Students are able to solve more complicated problems such as open-ended problems. Much of the relationships among objects are clarified through the interaction among students when making investigations.

Phase 5: Integration: Students survey and summarize what they have learned by making connections among the geometrical shapes. The teachers assist students in making a synthesis on each of the geometrical shapes.

Variation Theory explained as	**Researcher**
Learning through variation theory is sees as an expansion in awareness, in which students become aware of critical aspects of a disciplinary concept, skill, or practice that they had not previously noticed	Akerlind (2015)
Variation theory is simply a more coherent, explicit, and systematic framework for making use of variation and invariance	Marton and Pang (2016)
Variation theory-based mathematics teaching emphasizes variation as necessary condition for learners to be able to discern new aspects of an object of learning	Kullberg, Kempe & Marto (2017).
	Askew (2012)
Variation is the key to being able to discern	Askew

Patterns of variation Figure 1.

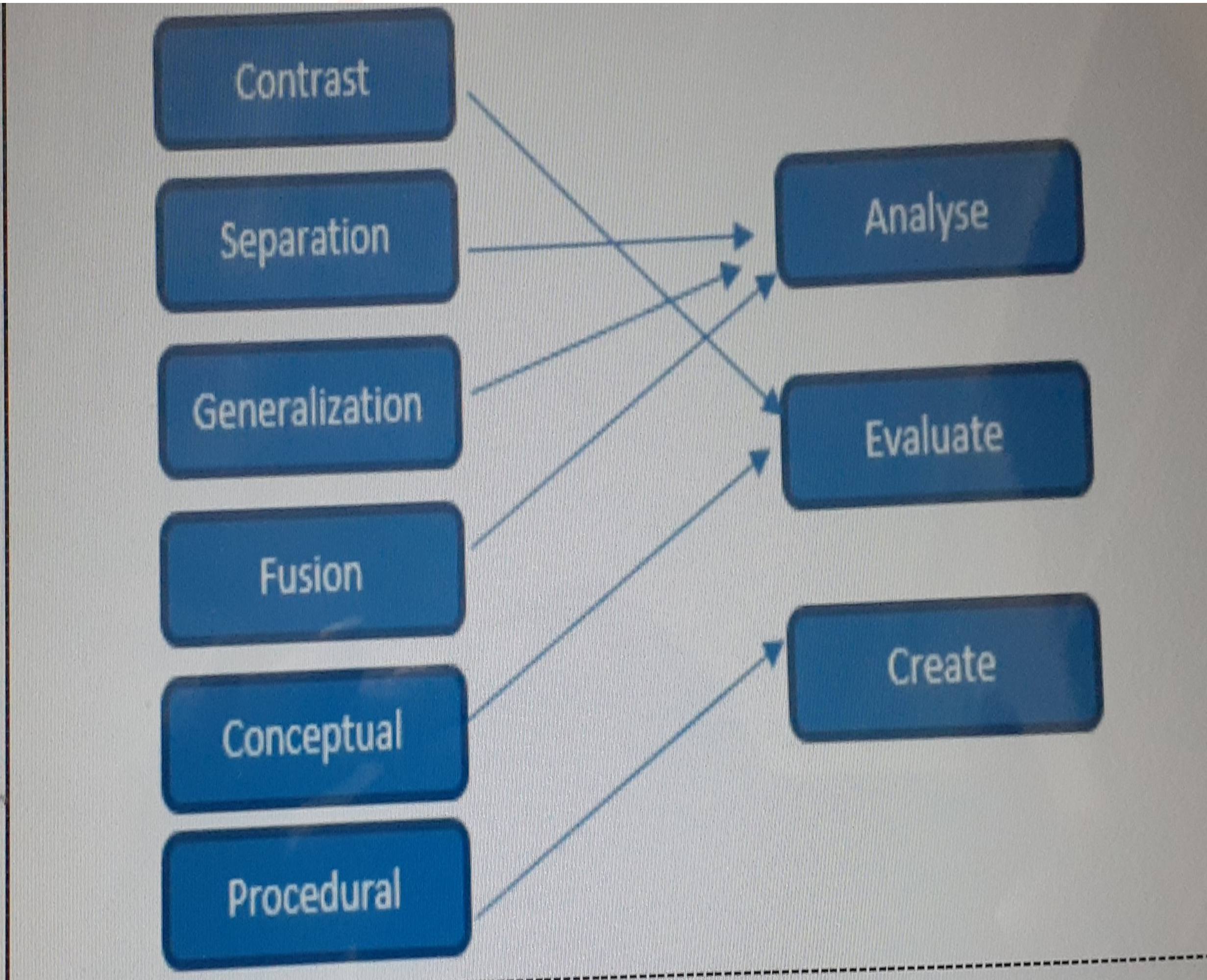
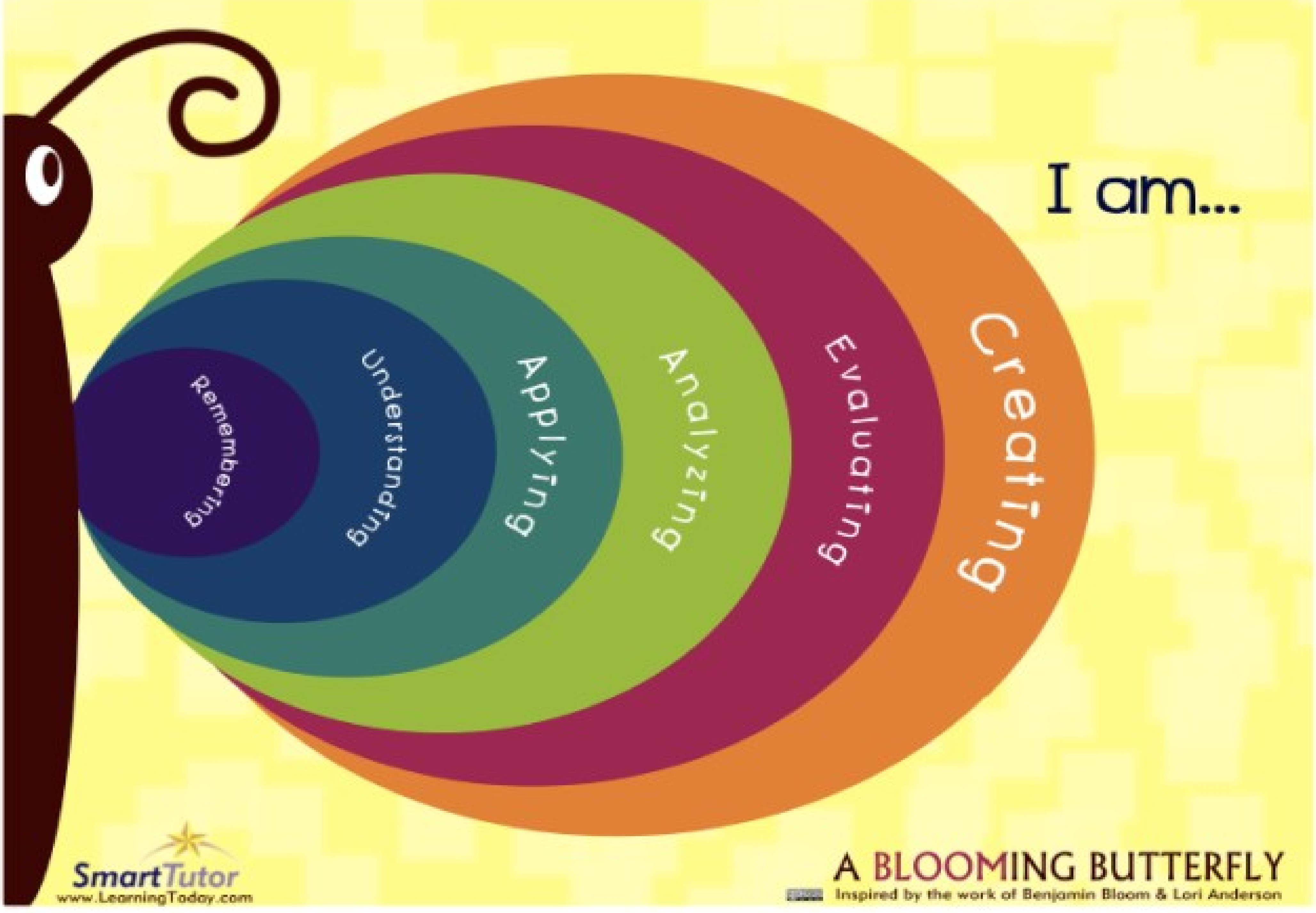


Figure 1. The connection between variation patterns and HOTS.

- Contrast. Comparing two concepts involves particular pattern of variation called contras In order to experience something, a person must experience something else to compare it with.
- Separation. In order to experience a certain aspect of something, and in order to separa this aspect from other aspects, it must vary while other aspects remain invariant.
- Generalization is all about experience varyin appearances. Using generalization, the teacher tried to visualize that the same principle is applied to different representations
- Fusion. If there are several critical aspects that the learner has to take into consideration at the same time,


I am...
Remembering
Understanding
Applying
Analyzing
Evaluating
Creating
SmartTutor
www.LearningToday.com
A BLOOMING BUTTERFLY
Inspired by the work of Benjamin Bloom & Lori Anderson

Research Design

Research on geometric thinking consists of two parts: determining the levels of geometric thinking and analyzing errors in solving geometric tasks. From an interpretivist paradigm, I considered a multiple case study in which qualitative approaches were used

Research Question:

What are the pedagogical practices in integration of IKS in the teaching and learning of mathematics?

Task 1

TASK A

Write the number of individual shapes in the picture (Figure a):

a. squares.............

b. rectangles............

c. rhombuses................

d. triangles..............

e. circles.................

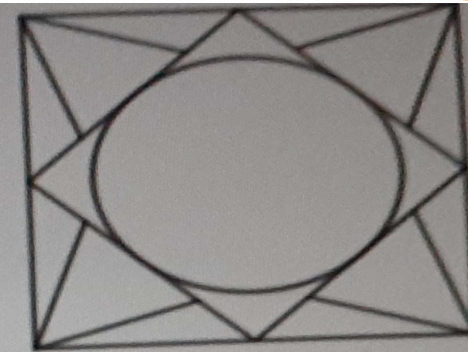


Figure a

TASK B

Circle all true statements connected with the picture (Figure b):

a. The area of quadrilateral EFGH is a half of the area ABCD quadrilateral.

b. One side of quadrilateral ABCD is double of quadrilateral EFGH side length.

c. The length of diagonal of quadrilateral EFGH is the same as a length of diagonal ABCD.

d. The radius of circle inscribed into quadrilateral EFGH is the same as a half of its side length.

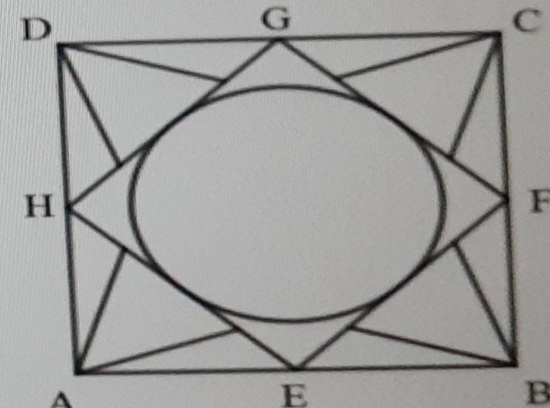


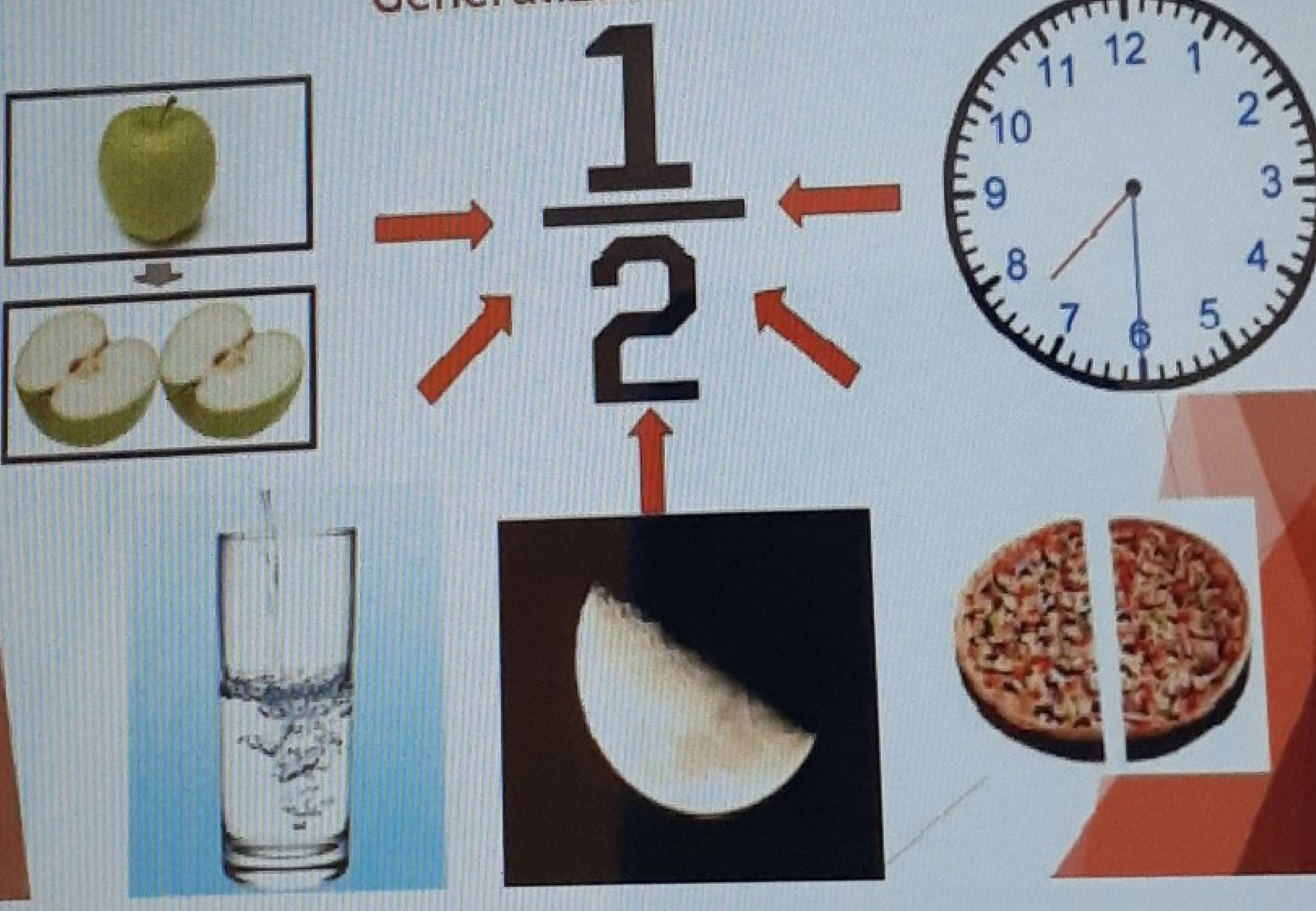
Figure b

Geometric representations in nature

- If one looks closely, one might find different geometrical shapes and patterns in leaves, flowers, stems, roots, bark, and the list goes on.
- The organisation of the human digestive system as a tube within a tube also ascertains the role of geometry.
- The leaves on the trees are of varying shapes, sizes, and symmetries.
- Different fruits and vegetables have different geometrical shapes; take the example of orange, it is a sphere and after peeling it, one might notice how the individual slices form the perfect sphere.

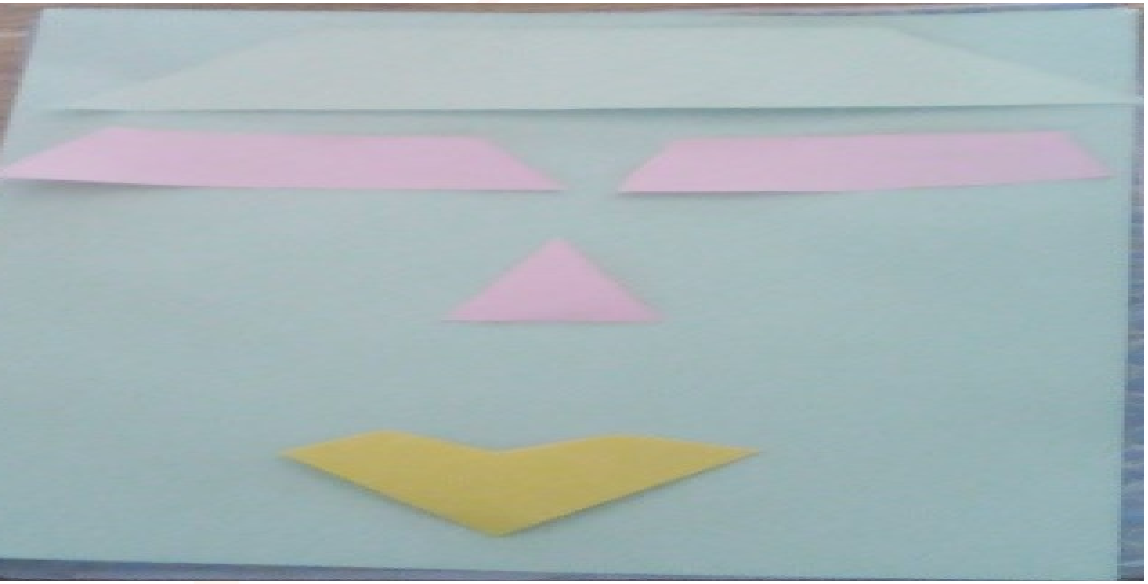
Drawings on Ndebele Huts

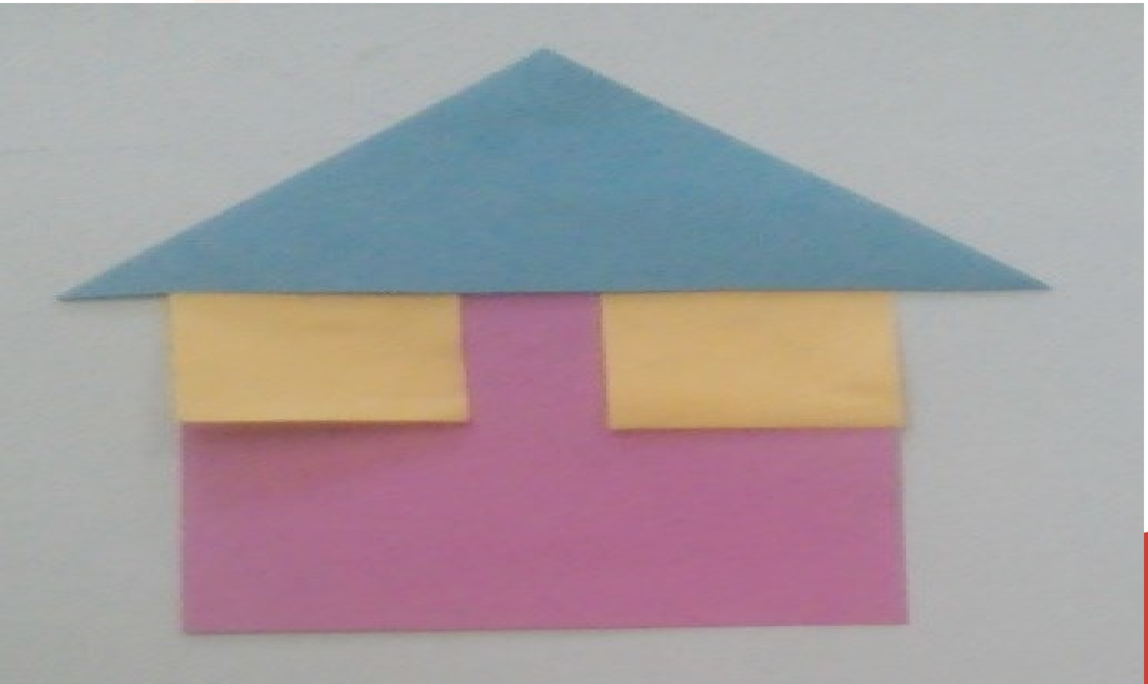
- Basic shapes in the diagram
- Symmetry
- Congruency of figures
- Similarity
- Parallelism
- Proportionality
- Symbolism in nature



1
2
12 1 2 3 4 5 6 7 8 9 10 11

Grade 4 learners’ work on sorting different shapes

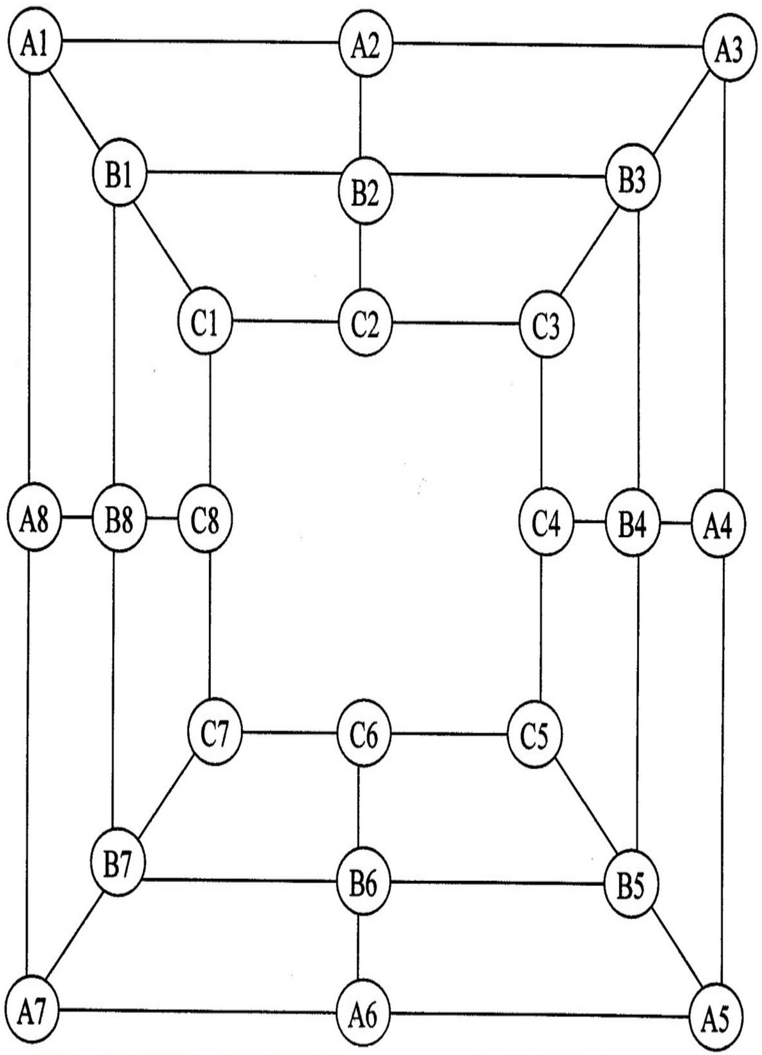




The learner described his illustration as a person’s face, with forehead, two eyes, a nose and a mouth. He justified that he thought he must illustrate something different from other learners as he noticed that many children were representing huts in their sorting.

The freedom given to the learners to sort the shapes in their own ways and describe how they see things without intimidation of being right or wrong, indicates to them that their views are important. This afforded them a chance to share ways in which geometric concepts can be understood.

Identification of indigenous games according to the potential of their use in the curriculum



select a particular game or numin Morabaraba

Identification of various quadrilaterals (squares) and the similarities and

Ratio and proportion between the lines and the squares making the complete

- Symmetry:

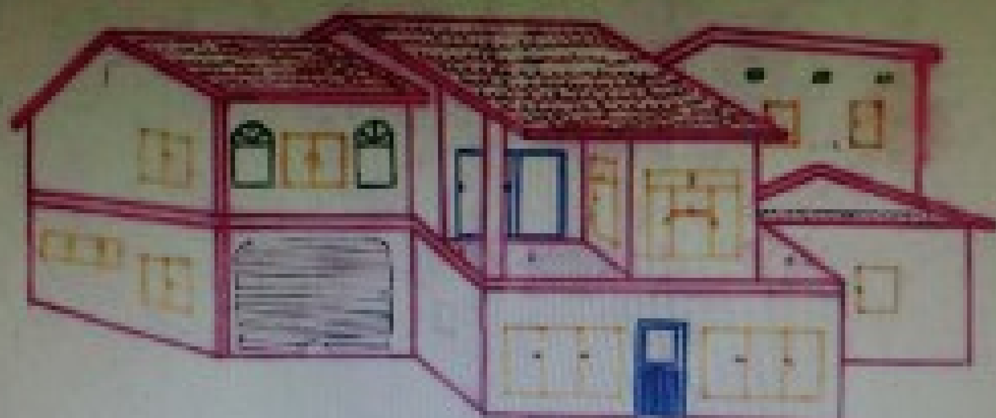
- Logical deductions in the execution of the various steps of the game;

- Counting of the tokens;

- Addition and subtraction of the tokens until a game is won on the basis

of the remaining number of tokens.

Figure 4.10. One of the houses drawn by the learners.



62

Problem

A goat is tied to a corner of a square field of side 21 m with a rope of length 14 m. What is the area of the square field that the goat cannot graze?

Building a rondavel

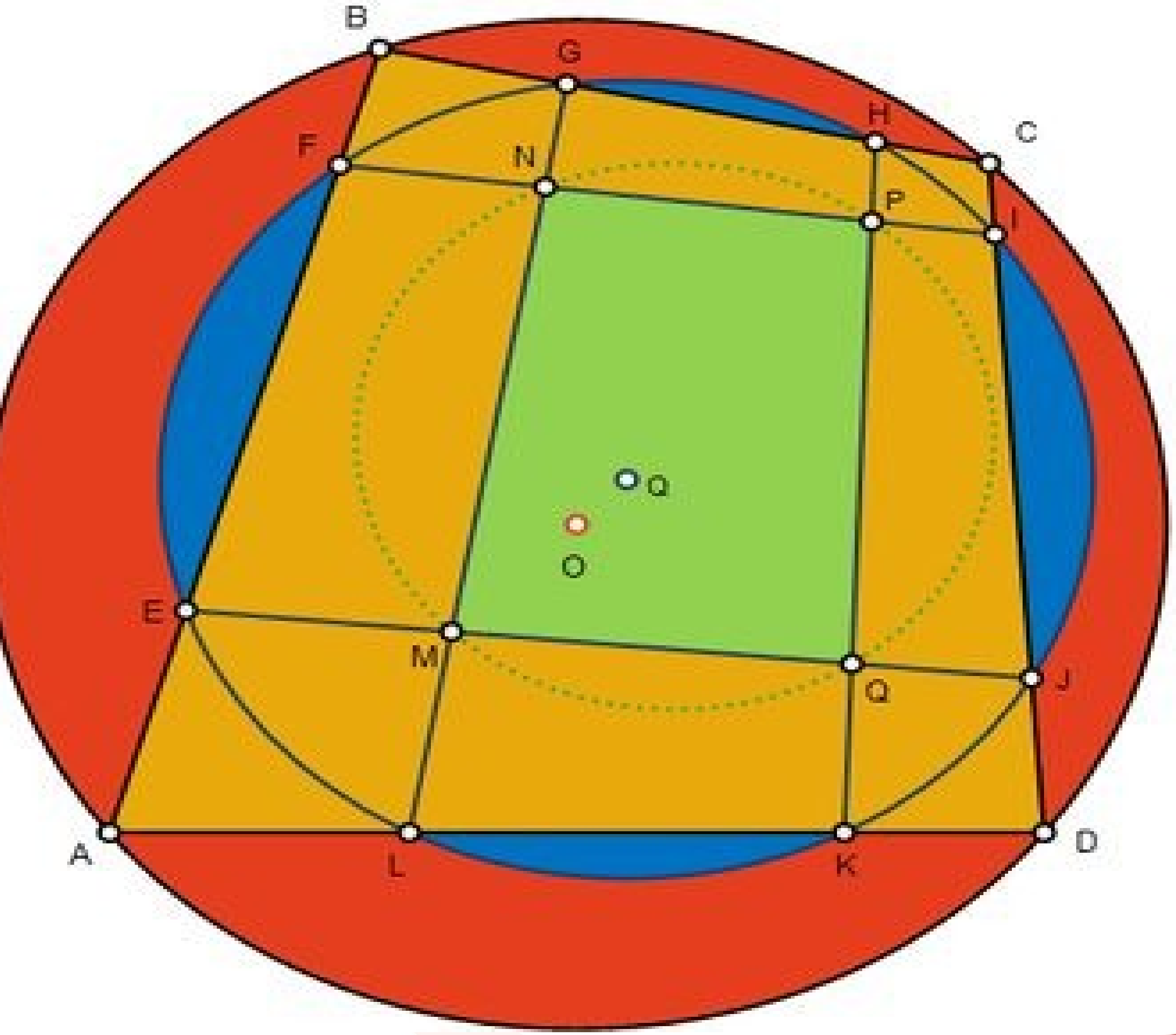
- **The rondavel is usually round in shape and is traditionally made with materials that can be locally found in raw form**
- **Its building starts with the identification of a centre of the complete structure of the hut**

Making rectangular prism bricks from mud

MUD BRICKS IN MAKING

A complete structure of a hut

Two Cyclic Quadrilateral, Cyclic Octagon, Circle



Given:

- Cyclic quadrilateral ABCD
- Circle Q meets AB, BC, CD, AD at E, F, G, H, I, J, K, L
- EJ intersects to LG and KH at M and Q
- FI intersects to LG and KH at N and P

Prove:

- MNPQ is a cyclic quadrilateral

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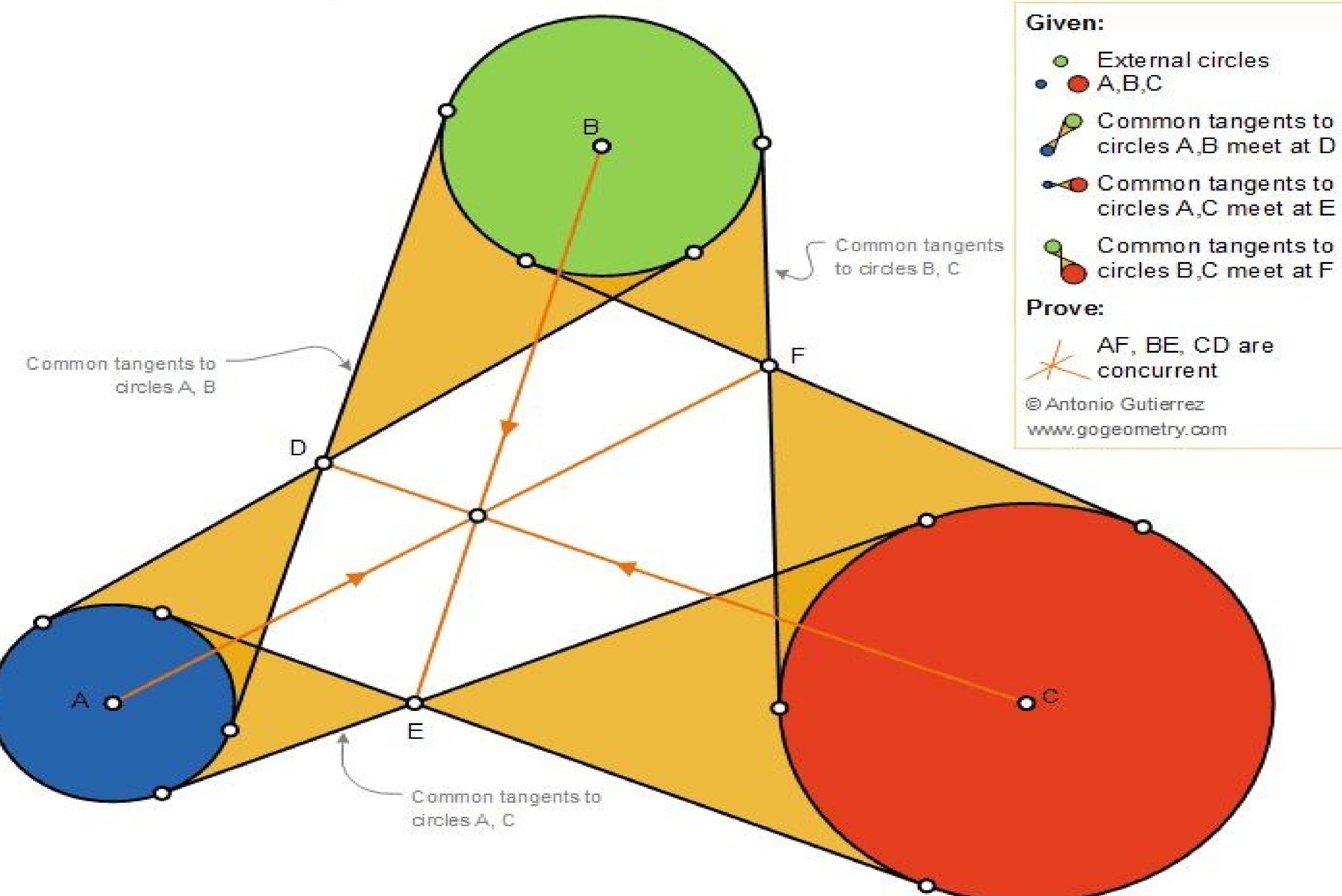
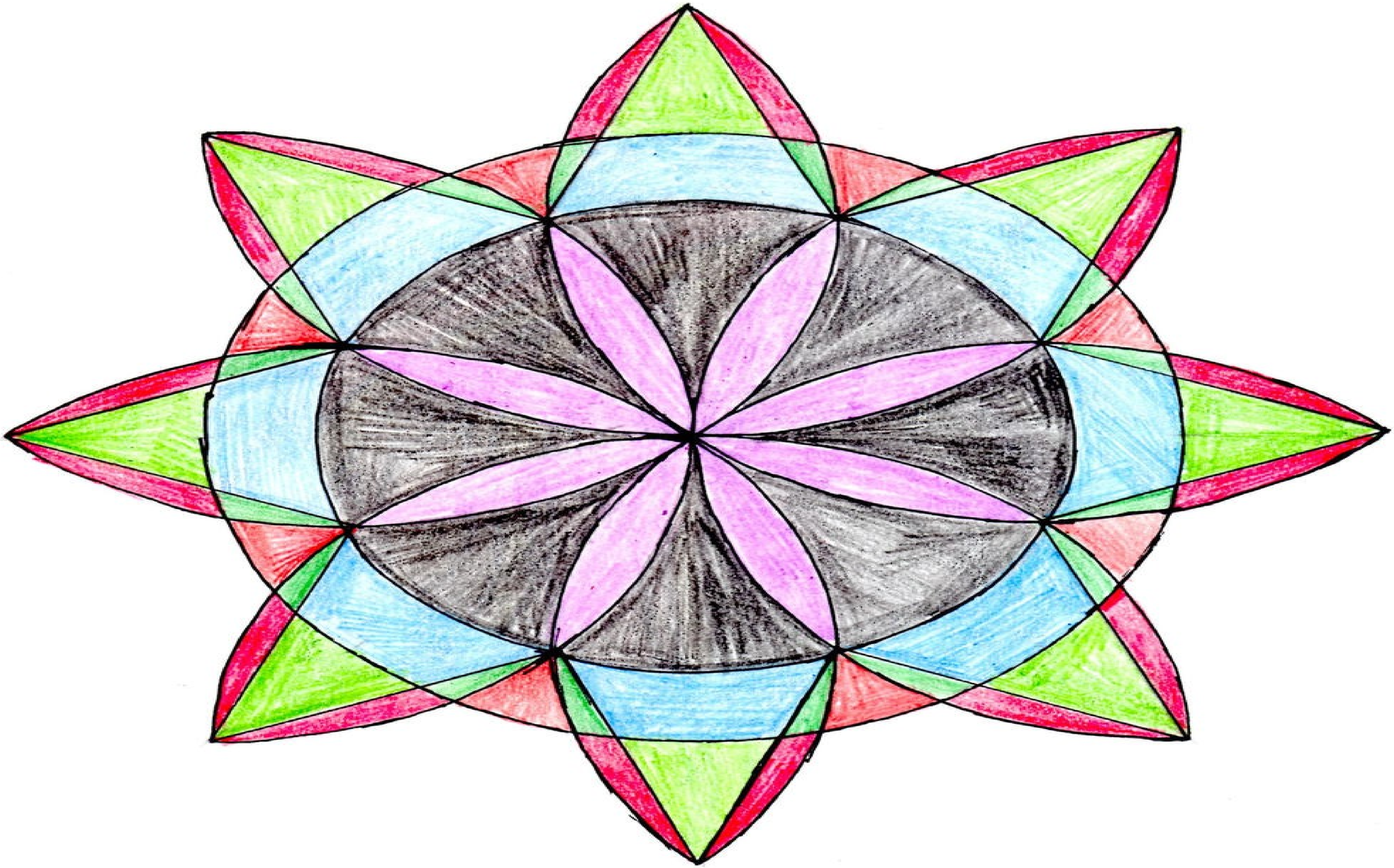
www.gogeometry.com

Three Circles and Three Pair of Common Internal Tangents, Concurrency, Centers





Investigative techniques- prob;em solving can be viewed as a situated and contextually determined activity

Representational techniques

Brainstorming techniques

Idea-generating techniques

Screening techniques

Decision-making techniques

Alignment techniques

Implementation techniques

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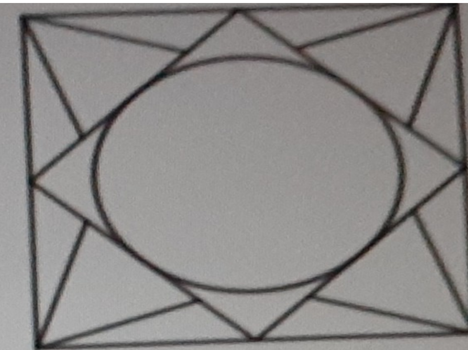

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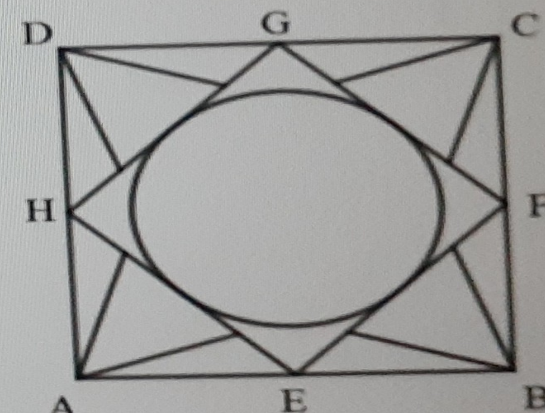

Figure b

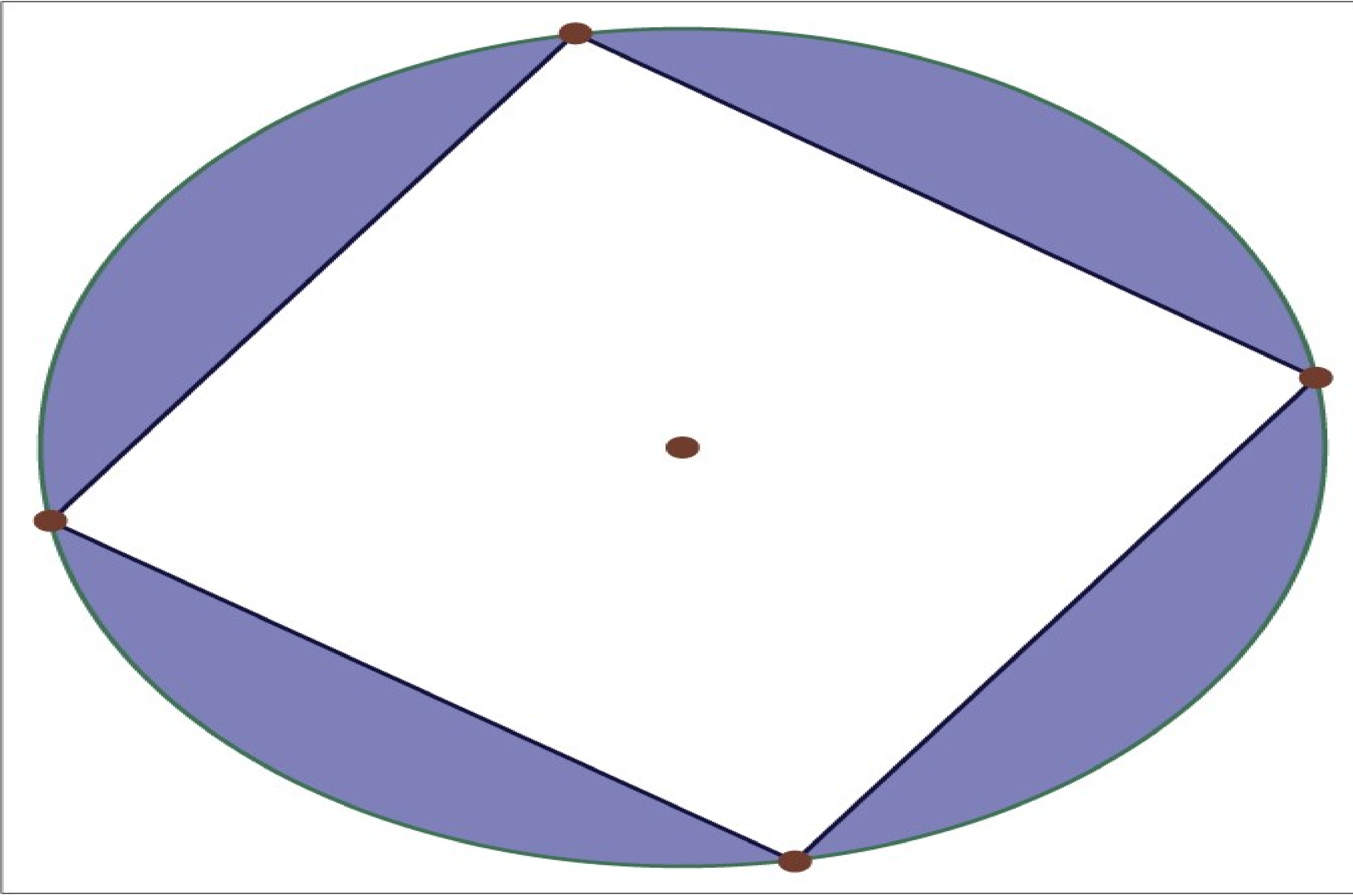

FIGURE 6: Areas of circles and squares.

Findings

Teachers used real world problems as opportunities for reflective thinking.

Teachers discerned that the value of using workplace and everyday tasks for teaching mathematics lies in its potential to shield learners from falling into the trap of focusing on the procedures at the expense of concepts.

Honor diversity and respect cultural heritages thus promoting the belief that all people are capable of doing mathematics in their own unique and personal perspective.

This study encourages teachers to be mindful of the fact that children from diverse backgrounds have different modes of thinking, possess diverse perceptual abilities and spend differential efforts on tasks depending on personal criteria which they deem useful.

Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.

— William Paul Thurston, American mathematician