Double-diffusive convection in the anisotropic porous layer under rotational modulation with internal heat generation

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Outline

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- Numerical result

Why study this problem?

- The study of rotating double-diffusive convection in internally heated porous media has received significant attention. The study of this problem was prompted by its industrial and environmental applications including:

geothermal power usage and storage, food processing and atmospheric pollution, to name a few

- The results for the unmodulated case have been discussed by Altawallbeh et al. [1], Bhadauria et al. [2], Gaikwad et al. [3].

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- Find the onset criteria for stationary and oscillatory double-diffusive convections using linear and nonlinear stability analyses.
- Study heat and mass transfers across the porous layer.
- Using a LQBHM to solve the coupled nonlinear Lorenz type equations.

The model problem

We consider a rotating anisotropic porous layer, confined between infinitely extended two horizontal parallel planes with distance d.

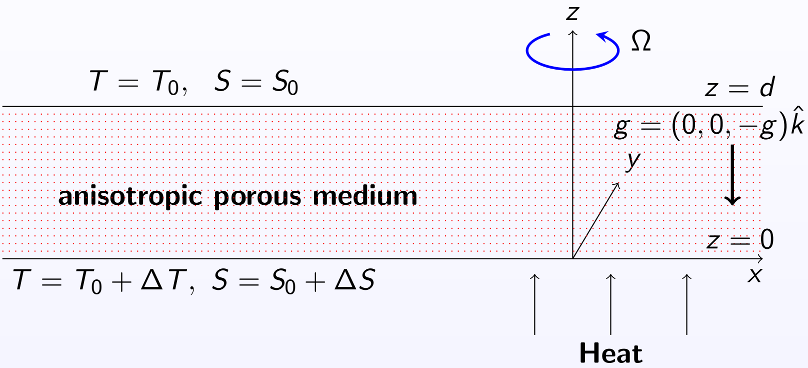

Figure: Geometry of the problem

The model problem

The Oberbeck-Boussinesq approximation takes into consideration the effect of density changes. Under these assumptions, the generalized Darcy model has been used for the momentum equation, [4, 2, 1]

Governing equation

$$\nabla \cdot q = 0, \tag{1}$$

$$\frac{\rho_0}{\gamma}\frac{\partial q}{\partial t} + 2\frac{\Omega}{\gamma} \times q = -\nabla P + \rho g - \frac{\mu}{\mathbf{K}} q, \tag{2}$$

$$\chi \frac{\partial T}{\partial t} + (q \cdot \nabla) T = \nabla \cdot (\kappa_T \cdot \nabla T) + Q(T - T_0), \tag{3}$$

$$\gamma \frac{\partial S}{\partial t} + (q \cdot \nabla) S = \kappa_S \nabla^2 S, \tag{4}$$

$$\rho = \rho_0[1 - \beta_1(T - T_0) + \beta_2(S - S_0)], \tag{5}$$

where q is the velocity, μ is the dynamic viscosity, Q is internal heat source, **K** denotes the permeability tensor, κ_T denotes the thermal diffusivity tensor, T is the temperature, κ_S is the concentration diffusivity, χ denotes heat capacity ratio, γ is the porosity, P is the pressure, $g = (0, 0, -g)\hat{k}$ is the gravitational acceleration, ρ is the density and ρ_0 is the reference density.

Together with the boundary conditions

$$\begin{aligned} T &= T_0 + \Delta T, \text{ at } z = 0, \text{ and } T = T_0, \text{ at } z = d, \\ S &= S_0 + \Delta S, \text{ at } z = 0, \text{ and } S = S_0, \text{ at } z = d. \end{aligned} \tag{6}$$

The time-varying rotational modulation term is given as

$$\Omega = \Omega_0[1 + \epsilon^2 \delta_1 \cos(\omega t)]\hat{k}, \tag{7}$$

The fluid is assumed to be quiescent at basic state and the corresponding quantities are

$$q = (0,0,0),\ \rho = \rho_b(z),\ P = P_b(z),\ S = S_b(z),\ T = T_b(z), \tag{8}$$

Using (8) in Eqns. (1) - (5), gives

The basic state solutions

$$T_b(z) = T_0 + \Delta T \frac{\sin d\sqrt{Q/\kappa_{T_z}}(1-\frac{z}{d})}{\sin d\sqrt{Q/\kappa_{T_z}}}, \tag{9}$$

$$S_b(z) = S_0 + \Delta S \left(1 - \frac{z}{d}\right). \tag{10}$$

To investigate the behaviour of infinitesimal disturbances, we perturb the basic state as

$$q = q' = (u', v', w'), \; T = T_b + T', \; S = S_b + S', \; P = P_b + p', \; \rho = \rho_b + \rho'. \tag{11}$$

Substituting Eq. (11) into Eqs. (1)-(5) and using basic states (8), after eliminated the pressure term and introducing stream function, the perturbed equations in dimensionless, are obtained as

Perturbed equations

$$\left[\frac{1}{Va}\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)+\left(\frac{\partial^2}{\partial x^2}+\frac{1}{\varsigma}\frac{\partial^2}{\partial z^2}\right)\right]\psi$$

$$+\sqrt{Ta}(1+\epsilon^2\delta_1\cos(\omega t))\frac{\partial \xi}{\partial z}=-Ra_T\frac{\partial T}{\partial x}+Ra_S\frac{\partial S}{\partial x}, \quad (12)$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}+\frac{1}{\varsigma}\right)\xi=\sqrt{Ta}(1+\epsilon^2\delta_1\cos(\omega t))\frac{\partial \psi}{\partial z}, \quad (13)$$

$$\left[\frac{\partial}{\partial t}-\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+R_i\right)\right]T=\frac{\partial \psi}{\partial x}h(z)+\frac{\partial(\psi,T)}{\partial(x,z)}, \quad (14)$$

$$\left[\frac{\partial}{\partial t}-\frac{1}{Le}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right]S=-\frac{\partial \psi}{\partial x}+\frac{\partial(\psi,S)}{\partial(x,z)}, \quad (15)$$

where

Parameters

where $Ta = \left(\dfrac{2\Omega_0 K_z}{\nu}\right)^2$ is the Taylor number, $Pr = \dfrac{\nu}{\kappa_{T_z}}$ is the Prandtl number, $Da = \dfrac{K_z}{d^2}$ is the Darcy number, $Va = \dfrac{Pr}{Da}$, is the Vadasz number, $\varsigma = \dfrac{K_x}{K_z}$ is the mechanical anisotropy parameter, $\eta = \dfrac{\kappa_{T_x}}{\kappa_{T_z}}$ is the thermal anisotropy parameter, $Ra_T = \dfrac{\beta_1 g K_z d \Delta T}{\nu \kappa_{T_z}}$ is the thermal Rayleigh number, $Ra_S = \dfrac{\beta_2 g K_z d \Delta S}{\nu \kappa_{T_z}}$ is the concentration Rayleigh number, $R_i = \dfrac{Qd^2}{\kappa_{T_z}}$ is the internal Rayleigh number, $\nu = \dfrac{\mu}{\rho_0}$ is the kinematic viscosity, $Le = \dfrac{\kappa_{T_z}}{\kappa_S}$ is the Lewis number,and $h(z) = \dfrac{dT_b(z)}{dz} = \dfrac{-\sqrt{R_i}\cos(\sqrt{R_i}(1-z))}{\sin\sqrt{R_i}}$.

Cont...

together with

The boundary conditions

$$\psi = T = S = \frac{\partial \xi}{\partial z} = \frac{\partial^2 \psi}{\partial z^2} = 0 \text{ at } z = 0, 1. \qquad (16)$$

Linear Stability analysis

Eqs. (12) - (15) are linearized by neglecting the nonlinear terms. We use the normal mode technique with

$$\Psi = \mathcal{A}_1 e^{\sigma t} sin(\pi \alpha x) sin(\pi z), \tag{17}$$

$$\xi = \mathcal{A}_2 e^{\sigma t} sin(\pi \alpha x) cos(\pi z), \tag{18}$$

$$T = \mathcal{A}_3 e^{\sigma t} cos(\pi \alpha x) sin(\pi z), \tag{19}$$

$$S = \mathcal{A}_4 e^{\sigma t} cos(\pi \alpha x) sin(\pi z), \tag{20}$$

where α is the wave number, $\sigma = \sigma_R + i\sigma_{im}$ represents complex growth rate, and $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4$ are constants.

Substituting Eqs. (17)-(20) in the linearized form of Equations (12)–(15) and eliminate the constants and solve it for Ra_T we obtain

$$Ra_T = \frac{(4\pi^2 - R_i)(\sigma + \lambda_2 - R_i)}{4\pi^4\alpha^2 Va}\left\{m^2\sigma + Va\lambda_1 + \frac{Va^2 Ta\varsigma\pi^2}{\varsigma\sigma + Va} + \frac{\pi^2\alpha^2 VaLeRa_S}{Le\sigma + m^2}\right\}, \tag{21}$$

where $m^2 = \pi^2(\alpha^2 + 1)$, $\lambda_1 = \pi^2(\alpha^2 + \varsigma^{-1})$, $\lambda_2 = \pi^2(\eta\alpha^2 + 1)$.

To have a clear analysis of the onset of instability in the fluid system, the real part of σ is set to zero and $\sigma = i\sigma_{im}$ in Eq. (21). Getting rid of the complex terms in the denominator, Eq. (21) yields

$$Ra_T = \Delta_1 + i\sigma_{im}\Delta_2, \tag{22}$$

where

$$\Delta_1 = \frac{4\pi^2 - R_i}{4\pi^4\alpha^2 Va}\Bigg\{ Va\lambda_1(\lambda_2 - R_i) + \frac{\pi^2\varsigma Va^2 Ta(Va(\lambda_2 - R_i) + \varsigma\sigma_{im}^2)}{Va^2 + \varsigma^2\sigma_{im}^2} - m^2\sigma_{im}^2 + \frac{\pi^2\alpha^2 LeVa(m^2(\lambda_2 - R_i) + Le\sigma_{im}^2)Ra_S}{m^4 + Le^2\sigma_{im}^2}\Bigg\}, \tag{23}$$

$$\Delta_2 = \frac{4\pi^2 - R_i}{4\pi^4\alpha^2 Va}\left\{ Va\lambda_1 + m^2(\lambda_2 - R_i) + \frac{\pi^2\varsigma Va^2 Ta(Va - \varsigma(\lambda_2 - R_i))}{Va^2 + \varsigma^2\sigma_{im}^2} \right.$$
$$\left. + \frac{\pi^2\alpha^2 LeVa(m^2 - Le(\lambda_2 - R_i))Ra_S}{m^4 + Le^2\sigma_{im}^2} \right\}. \quad (24)$$

Substituting $\sigma_{im} = 0$ into Eq. (22), we get the stationary thermal Rayleigh number $Ra_T = Ra_T^{st}$ as

$$Ra_T^{st} = \frac{(\lambda_2 - R_i)(4\pi^2 - R_i)\left\{m^2(\lambda_1 + \varsigma\pi^2 Ta) + \pi^2\alpha^2 LeRa_s\right\}}{4\pi^4\alpha^2 m^2}. \tag{25}$$

Oscillatory convection ($\sigma_{im} \neq 0$, $\Delta_2 = 0$)

From Eq. (22), when $\Delta_2 = 0$, we observe that oscillatory convection may occur when $Ra_T = Ra_T^{os}$ where

Oscillatory Rayleigh number

$$Ra_T^{os} = \frac{4\pi^2 - R_i}{4\pi^4\alpha^2 Va}\Bigg\{ Va\lambda_1(\lambda_2 - R_i) - m^2\sigma_{im}^2 + \frac{\pi^2\varsigma Va^2 Ta(Va(\lambda_2 - R_i) + \varsigma\sigma_{im}^2)}{Va^2 + \varsigma^2\sigma_{im}^2} + \frac{\pi^2\alpha^2 LeVa(m^2(\lambda_2 - R_i) + Le\sigma_{im}^2)Ra_S}{m^4 + Le^2\sigma_{im}^2}\Bigg\}. \qquad (26)$$

Weakly nonlinear stability analysis

The main focus of the study is on investigating the weakly nonlinear stability with rotation modulation. The nonlinear analysis provides information on the heat and mass transfer rates which the linear stability analysis cannot provide.

The nonlinear stability analysis is carried out using the system of equations (12) - (15) which satisfy boundary conditions (16) by means of a truncated minimal Fourier series. The stream function, vorticity, temperature and concentration distributions are represented as

$$\psi = A_1(t)\sin(\alpha x)\sin(\pi z), \tag{27}$$

$$\xi = B_1(t)\sin(\alpha x)\cos(\pi z) + B_2(t)\sin(2\alpha x), \tag{28}$$

$$T = C_1(t)\cos(\alpha x)\sin(\pi z) + C_2(t)\sin(2\pi z), \tag{29}$$

$$S = D_1(t)\cos(\alpha x)\sin(\pi z) + D_2(t)\sin(2\pi z), \tag{30}$$

The generalized Lorenz model is obtained by applying the truncated Fourier series to non-dimensionalised Eqs. (12) - (15) and define new variables as

$$X_1 = \frac{\alpha\pi}{\delta^2}A_1(t),\ X_2 = \frac{\pi^2\alpha\sqrt{Ta}}{\delta^6}B_1(t),\ X_3 = \frac{\pi^2\alpha_c\sqrt{Ta}}{\delta^6}B(t)_2,$$
$$X_4 = -R\pi C_1(t),\ X_5 = -R\pi C_2(t),\ X_6 = -R\pi D_1(t),$$
$$X_7 = -R\pi D_2(t),\ R = \frac{\alpha_c^2 Ra_T}{\delta^6},\ \tau = t\delta^2.$$

This leads to the following coupled nonlinear Lorenz-type system of equations

Lorenz-type system

$$\frac{dX_1}{d\tau} = -\frac{Vaa_1^2}{\delta^2}X_1 - Va(1+\epsilon^2\delta_1 f)X_2 + VaX_4 - NVaX_6, \qquad (31)$$

$$\frac{dX_2}{d\tau} = -Ta^\star Va(1+\epsilon^2\delta_1 f)X_1 - \frac{Va}{\varsigma\delta^2}X_2, \qquad (32)$$

$$\frac{dX_3}{d\tau} = -\frac{Va}{\varsigma\delta^2}X_3, \qquad (33)$$

$$\frac{dX_4}{d\tau} = -2RHX_1 + \frac{1}{\delta^2}(R_i - a_2^2)X_4 - X_1X_5, \qquad (34)$$

$$\frac{dX_5}{d\tau} = \frac{1}{\delta^2}(R_i - 4\pi^4)X_5 + \frac{X_1X_4}{2}, \qquad (35)$$

$$\frac{dX_6}{d\tau} = RX_1 - \frac{X_6}{Le} - X_1X_7, \qquad (36)$$

$$\frac{dX_7}{d\tau} = -\frac{4\pi^2}{\delta^2}\frac{X_7}{Le} + \frac{X_1X_6}{2}, \qquad (37)$$

where $\delta^2 = \alpha^2 + \pi^2,\ a_1^2 = \alpha^2 + \dfrac{\pi^2}{\varsigma},\ \delta^* = \dfrac{\delta_1}{\epsilon^2},\ a_2^2 = \eta\alpha^2 + \ \pi^2,$ and $H = \int_0^1 h(z)\sin^2(\pi z)dz$. The buoyancy ratio is denoted by $N = \dfrac{Ra_S}{Ra_T}$ and $Ta^{\star} = \dfrac{\pi^2 Ta}{\delta^6}$ is the revised Taylor number.

The above nonlinear system of autonomous differential equations is not suitable to be solved analytically, and thus it is to be solved using a numerical method. After determining the numerical values of the amplitude functions, the Nusselt number and Sherwood number can be obtained as functions of time

- **Heat and mass transfer**

Heat and mass movements are measured in terms of Nusselt number Nu and Sherwood number Sh respectively The rates of heat and mass transport per unit area are represented by H and J respectively, where

$$H = -\kappa_T \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}, \tag{38}$$

$$J = -\kappa_S \left\langle \frac{\partial S_{total}}{\partial z} \right\rangle_{z=0}, \tag{39}$$

where the angular brackets refer to the horizontal average at $z = 0$.

Substituting Eqs. (27)-(30) into (38)-(39), we get

$$Nu = \frac{H}{\kappa_T \Delta T} = 1 + \frac{2}{R} X_5, \tag{40}$$

$$Sh = \frac{J}{\kappa_S \Delta S} = 1 + \frac{2}{R} X_7. \tag{41}$$

To investigate the effect of various parameters on the Nusselt and Sherwood numbers, we solved the system of Eqs. (31)–(37) using the Local quasilinearisation block hybrid method LQBHM.

Introduction

This is a novel method for solving coupled nonlinear initial value problems. The development of block hybrid linear multistep method considers off-step points. The additional off-step points enhance the accuracy of the methods and ensure consistency, zero-stability, and convergence. The accuracy is increased by adding extra off-step points while keeping the grid size constant.

Method of solution

A system of non-linear first order differential equation is assumed to take the form

$$\begin{aligned}
\dot{X}_1 &= f_1(\tau, X_1, X_2, \ldots, X_M) = \mathcal{L}_1(\tau, X_2, X_3, \ldots, X_M)X_1 + \mathcal{N}_1(\tau, X_1, X_2, \ldots, X_M), \\
\dot{X}_2 &= f_2(\tau, X_1, X_2, \ldots, X_M) = \mathcal{L}_2(\tau, X_1, X_3 \ldots, X_M)X_2 + \mathcal{N}_2(\tau, X_1, X_2, \ldots, X_M), \\
&\vdots \\
\dot{X}_k &= f_k(\tau, X_1, X_2, \ldots, X_M) = \mathcal{L}_k(\tau, X_1, \ldots, X_{k-1}, X_M)X_k + \mathcal{N}_k(\tau, X_1, X_2, \ldots, X_M), \\
&\vdots \\
\dot{X}_M &= f_M(\tau, X_1, X_2, \ldots, X_M) = \mathcal{L}_M(t, X_1, \ldots, X_{M-1}, X_M)X_M + \mathcal{N}_M(\tau, X_1, X_2, \ldots, X_M),
\end{aligned} \tag{42}$$

where $\mathcal{L}_k(\tau)$ is the non-linear function component which is a coefficient to X_k in the k-th equation and $\mathcal{N}_k(\tau)$ is the remaining component which may or may not be a non-linear function for each $k = 1, 2, \ldots, M$.

We then consider the quasilinearisation method *QLM* iteration. The quasilinearisation technique is based on Taylor series expansion of the non-linear term $\mathcal{N}_k(\tau, X_k)$ based on the assumption that the difference between the current and previous iteration $(X_{k,r+1} - X_{k,r})$ is small. Thus,

$$\mathcal{N}_k(t, X_{k,r+1}) \approx \mathcal{N}(t, X_{k,r}) + \frac{\partial \mathcal{N}_k}{\partial y_k}(X_{k,r+1} - X_{k,r}).$$

A quasilinearisation scheme [5, 6, 7] which has good convergence rate is developed by applying sequential linearisation in X_k to obtain

$$\begin{aligned}\dot{X}_{k,r+1} = {} & \mathcal{L}_k(\tau, X_{1,r+1}, X_{2,r+1}, \ldots, X_{k-1,r+1}, X_{k+1,r}, \ldots, X_{M,r})X_{k,r+1} \\ & + \mathcal{N}_k(\tau, X_{1,r+1}, X_{2,r+1}, \ldots, X_{k-1,r+1}, X_{k,r}, \ldots, X_{M,r}) \\ & + \frac{\partial \mathcal{N}_k}{\partial X_k}(X_{k,r+1} - X_{k,r}). \end{aligned} \tag{43}$$

The LQBHM is now applied with

$$\varphi = \mathcal{L}_k + \frac{\partial \mathcal{N}_k}{\partial X_k}, \quad \upsilon = \mathcal{N}_k - X_{k,r}\frac{\partial \mathcal{N}_k}{\partial X_k}. \tag{44}$$

Eqs. (31) - (37) are now expressed as

$$\dot{X}_1 = \mathcal{L}_1(\tau, X_2, X_3, X_4, X_5, X_6, X_7)X_1 + \mathcal{N}_1(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{45}$$
$$\dot{X}_2 = \mathcal{L}_2(\tau, X_1, X_3, X_4, X_5, X_6, X_7)X_2 + \mathcal{N}_2(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{46}$$
$$\dot{X}_3 = \mathcal{L}_3(\tau, X_1, X_2, X_4, X_5, X_6, X_7)X_3 + \mathcal{N}_3(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{47}$$
$$\dot{X}_4 = \mathcal{L}_4(\tau, X_1, X_2, X_3, X_5, X_6, X_7)X_4 + \mathcal{N}_4(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{48}$$
$$\dot{X}_5 = \mathcal{L}_5(\tau, X_1, X_2, X_3, X_4, X_6, X_7)X_5 + \mathcal{N}_5(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{49}$$
$$\dot{X}_6 = \mathcal{L}_6(\tau, X_1, X_2, X_3, X_4, X_5, X_7)X_6 + \mathcal{N}_6(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{50}$$
$$\dot{X}_7 = \mathcal{L}_7(\tau, X_1, X_2, X_3, X_4, X_5, X_6)X_7 + \mathcal{N}_7(\tau, X_1, X_2, X_3, X_4, X_5, X_6, X_7), \tag{51}$$

with

$$\mathcal{L}_1 = -\frac{Vaa_1^2}{\delta^2},\ \mathcal{N}_1 = -Va(1+\epsilon^2\delta_1 f)X_2 + VaX_4 - NVaX_6,$$

$$\mathcal{L}_2 = -\frac{Va}{\varsigma\delta^2},\ \mathcal{N}_2 = -Ta^* Va(1+\epsilon^2\delta_1 f)X_1,$$

$$\mathcal{L}_3 = -\frac{Va}{\varsigma\delta^2},\ \mathcal{N}_3 = 0,$$

$$\mathcal{L}_4 = \frac{1}{\delta^2}(R_i - a_2^2),\ \mathcal{N}_4 = -2RHX_1 - X_1X_5,$$

$$\mathcal{L}_5 = \frac{1}{\delta^2}(R_i - 4\pi^4),\ \mathcal{N}_5 = \frac{X_1X_4}{2},$$

$$\mathcal{L}_6 = -\frac{1}{Le},\ \mathcal{N}_6 = RX_1 - X_1X_7,$$

$$\mathcal{L}_7 = -\frac{4\pi^2}{\delta^2 Le},\ \mathcal{N}_7 = \frac{X_1X_6}{2}.$$

The parameters for the LQBHM becomes

$$\varphi_1 = -\frac{Vaa_1^2}{\delta^2},\ \upsilon_1 = -Va(1+\epsilon^2\delta_1 f)X_2 + VaX_4 - NVaX_6,$$
$$\varphi_2 = -\frac{Va}{\varsigma\delta^2},\ \upsilon_2 = -Ta^* Va(1+\epsilon^2\delta_1 f)X_1,$$
$$\varphi_3 = -\frac{Va}{\varsigma\delta^2},\ \upsilon_3 = 0,$$
$$\varphi_4 = \frac{1}{\delta^2}(R_i - a_2^2),\ \upsilon_4 = -2RHX_1 - X_1X_5,$$
$$\varphi_5 = \frac{1}{\delta^2}(R_i - 4\pi^4),\ \upsilon_5 = \frac{X_1X_4}{2},$$
$$\varphi_6 = -\frac{1}{Le},\ \upsilon_6 = RX_1 - X_1X_7,$$
$$\varphi_7 = -\frac{4\pi^2}{\delta^2 Le},\ \upsilon_7 = \frac{X_1X_6}{2}.$$

Validation of the method

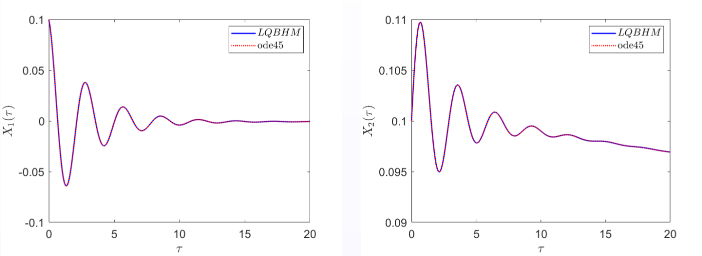

Figure: Comparison between the LQBHM and ode45 results for $X_1, \text{and } X_2$ profiles.

The figure displays the pictorial comparison between the selected time series LQBHM solution and the ode45 generated solution of the coupled nonlinear Lorenz type equations (31) - (37). It is found that these two results are in acceptable agreement, demonstrating the validity of the LQBHM.

- The goal of this work is to focus on three mechanisms, namely internal heating, the presence of a solute concentration, and rotational modulation, for promoting or inhibiting convective heat and mass transport. To analyze heat and mass transfer we use the nonlinear stability theory. The Nusselt number Nu and Sherwood number Sh are employed to represent heat and mass transmission, respectively.

Results and discussion

- The goal of this work is to focus on three mechanisms, namely internal heating, the presence of a solute concentration, and rotational modulation, for promoting or inhibiting convective heat and mass transport. To analyze heat and mass transfer we use the nonlinear stability theory. The Nusselt number Nu and Sherwood number Sh are employed to represent heat and mass transmission, respectively.
- The other interesting part of this work is the use of the newly developed Local Quasilinearisation Block Hybrid Method LQBHM to solve the nonlinear Lorenz-type system of equations and this method necessitated the analysis of heat and mass transport in the fluid system.

Results and discussion

The linear stability analysis is performed to determine the stability criteria in terms of the critical Rayleigh number Ra, below which the system is stable and above which it is unstable.

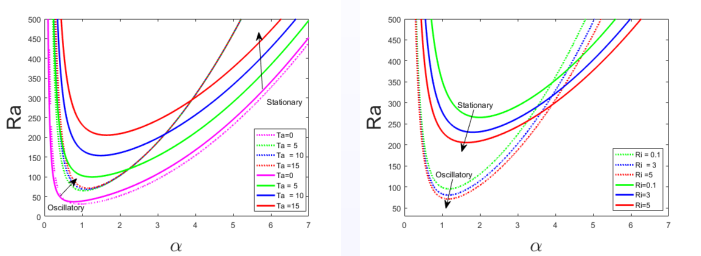

Figure: Effect of Ta and R_i on the stability curves for stationary and oscillatory convection against the wave number α with fixed parameters $Le = 2,\ R_i = 5,\ \eta = 1,\ \varsigma = 1,\ Ra_S = 10$ and $Ta = 15$.

Results and discussion

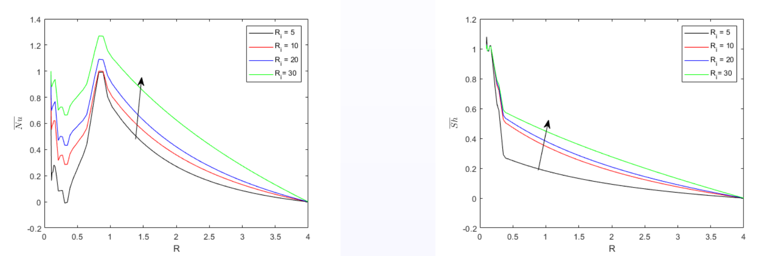

Figure: Changes in $\overline{Nu}$ and $\overline{Sh}$ with revised Rayleigh number R for internal Rayleigh numbers $R_i = 5,\ 10,\ 20$ and 30 with fixed parameters $\delta_1 = 2,\ Le = 4,\ Va = 2,\ \omega = 2,\ \eta = 0.4,\ \varsigma = 0.6,\ Ra_S = 100$ and $Ta = 25$.

Results and discussion

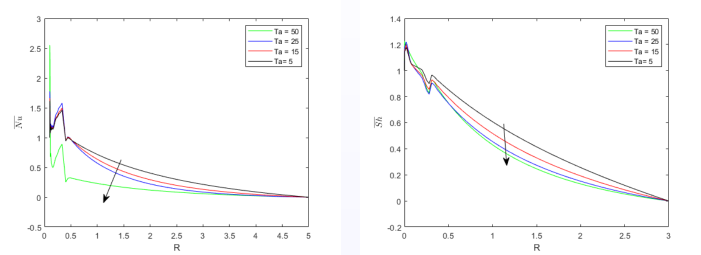

Figure: Changes in $\overline{Nu}$ and $\overline{Sh}$ with revised Rayleigh number R for Taylor numbers $Ta = 5,\ 15,\ 25$ and 50 with fixed parameters $\delta_1 = 2,\ Le = 4,\ Va = 2,\ \omega = 2,\ \eta = 0.4,\ \varsigma = 0.6,\ Ra_S = 100,\ N = 5$ and $Le = 4$.

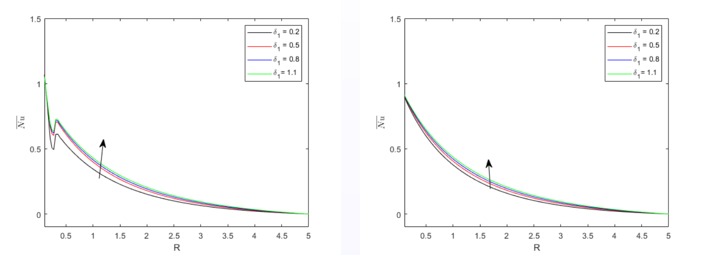

Figure: Changes in $\overline{Nu}$ and $\overline{Sh}$ with revised Rayleigh number R for modulation amplitudes $\delta_1 = 0.2,\ 0.5,\ 0.8$ and 1.1 with fixed parameters $Va = 2,\ Le = 4,\ R_i = 20,\ \omega = 2,\ \eta = 0.4,\ \varsigma = 0.6,\ Ra_S = 100$ and $Ta = 25$.

Conclusion

- Increasing the rotation has the effect of reducing the transfer of heat and mass hence delaying the onset of convection.

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- LQBHM results were in good agreement with the ode45 results.

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Thank you!!!
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