Modelling Blood Flow During Syringing

Jaimé Goedhals (MSc Candidate), Andie de Villiers (Supervisor), Francois Smit (Co-Supervisor)
Department of Applied Mathematics, Applied Mathematics Division, Stellenbosch University

Introduction

Blood is a crucial fluid within the body that is made up of four main components – red blood cells, white blood cells, platelets and plasma [1]. In the event that an emergency rapid transfusion needs to be administered, it is preferable to use a pressure bag to speed up the transfusion. However in low-resource settings, such as in rural hospitals, these pressure bags are not always readily available and the transfusion is instead administered via syringing. This has been shown to cause significant hemolysis as compared to the use of a pressure bag, which can have negative effects on the patient receiving the transfusion [2].
This project therefore aims to model blood flow during syringing, to determine under what conditions hemolysis is most likely to occur. This will be achieved using numerical simulations together with various hemolysis models, implemented in the open source finite element software deal.II [3].

The Navier-Stokes Equations

The incompressible Navier-Stokes equations together with a viscosity equation are

$$\rho(\mathbf{u}\cdot\nabla)\mathbf{u} - \mu\Delta\mathbf{u} - 2\nabla^s\mathbf{u}\,\nabla\mu + \nabla p = \mathbf{f},$$
$$\nabla\cdot\mathbf{u} = 0,$$
$$\eta_1 + \eta_2\left[\kappa + (\lambda\dot{\gamma})^a\right]^{\frac{n-1}{a}} = \mu,$$

where $\nabla^s\mathbf{u} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$, and $\dot{\gamma} = 2\sqrt{\frac{1}{2}\nabla^s\mathbf{u} : \nabla^s\mathbf{u}}$, and the parameters $a, n, \eta_1, \eta_2, \kappa$ and λ are material constants.

Setting $\eta_2 = 0$ gives the Newtonian viscosity model, whereas the Power-Law model is obtained by setting $\kappa = \eta_1 = 0$ [4].

Computational Implementation

The equations are solved in deal.II using the Newton-Raphson method:

Algorithm for Newton-Raphson

Initialise initial guess ($\mathbf{x}_0$) and tolerance (ϵ)

while $||\mathbf{R}|| \geq \epsilon$ **do**
  $\mathbf{F}\delta\mathbf{x}^k = -\mathbf{R}$
  $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha\delta\mathbf{x}^k$
  calculate $||\mathbf{R}||$
end while

Modelling Blood as a Newtonian Fluid

Blood is modelled as an incompressible Newtonian fluid in a syringe-like structure. These results will be used as reference for future non-Newtonian simulations. The dimensions of the structure are approximated according to a 20 ml syringe ($\pm$20 mm in diameter) and an 18-gauge IV cannula ($\pm$0.8 mm in diameter) [2].

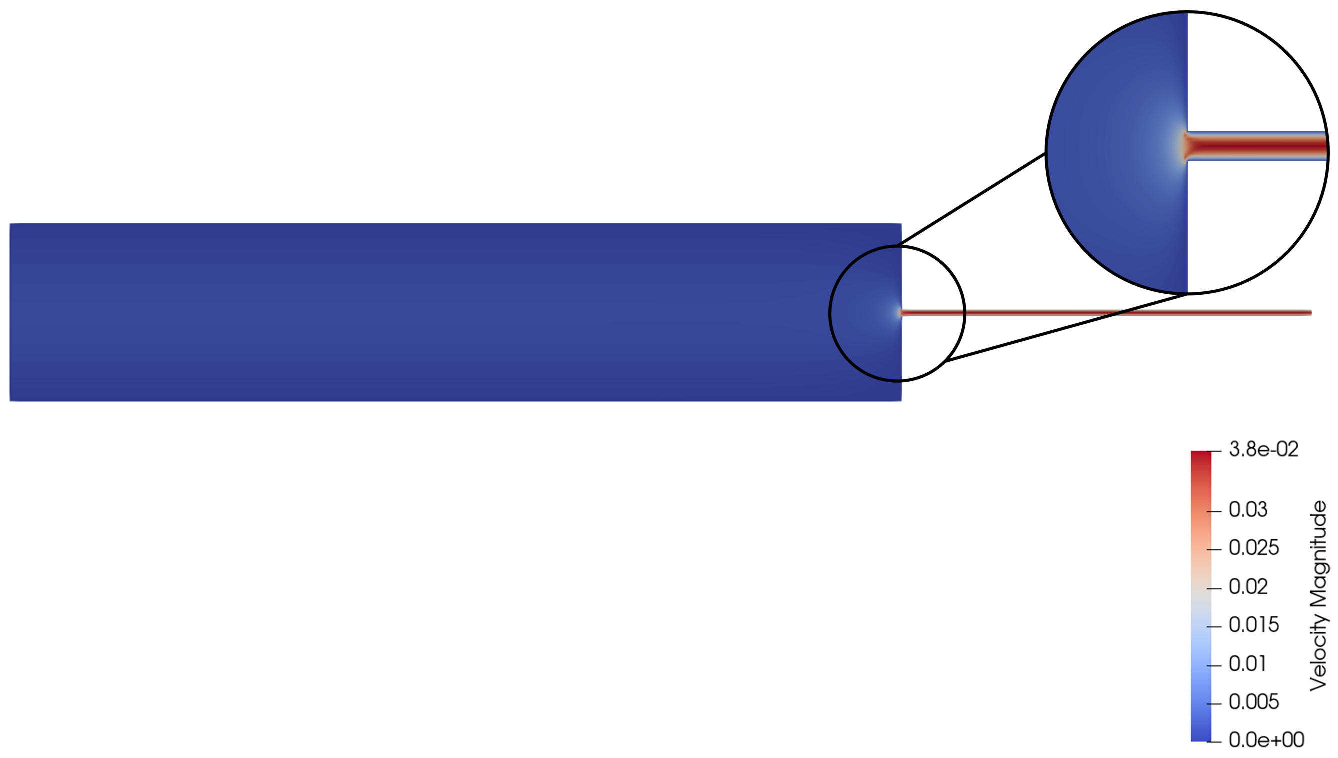

Figure: The velocity magnitude obtained from the implementation of the equations in deal.II.

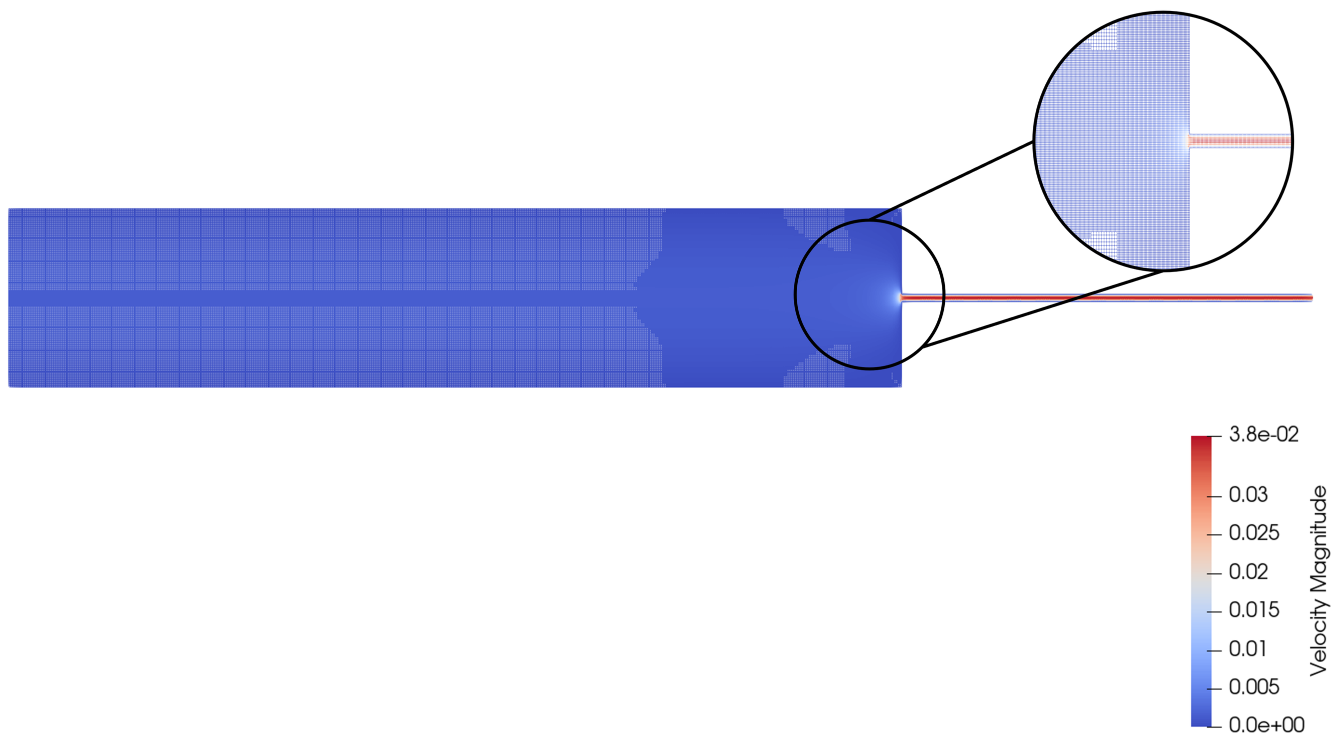

Figure: The adaptive meshing implemented in deal.II.

Verification of Numerical Results

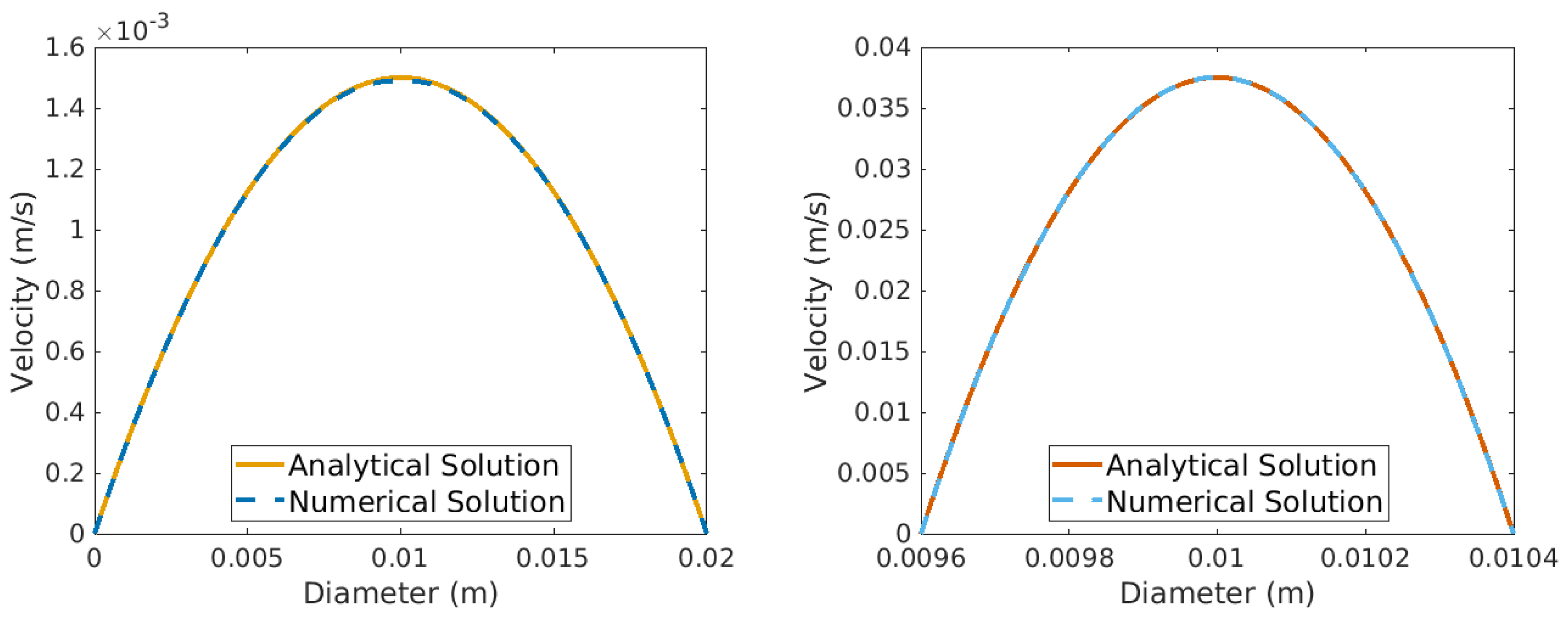

Figure: Comparison between the numerical and analytical solutions both before (left) and after (right) the contraction of the syringe in the respective fully developed regions.

Shear Rate Results

Since hemolysis is linked to shear stresses, we are also interested in the shear rate across the syringe and needle.

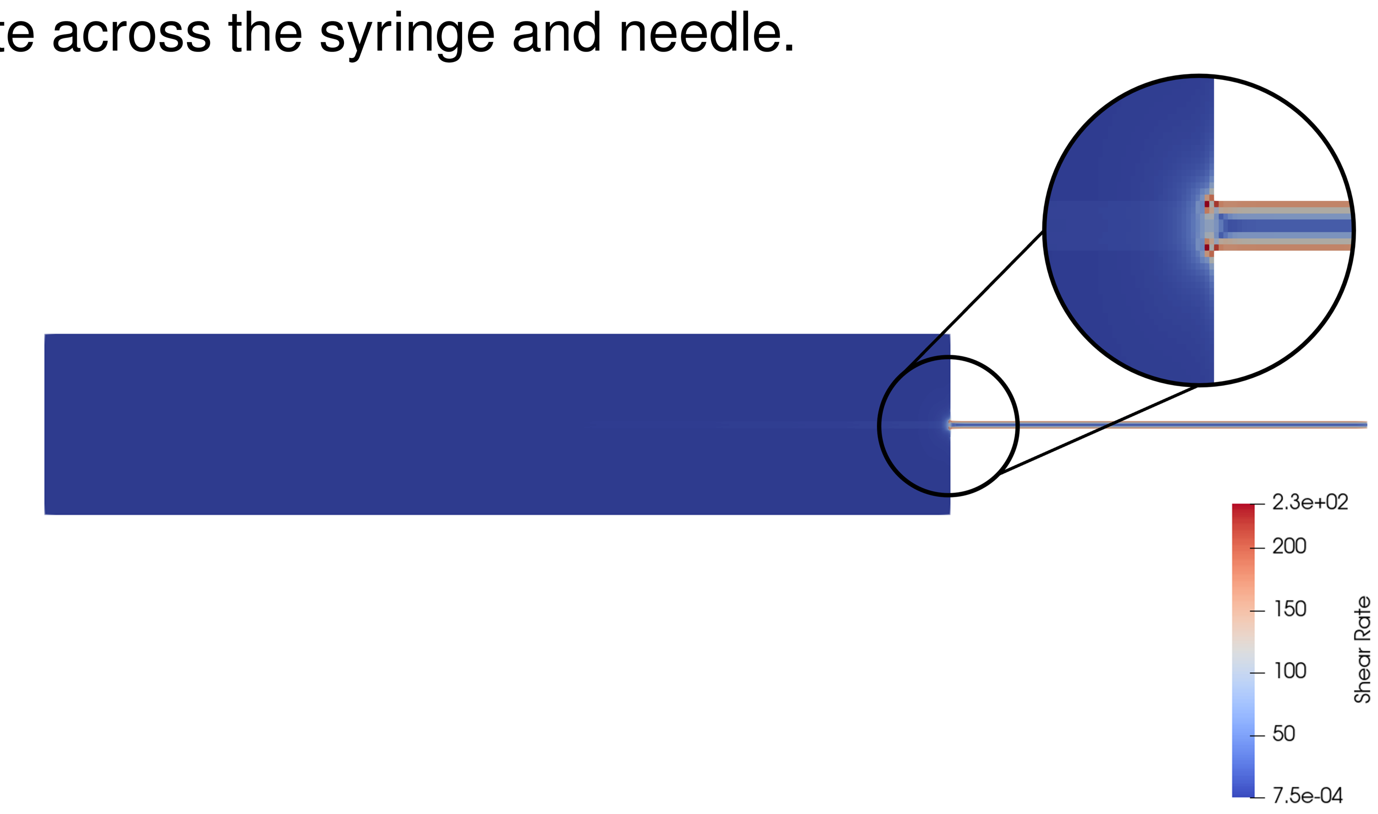

Figure: The cell averaged shear rate across the syringe and needle.

Future Work

Future work includes the following:

- Modelling blood as a non-Newtonian fluid and comparing different viscosity models (eg. Power-law vs. Carreau-Yasuda),
- Implementing hemolysis models and investigating the effect of various parameters.

References

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[3] D. Arndt, W. Bangerth, M. Feder, M. Fehling, R. Gassmöller, T. Heister, L. Heltai, M. Kronbichler, M. Maier, P. Munch, J.-P. Pelteret, S. Sticko, B. Turcksin, and D. Wells, "The deal.II library, version 9.4," *Journal of Numerical Mathematics*, vol. 30, no. 3, pp. 231–246, 2022.
[4] D. R. Pacheco, R. Schussnig, and T. P. Fries, "An efficient split-step framework for non-newtonian incompressible flow problems with consistent pressure boundary conditions," *Computer Methods in Applied Mechanics and Engineering*, vol. 382, p. 113888, 2021.