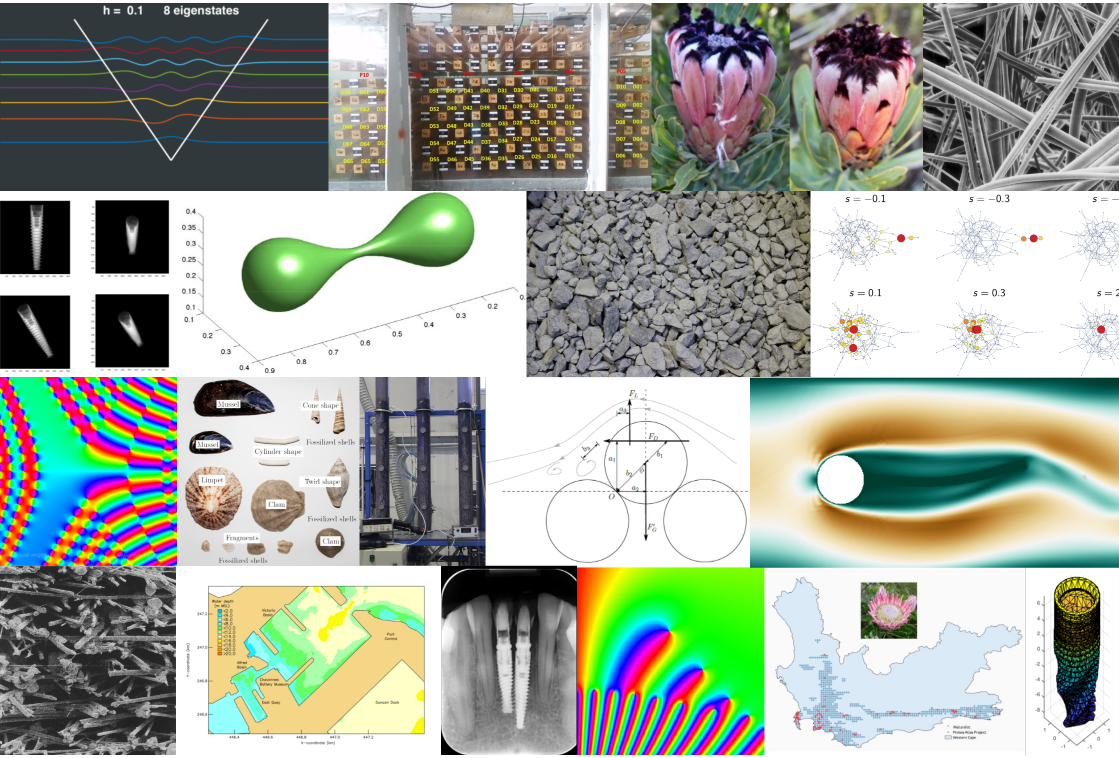
# *Applied Mathematics*

## Department of Mathematical Sciences

## Research Projects

## 2018-2019

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# *Forecasting solar and wind power outputs using deep learning approaches*

**Research Team:**

Dr Bubacarr Bah

In collaboration with University of Pretoria

## *Highlights*

- Identification of suitable locations of wind and solar farms in South Africa.
- Prediction of wind (solar) power from historical wind (solar) data, and other environmental factors, of South Africa using recurrent neural networks.



Figure. Wind turbines at Darling, Western Cape.

## *Applications*

- Informed decision making by power producer in terms of integration of renewable energies into existing electricity grids.
- Forecasting of energy prices.

# *Machine learning outcome prediction for coma patients*

**Research Team:**

Dr Bubacarr Bah

In collaboration with University of Cape Town

## *Highlights*

- Outcome prediction with Serial Neuron-Specific Enolase (SNSE) measurements from coma patients.
- The use of k-nearest neighbors (k-NN) for imputation of missing data shows promising results.
- Outcome prediction with EEG signals from coma patients.
- Using machine learning to determine the prognostic power of SNSE and EEG

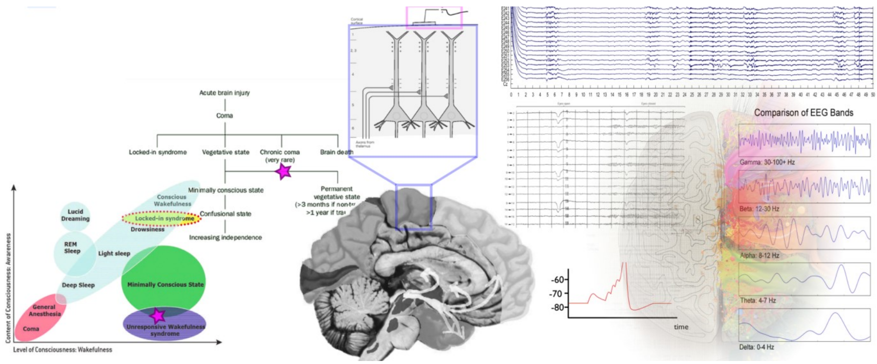



Figure. EEG and the determination of level of consciousness in coma patients.

## *Applications*

- Potential application in medical and health systems to improve diagnosis (and treatment) of coma.

# *Data-driven river flow routing using deep learning*

**Research Team:**

Dr Willie Brink and Prof Francois Smit

MSc Student: Jaco Briers

## *Highlights*

- Predicting flow along the lower Orange River in South Africa, for improved release scheduling at the Vanderkloof Dam, using recurrent neural networks as well as feedforward convolutional neural networks trained on historical time series records.



Figure. A photo of Vanderkloof Dam.

## *Applications*

- Better release scheduling can lead to enormous savings of fresh water in South Africa, which could be utilized for agricultural irrigation. Approximately 280 million cubic metres of water is lost from the Vanderkloof Dam each year because of inefficient releases.

# *Deep knowledge modelling*

**Research Team:**

Dr Willie Brink

MSc Student: Luyolo Magangane

## *Highlights*

- Developing deep learning solutions to logical reasoning, to improve upon formal reasoners that struggle with incomplete, conflicting, or uncertain information, as well as scalability.
- Trained models using word embeddings of entity relationship tuples from ontological knowledge bases (WordNet and Freebase), and achieved accuracy of 72% on a test set of yes-no questions.

```
Wordnet:

film_maker_1  has_instance director_4  1
film_maker_1  has_instance apothecary_s_shop_1 -1
mediterranean_1 has_part malta_2 1
mediterranean_1 has_part carrier_4 -1
military_capability_1 type_of  capability_1  1
military_capability_1 type_of  pastry_2  -1
heart_2 has_part valve_1 1
heart_2 has_part quack_2 -1


Freebase:

pope_leo_viii profession  pope  1
pope_leo_viii profession  record_producer -1
bruce_lee profession  actor 1
bruce_lee profession  insurance_agent -1
winston_churchill profession  member_of_parliament  1
winston_churchill profession  human_computer  -1
michael_jackson profession  singer  1
michael_jackson profession  soldier -1
```

Figure. Examples of yes-no questions in the form of entity relationship tuples, answered by the trained model ("-1" means no and "1" means yes).

## *Applications*

- Artificial intelligence: general question answering and knowledge discovery.

# *Image and attribute-based identification of Protea species using machine learning*

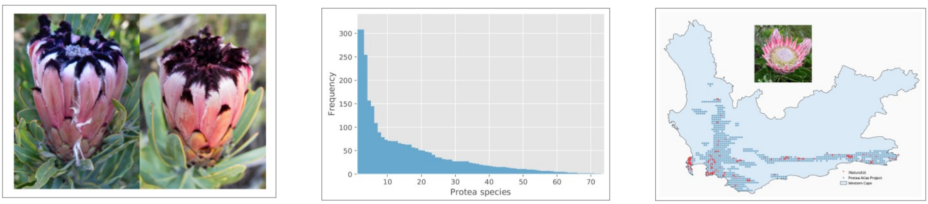
**Research Team:**

Dr Willie Brink

MSc Student: Peter Thompson

## *Highlights*

- Building systems of convolutional neural networks and probabilistic graphical models that are able to identify Protea species from digital images, optionally enriched by a small set of attributes such as location, elevation and time of year.
- It is a challenging image recognition problem, given extreme visual similarities between two species (a), and heavy class imbalance in the available observations data (b).




(a) (b) (c)

Figure. (a) A comparison of Protea neriifolia (left) and Protea laurifolia (right) shows the striking visual similarities of the two different species. (b) A histogram of the frequency of observations per species, showing the large class imbalance in the dataset. (c) A map of the Western Cape with locations of observations used in this project.

## *Applications*

- Support for crowd-sourced observational surveying, crucial for the conservation of our biodiversity.
- Ideas around fine-grained image classification trained on a limited supply of data can be extended to other types of fauna and flora.

# *Connection type recognition for dental implants*

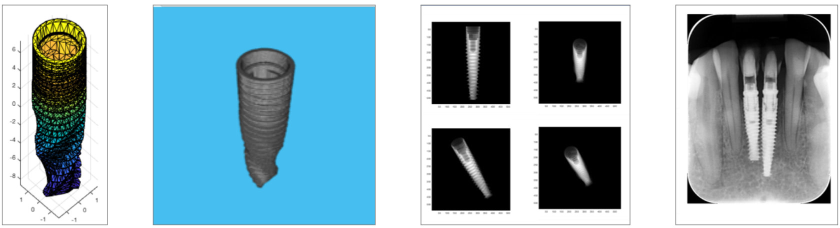
**Research Team:**

Dr Hanno Coetzer

In collaboration with KU Leuven, Belgium

## *Highlights*

- A three-dimensional volumetric representation is constructed from each triangulated surface model associated with a specific dental implant.
- Simulated x-ray images are generated from a number of angles and used to train a convolutional neural network (CNN).
- A questioned dental implant is extracted from an actual x-ray image and presented to the trained CNN to determine its connection type.



(a) (b) (c) (d)

Figure. (a) The triangulated surface model. (b) A 3D volumetric representation. (c) Simulated x-ray images (training data). (d) Actual x-ray image (test data).

## *Applications*

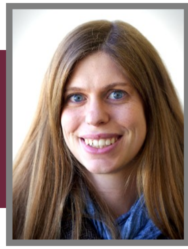
- A dental implant is typically embedded into the bone and cartilage of a patient’s jaw and is difficult to access manually in order to ascertain the connection type.
- Knowledge of the connection type is important in order to determine a suitable abutment and crown.

# *Computational Biomechanics*

**Research Team:**

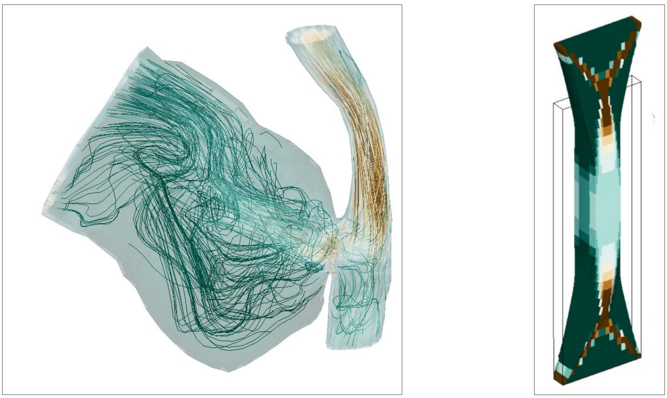
Dr Andie de Villiers

In collaboration with University of Cape Town and University College London, UK



## *Highlights*

- Modelling complex materials such as blood vessels and skin.
- Developing a robust numerical framework to solve these models on patient specific geometries using the C++ based finite element library deal.ii.



De Villiers, A.M et al., Biomechanics and modeling in mechanobiology (2018)

(a) (b)

Figure. (a) Streamlines in patient-specific fistula. (b) Simulated tensile test of vessel wall.

## *Applications*

- Simulating blood flow in arterio-venous fistula and analysing stress in vessel walls.
- Simulating waves propagating through skin to diagnose melanoma.

# *Interaction of waves with porous structures*

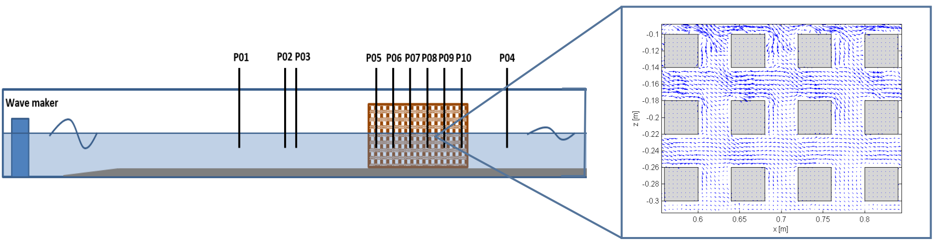

**Research Team:**

Dr Hardus Diedericks

Prof Francois Smit


## *Highlights*

- Conducting detailed interstitial flow field measurements of a monochromatic wave in a two-dimensional, rectangular, staggered and non-staggered porous structure.
- Using process-based models to perform simulations of the flow field within the two-dimensional staggered and non-staggered porous structures.
- Developing a mathematical model that will allow for fast, yet accurate, simulations of the interaction of waves with porous structures.




Terblance L, Diedericks GPJ, Smit GJF and Troch CNA, Proceedings Coastlab18 (2018)

Figure. Layout of the wave flume and flow field.

## *Applications*

- Reduce the propagation of waves through breakwaters.
- The absorption of long waves in ports to prevent resonance and unwanted ship motions when moored.

# *Process-based hydrodynamic modelling*

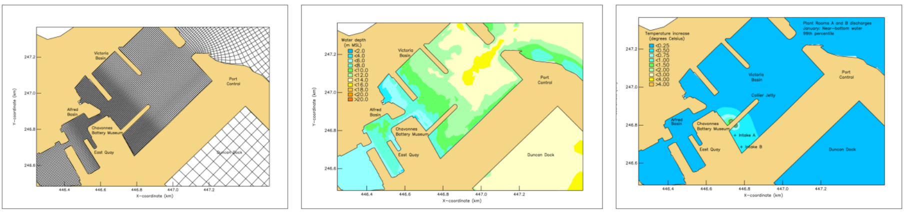
**Research Team:**

Dr Hardus Diedericks

Prof Francois Smit

## *Highlights*

- Setting up numerical models at various locations e.g. Table Bay (Cape Town), False Bay, Cameroon and Ireland.
- Determining which physical processes are dominant and how to simulate them.
- Formulating a scaling method to show the differences between incompressible fluids and incompressible flows for situations with variable density.



(a) Computational domain. (b) Model inputs. (c) Model results.

Figure. A model of the V&A Waterfront in Cape Town.

## *Applications*

- Perform modelling as specialists for Environmental Impact Assessments (EIAS).
- Assess the behaviour of dredged and disposed materials, thermal discharges and effluents released from Salmon cage farming.

# *Analytical determination of the effect of biofilm growth on the pressure drop in a biofilter*

**Research Team:**

Dr Sonia Fidder-Woudberg

In collaboration with IMT Atlantique in Nantes, France

## *Highlights*

- The pressure drop prediction is adapted to take biofilm thickness into account. Predictive equations are also provided for the biofilm affected specific surface area of the packing material and porosity as a function of the biofilm thickness.

(a) (b) (c)

Figure 1. (a) Three biofilters used to remove toxic $H_2S$ from an airstream. (b) Expanded schist used as packing material. (c) UP20 (left) and schist (right) used in biofilters.

## *Applications*

- A biofilter removes toxic gases such as hydrogen sulphide produced by several industries. Knowledge and understanding of the underlying physical transport processes in a biofilter can optimize the process.

# *Predicting the permeability of fibrous porous media*

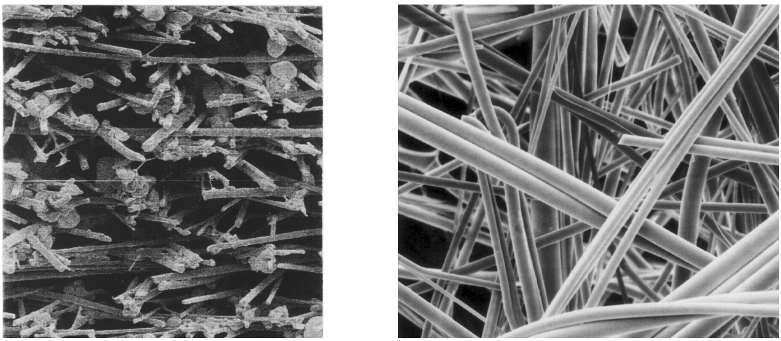
**Research Team:**

Dr Sonia Fidder-Woudberg

In collaboration with IMT Atlantique in Nantes, France ; MSc Student: Esmari Maré

## *Highlights*

- The permeability of a fibrous filter is an important parameter since it determines filter performance.
- The permeability prediction is adapted to take the effect of local porosity variation and compression of fibrous filters into account.



Woudberg, S. Van Heyningen, M.C., Le Coq, L., Legrand, J. and Du Plessis, J.P. , Chemical Engineering Science (2014)

(a) (b)

Figure. (a) Horizontal and (b) vertical sections of the porous micro-structure of a non-woven glass fibre filter.

## *Applications*

- Fibrous filters are used for air filtration and the use thereof positively influence the health of humans.

# *Numerical methods for fractional differential equations*

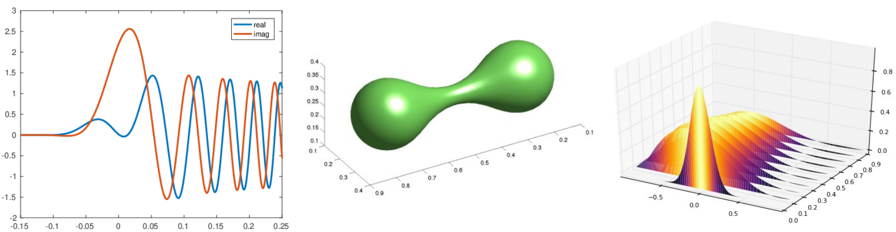

**Research Team:**

Dr Nick Hale

In collaboration with Imperial College London, UK


## *Highlights*

- Fractional differential equations are playing an ever-increasing role in the mathematical modelling of real-world phenomena.
- This project aims to develop robust and efficient numerical methods to solve such equations and apply them to practical problems.




(a) (b) (c)

Figure. (a) Solution to the 1D fractional Airy equation. (b) Allen-Cahn solution for a 3D dumbbell test problem. (c) Fractional heat equation.

## *Applications*

- Space-fractional derivatives ("super-diffusion") appear in the modelling of epidemics, financial markets, and other processes where concentrations can take rare, but large jumps.
- Time-fractional derivatives ("anomalous diffusion") are used to model processes with a 'memory', such as neural synapse responses and DNA sequences.

# *Mathematical modelling of simple mechanical systems with complicated behaviour*

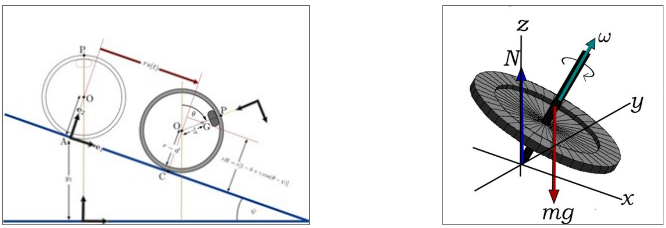
**Research Team:**

Dr Milton Maritz and Dr Marèt Cloete

In collaboration with Willie Theron

## *Highlights*

- There is currently a keen interest in simple mechanical systems that exhibit surprisingly complicated behaviour. Some typical examples are: the double pendulum, the loaded hoop, and the celtic stone (also called the 'rattleback').
- This project entails the mechanical modelling of the simple system, i.e. obtaining the equations of motion, and solving them using numerical solution techniques.
- Some physical experiments are also performed and photographed with a high speed camera, and the resulting frames are then digitized to be compared to the result predicted by the model.




(a) (b)

Figure. (a) Diagrammatic setup for a rolling loaded hoop. (b) Diagrammatic setup for a spinning top.

## *Applications*

- An extensive study of the loaded hoop has been undertaken.
- A study of the spinning top is currently undertaken, with the aim of preparing the field for a comprehensive study of the celtic stone.

# *Deconvolution for performing photo deblur*

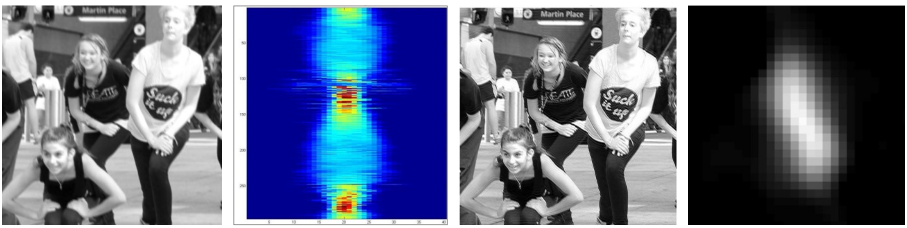
**Research Team:**

Dr Milton Maritz

In collaboration with Jan-At Engelbrecht

## *Highlights*

- Photos may be blurred because the optics was out of focus, or because the camera moved during the acquisition process (or both).
- A blur kernel is a function that encapsulates the entire blurring process. Deblurring can be performed digitally if the kernel is available.
- If the kernel is not available, it can be estimated from the blurred photo itself – so called 'blind deconvolution'.
- This project entails the investigation of blind deconvolution techniques using tomographic reconstruction of the blur kernel from sampling edges in the image.



(a) (b) (c) (d)

Figure. (a) A blurred image. (b) The reconstructed sonogram. (c) The reconstructed blur kernel. (d) The deblurred image.

## *Applications*

- Current deblurring techniques still suffer from some artifacts.
- There is still a need for improvement of the current blind deconvolution techniques.

# Reconfiguration problems in graphs

**Research Team:**

Dr Riana Roux
In collaboration with University of Victoria, Canada

## Highlights

- Investigate the structure of the solution space of the domination problem and the irredundance problem in graphs.
- Determine which type of graphs describes the solution space of a given problem.

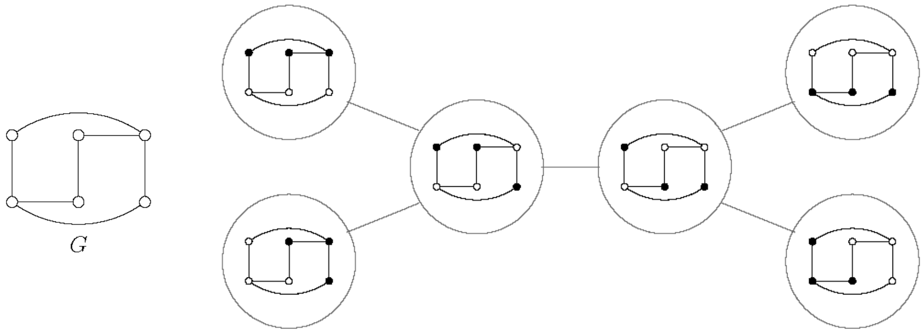



Figure. The graph G and a depiction of all possible maximum irredundant sets of G and the relationships between them.

## Applications

- The domination problem is a mathematical model for the facility location problem.
- The structure of the solution space will give an indication of how one solution can be changed into another.

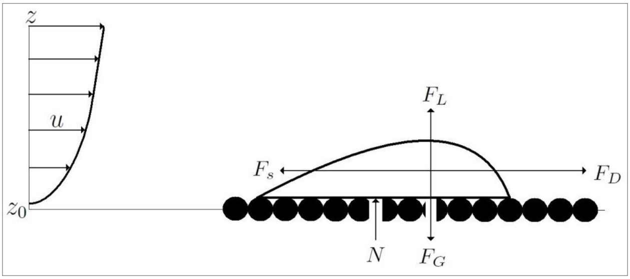
# *Initiation of sediment motion in coastal dynamics*

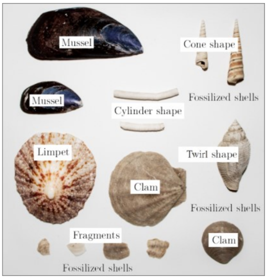
**Research Team:**

Prof Francois Smit
Dr Hardus Diedericks

## *Highlights*

- Developing a methodology to assess the incipient motion of shells and shell gravel.
- Discovering how the presence of mud influences the transport of sand-mud mixtures.
- Distinguishing between the initiation of motion mechanisms of shear, lift, force moments and buoyancy.




**Diedericks GPJ, Troch CNA and Smit GJF, J. Hydraul. Eng., (2018)**

Figure. A depiction of different shells considered in the study and a force diagram of a shell on the seabed.

## *Applications*

- The selection of the relevant processes for numerical modelling which includes identifying the relevant drivers, sources and parameters and understanding the importance of each.
- Specifically aid determining the sediment transport parameters for dredging studies.

# Adaptive algorithms for rare event sampling

**Research Team:**

Prof Hugo Touchette

## Highlights

- Developed new algorithm for sampling very rare (failure-type) events characterized by probability smaller than $10^{-5}$.
- Adaptive algorithm 'learns on the fly' the optimal sampler known to be efficient.
- Applied algorithm to physical processes modelled by stochastic differential equations.

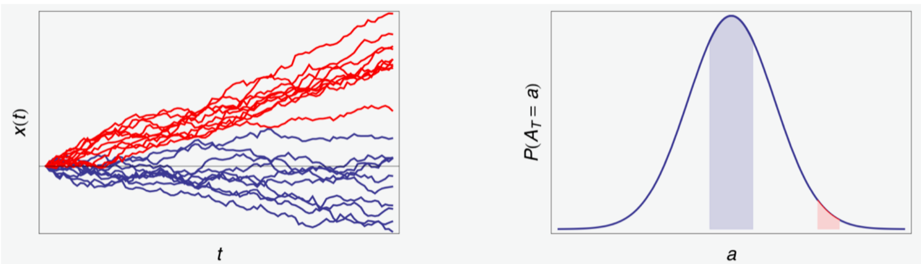



Figure. Schematic illustration of different trajectories of a stochastic process giving rise to typical events (blue) or rare events (red).

## Applications

- Numerical estimation of fluctuation distributions in noisy physical systems.
- Estimation of failure probability in control systems and other man-made systems (e.g. overflow in queues).
- Response characterisation of nonequilibrium systems based on their fluctuation properties.

# *Random walks on random graphs*

**Research Team:**

Prof Hugo Touchette

## *Highlights*

- Studied random walkers evolving on random graphs.
- Predicted stationary distribution as well as probability of rare events such as reaching nodes with low degree/connectivity.
- Constructed biased random walker models that explain how rare events happen on a random graph.

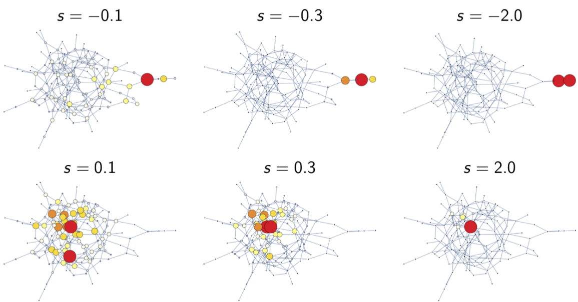



Figure. Stationary distribution of a random walk biased on reaching low connected nodes (s negative) or high connected nodes (s positive).

## *Applications*

- Fast detection/characterization of graph properties (e.g. central nodes, communities, etc., similar to Google's PageRank).
- Estimation of failure probability for detection/characterization.
- Localization of sparse regions in real complex networks.

# *Numerical computation of special functions*

**Research Team:**

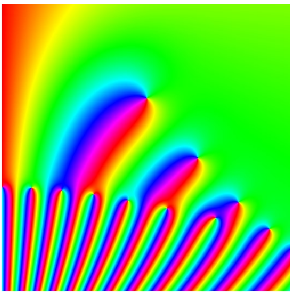
Prof André Weideman

In collaboration with University of Colorado, Boulder, USA

## *Highlights*

- Certain special mathematical functions arise often in scientific and engineering investigations.
- To compute such functions, one can use an integral representation or a differential equation:

$$w(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}}{z-t}dt \qquad u''(z) = 2u^3 + zu$$



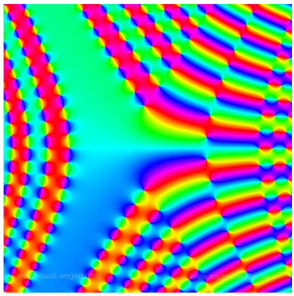


Figure. The diagrams show phase plots of $w(z)$ (left) and $u(z)$ (right) in the complex $z$-plane, as computed by methods developed in this research.

## *Applications*

- The function $w(z)$ is the plasma dispersion function, also known as the complex error function, and is used in astrophysics and spectroscopy.
- The function $u(z)$ is a Painlevé function, and one of its applications is in the computation of a certain probability distribution.

# *Software for numerical solution of differential equations: Chebfun and DMSUITE*

**Research Team:**

Prof André Weideman and Dr Nick Hale

In collaboration with Oxford University, UK

## *Highlights*

- Well-written and well-documented codes for numerical solutions of differential equations.
- Automatic discretisation and adaptive grid refinement.
- Intuitive and elegant user interface.
- www.chebfun.org
- www.mathworks.com/matlabcentral/fileexchange/29-dmsuite

```
% Create a chebfun on the interval [-3,3]
x = chebfun('x', [-3 3]);
% Define a potential function
V = abs(x);
% Plot the first 8 eigenstates of
% the Schrodinger operator
quantumstates(V, 8)
```

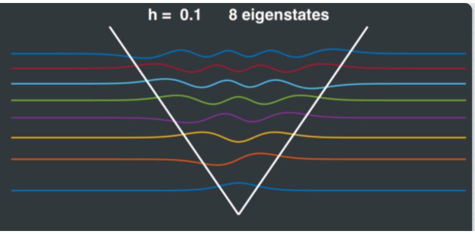



Figure. Eigenstates of the Schrödinger equation for a triangular potential.

## *Applications*

- Numerical function approximation and optimisation.
- ODEs and PDEs on rectangles, spheres, and disks.
- ODE eigenvalue problems (e.g. Schrödinger equation).

# Contact information

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