[9.13] SURFACE INTEGRALS

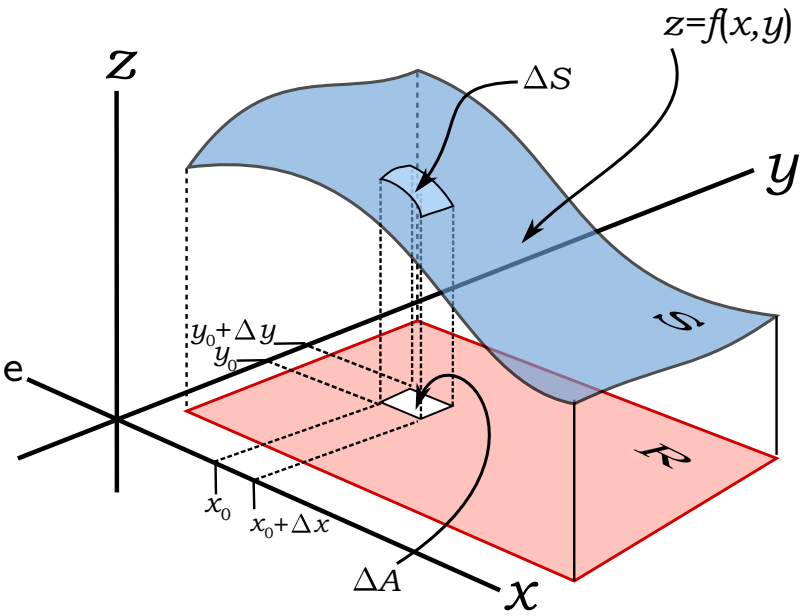
Review:

Surface Integral: $\iint_S G(x,y,z)dS$,

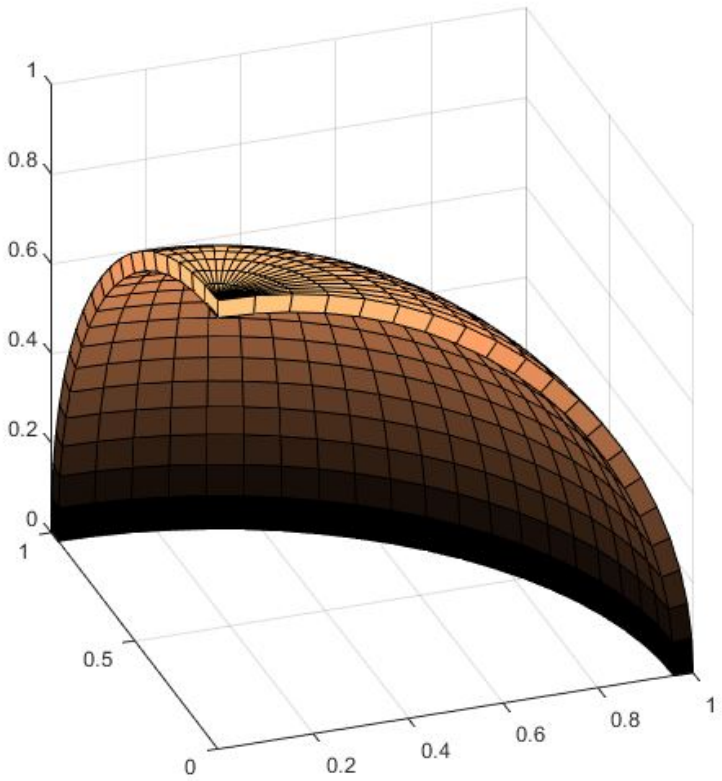
$G(x,y,z) =$ .... a volume function

S: $\begin{cases} z = f(x,y) \ldots \text{ a surface} \\ (x,y) \in \text{R}, \text{R: .... a region in the } xy\text{-plane} \end{cases}$

$$dS = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$



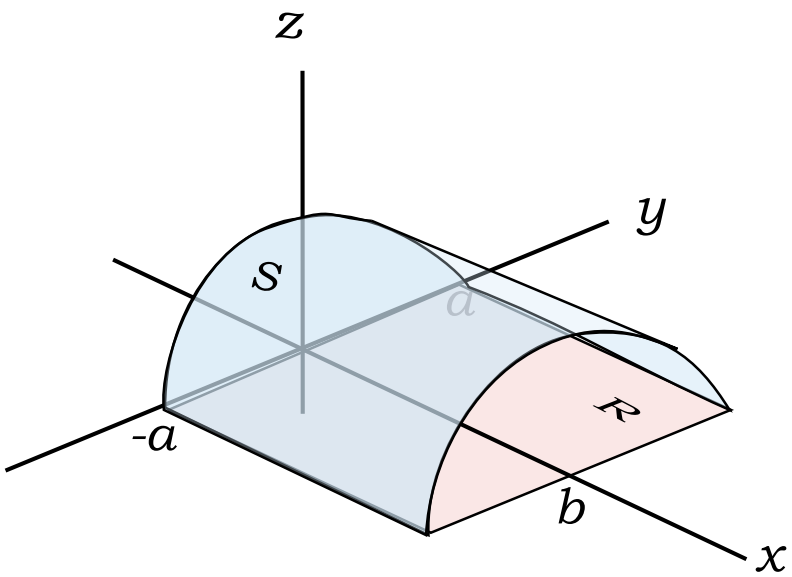
Uses for surface integrals:



- Surface area $= \iint_S 1 \, dS$,

Example 1: Find the surface area of S. a and b are positive constants.

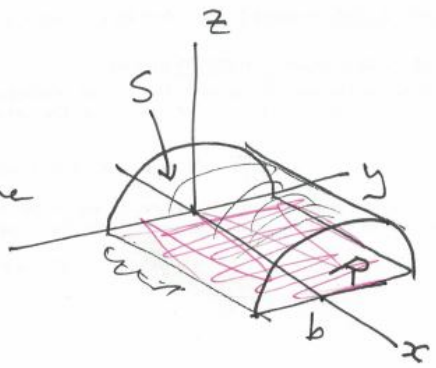
$$\text{S: } \begin{cases} z^2 + y^2 = a^2 \\ x \in [0, b] \\ z \geq 0 \end{cases}$$



EXAMPLE [1]

$z^2+y^2=a^2$

→ circ, r=a in zy-plane



$z \geq 0$

$x \in [0,b]$

$0 \leq x \leq b$

$$\text{area} = \iint_S 1 \, dS$$

$$z = f(x,y) = \sqrt{a^2-y^2}$$

$$dS = \sqrt{1+0^2+\left(\frac{-2y}{2\sqrt{a^2-y^2}}\right)^2}\, dA$$

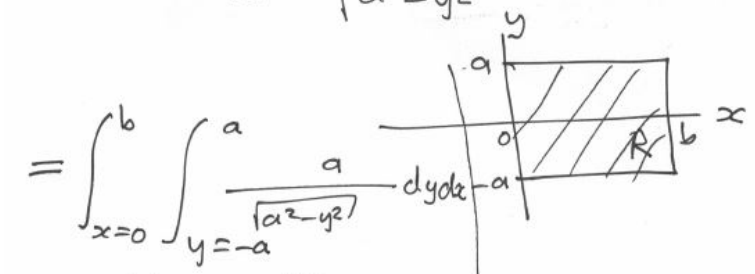
$$= \sqrt{1+\frac{y^2}{a^2-y^2}}\, dA$$

$$= \sqrt{\frac{a^2-y^2+y^2}{a^2-y^2}}\, dA = \frac{a}{\sqrt{a^2-y^2}}\, dA$$

[2]

$$\text{area} = \iint_S 1 \, dS$$

$$= \iint_R 1 \cdot \frac{a}{\sqrt{a^2-y^2}}\, dA$$

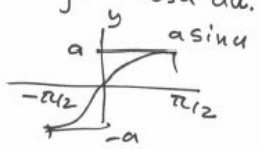


$$= \int_{x=0}^{b} \int_{y=-a}^{a} \frac{a}{\sqrt{a^2-y^2}}\, dy\, dx$$

$$= a \int_{x=0}^{b} dx \cdot \int_{y=-a}^{a} \frac{1}{\sqrt{a^2-y^2}}\, dy$$

$y = a\sin u$

$dy = a\cos u \, du$

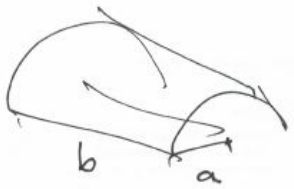


$$= a\left[x\right]_0^b \int_{-\pi/2}^{\pi/2} \frac{a\cos u \, du}{\sqrt{a^2-a^2\sin^2 u}}$$

$$\sqrt{a^2-a^2\sin^2 u} = a\cos u$$

$$= ab \int_{-\pi/2}^{\pi/2} 1 \, du = ab\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]$$

$$= \pi ab$$



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$$\frac{2\pi a}{2} \cdot b = \pi a b.$$