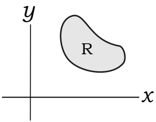
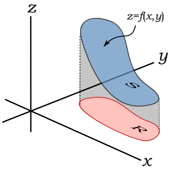
[9.13] SURFACE INTEGRALS





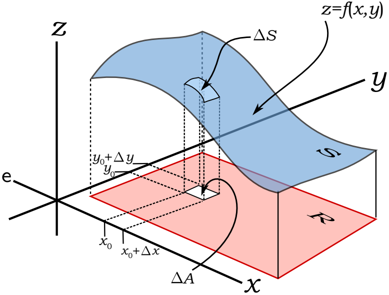
Double Integral: $\iint_R f(x,y)dA$

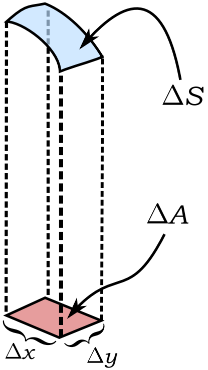
Surface Integral: $\iint_S G(x,y,z)dS,$

$G(x,y,z) =$ .... a volume function

S: $\begin{cases} z = f(x,y) \text{... a surface} \\ (x,y) \in \text{R, R: .... a region in the } xy\text{-plane} \end{cases}$

$$dS = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$





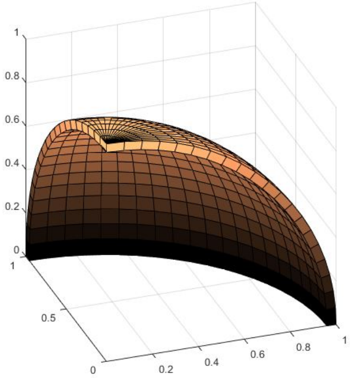
$$dS = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$

dS is a differential *surface element*.

$dS =$ some 'weight' $\times dA$

The 'weight' is always larger or equal to 1, since it is a 'projection' of ΔS on the xy-plane.

Uses for surface integrals:



- Surface area $= \iint_S 1 \, dS$,
- Surface mass $= \iint_S \rho(x, y) \, dS$,
  .... where $\rho(x, y)$ is an area-density
- Centroid x-coordinate:
  $\bar{x} = \frac{1}{m} \iint_S x\rho(x, y) \, dS$,
- Moment of inertia about x-axis
  $= \iint_S (y^2 + z^2)\rho(x, y) \, dS$,
- .... etcetera....

Example 1: Find the surface area of S. a and b are positive constants.

$$\text{S:} \quad \begin{cases} z^2 + y^2 = a^2 \\ x \in [0, b] \\ z \geq 0 \end{cases}$$