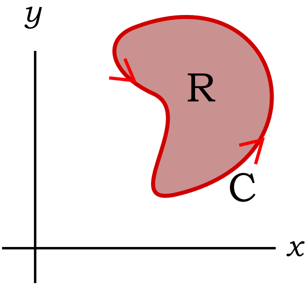
Green's Theorem:



$$\mathbf{F} = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

View of Green's Theorem in 3D:
Let

$$\mathbf{F} = \begin{bmatrix} P(x,y) \\ Q(x,y) \\ 0 \end{bmatrix}$$

$$\nabla \times \mathbf{F} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} P(x,y) \\ Q(x,y) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & - & 0 \\ 0 & - & 0 \\ \frac{\partial Q}{\partial x} & - & \frac{\partial P}{\partial y} \end{bmatrix}$$

but

$$(\nabla \times \mathbf{F}) \cdot \mathbf{k} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} \, dA$$