TW/AM 20753-242	TUTTOETS 12 / *TUT TEST 12*	2023
Van, Voorl's / *Surname, Init's:*	MEMO	
Studentenommer/ *Student number:*		

$\frac{10}{10}$

[9.16] The Divergence Theorem

Wenk /*Hint*: $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_D \nabla\cdot\mathbf{F}\,dV$

G is die totale vloed van $\mathbf{F}$ uit D. S is die buite-oppervlak van D. (a) Om G direk te bereken, moet die vloed oor vier verskillende oppervlakke gevind word. Beskryf/teken slegs die vier oppervlakke (maar moenie dit so bereken nie). (b) Gebruik die divergensie-stelling om G te bereken. Integrasie in silindriese koördinate is die maklikste hier. Gebruik die agterkant van die vraestel indien nodig.

G is the total flux of $\mathbf{F}$ out of D. S is the outer surface of D. (a) In order to calculate G directly, one needs to find the flux through four surfaces. Only describe/draw these four surfaces (but do not calculate the flux in this way.) (b) Use the divergence theorem to calculate G. Integration in cylindrical coordinates is the easiest here. Use the reverse side of the paper if necessary. floor, above

$$G = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS, \qquad \mathbf{F} = 2x^3z\,\mathbf{i} + 2y^3z\,\mathbf{j} + y\,\mathbf{k}, \qquad \text{D: } \begin{cases} 0 \le z \le \sqrt{x^2+y^2}, \\ 1 \le x^2+y^2 \le 2. \end{cases}$$

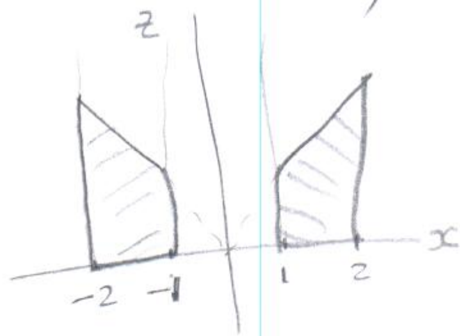
cone, below.

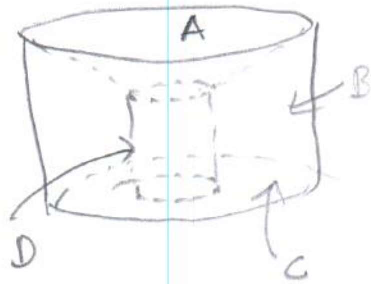
cylinders, between

(a)

A: truncated cone

B: cylinder, radius = 2
height = 2





C: annulus, inner radius = 1
outer radius = 2

D: cylinder, radius = 1
height = 1

(b) Use divergence th:

$$\underline{\nabla} \cdot \underline{F} = \begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} \cdot \begin{bmatrix} 2x^3 z \\ 2y^3 z \\ y \end{bmatrix} = 6x^2 z + 6y^2 z + 0$$
$$= 6z(x^2+y^2)$$

$$\text{Flux} = \iiint_D \underline{\nabla} \cdot \underline{F} \, dV = \int_{\theta=0}^{2\pi} \int_{r=1}^{2} \int_{z=0}^{r} (6r^2 z) \, dz \, r \, dr \, d\theta$$

$$= \Big[\theta\Big]_0^{2\pi} \int_{r=1}^{2} \left[6r^2 \frac{z^2}{2}\right]_0^r r \, dr$$

$$= 2\pi . 3 \int_1^2 \left[r^5 - 0\right] dr$$

$$= 6\pi \left[\frac{r^6}{6}\right]_1^2 = \pi \left[64 - 1\right]$$

$$= \underline{\underline{63\pi}}$$