

TW/AM 20753-242	TUTTOETS 11 / TUT TEST 11	2023
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[9.15] Triple Integrals

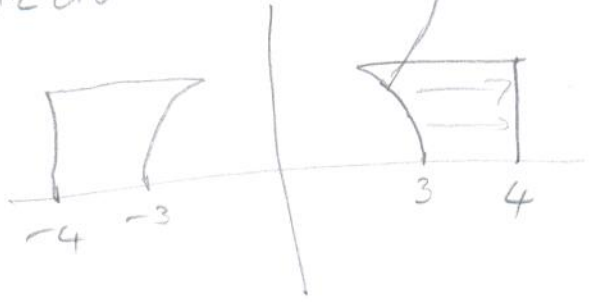
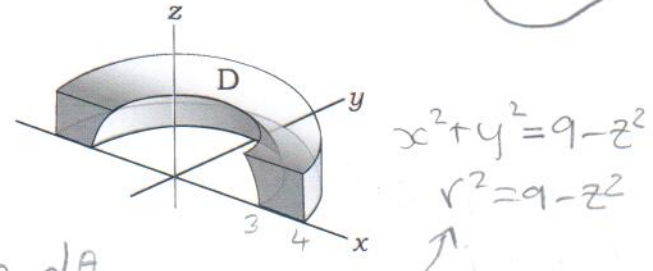
Bereken die massa van D. Dit is 'n halwe silinder met hoogte 1 en radius 4, waaruit 'n afgesnyde sferiese gat met radius 3 uitgeboor is. Die digtheid word gegee deur ρ . Integreer in silindriese koördinate.

Calculate the mass of D. It is half a cylinder with height 1 and radius 4, out of which a truncated spherical hole with radius 3 has been drilled. The density is given by ρ . Integrate in cylindrical coordinates.

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$\rho = 15z^2 + 3,$
 $D: \begin{cases} x^2 + y^2 + z^2 \geq 9, \\ 0 \leq z \leq 1, \\ x^2 + y^2 \leq 16, \\ y \geq 0. \end{cases}$

$$\text{mass} = \int_{\theta=0}^{\pi} \int_{z=0}^1 \int_{r=\sqrt{9-z^2}}^4 (15z^2+3) r dr dz d\theta$$



$$= [\theta]_0^{\pi} \int_{z=0}^1 (15z^2+3) \frac{r^2}{2} \Big|_{\sqrt{9-z^2}}^4 dz$$

$$= \pi \int_0^1 \left(\frac{15z^2+3}{2} \right) (4^2 - (9-z^2)) dz$$

$$= \pi \int_0^1 \left(\frac{15z^2+3}{2} \right) (7+z^2) dz$$

$$= \frac{\pi}{2} \int_0^1 (15z^4 + 108z^2 + 21) dz$$

mass = 30π

$$= \frac{\pi}{2} \left[\frac{15z^5}{5} + \frac{108z^3}{3} + 21z \right]_0^1 = \frac{\pi}{2} 60 = 30\pi$$