TW/AM 20753-242	TUTTOETS 10 / *TUT TEST 10*	2023
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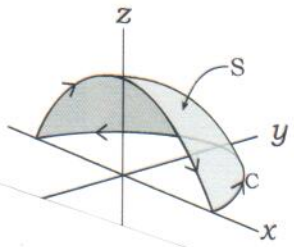
[9.14] Stokes' theorem

Bereken W hieronder deur van Stokes se stelling gebruik te maak. C is die rand van S.

Calculate W below by using Stokes's theorem. C is the boundary of S.

$\mathbf{F} = (2y+3z)\,\mathbf{i} + 6x\,\mathbf{j}$ $\quad W = \oint_C \mathbf{F}\cdot d\mathbf{r},$ $\quad S: \begin{cases} z = 1-x^2-y^2, \\ z \geq 0, \\ y \geq 0. \end{cases}$



Stokes: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F})\cdot \mathbf{n}\, dS$

Applying Stokes: $W = \iint_S (\nabla \times \underline{F})\cdot \underline{n}\, dS$

$$\nabla \times \underline{F} = \begin{bmatrix} d/dx \\ d/dy \\ d/dz \end{bmatrix} \times \begin{bmatrix} 2y+3z \\ 6x \\ 0 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 3-0 \\ 6-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \checkmark\checkmark$$

Normal: $G = z + x^2 + y^2 - 1$ ✓✓

$$\nabla G = \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix}, \quad \underline{n} = \frac{1}{\sqrt{1+4x^2+4y^2}} \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix}$$

$dS = \checkmark$ $\sqrt{1+4x^2+4y^2}\, dA$

$$W = \iint_R \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix} dA \quad \checkmark$$

$$= \iint_R (6y+4)\, dA$$

$$= \int_{\theta=0}^{\pi} \int_{r=0}^{1} (6r\sin\theta + 4)\, r\, dr\, d\theta \quad \checkmark\checkmark = \int_0^{\pi} \left[\frac{6r^3}{3}\sin\theta + \frac{4r^2}{2}\right]_0^1 d\theta$$

$$= \int_0^{\pi} (2\sin\theta + 2)\, d\theta = \left[-2\cos\theta + 2\theta\right]_0^{\pi} = 4 + 2\pi \quad \checkmark\checkmark$$