

TW/AM 20753-242	TUTTOETS 9 / TUT TEST 9	2023
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[9.13] Surface Integrals

Maak 'n skets van die dop P. Vind dan die area sowel as die massa van die dop P. Die area-digtheid van P is  $\rho = z$ .

Sketch the shell P. Then find the area as well as the mass of the shell P. The area density of P is  $\rho = z$ .

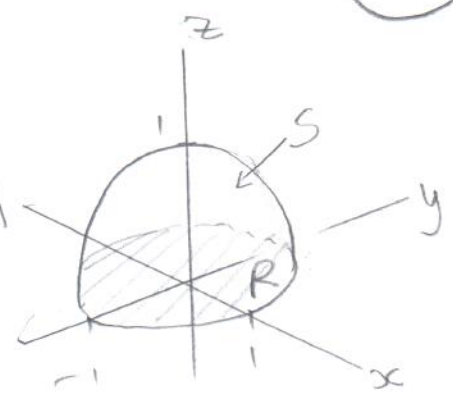
SURF ELEMENT:

$$dS = \sqrt{1 + \left(\frac{-2x}{2\sqrt{1-x^2-y^2}}\right)^2 + \left(\frac{-2y}{2\sqrt{1-x^2-y^2}}\right)^2} dA$$

$$= \sqrt{1 + \frac{x^2}{1-x^2-y^2} + \frac{y^2}{1-x^2-y^2}} dA$$

$$= \frac{1}{\sqrt{1-x^2-y^2}} dA \checkmark$$

$$P: \begin{cases} z = \sqrt{1-x^2-y^2}, \\ z \geq 0. \end{cases}$$



AREA:

$$\text{area} = \iint_S 1 dS$$

$$= \iint_R \frac{1}{\sqrt{1-x^2-y^2}} dA$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^1 \frac{1}{\sqrt{1-r^2}} r dr d\theta$$

$$= \int_0^{2\pi} 1 d\theta \cdot \int_0^1 \frac{r}{\sqrt{1-r^2}} dr$$

$$= [\theta]_0^{2\pi} \cdot [-\sqrt{1-r^2}]_0^1$$

$$= 2\pi \cdot (0 - (-1)) = 2\pi$$

MASS:

$$\text{mass} = \iint_S z dS$$

$$= \iint_R \frac{\sqrt{1-x^2-y^2}}{\sqrt{1-x^2-y^2}} dA$$

$$= \int_0^{2\pi} \int_{r=0}^1 1 \cdot r dr d\theta$$

$$= [\theta]_0^{2\pi} \cdot \left[\frac{r^2}{2}\right]_0^1$$

$$= 2\pi \cdot \left(\frac{1}{2}\right) = \pi$$

area =  $2\pi \checkmark$

m =  $\pi \checkmark$