

TW/AM 20753-242	TUTTOETS 8 / TUT TEST 8	2023
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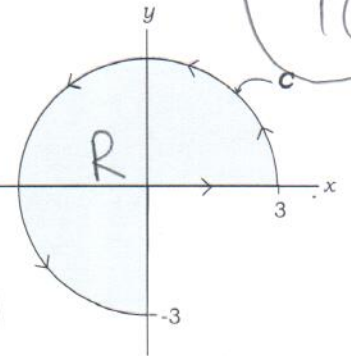
[9.12] Green's theorem

Bereken die integraal W hieronder. Gebruik Green'se stelling.

Calculate the integral W below. Use Green's theorem.

$$W = \oint_C \underbrace{y(x-y \sin x)}_P dx + \underbrace{(2y \cos x - x^2)}_Q dy$$

$$\begin{aligned} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} &= \frac{\partial}{\partial x} (2y \cos x - x^2) - \frac{\partial}{\partial y} (yx - y^2 \sin x) \\ &= -2y \sin x - 2x - (x - 2y \sin x) \\ &= -2y \sin x - 2x - x + 2y \sin x = -3x \end{aligned}$$



10
10

Using Green: $\oint_C P dx + Q dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$

$$W = \iint_R -3x dA = \int_{\theta=0}^{3\pi/2} \int_{r=0}^3 (-3r \cos \theta) r dr d\theta$$

$$= \int_0^{3\pi/2} \cos \theta d\theta \cdot \int_0^3 -3r^2 dr$$

$$= -\left[\sin \theta \right]_0^{3\pi/2} \left[\frac{3r^3}{3} \right]_0^3 = -[-1 - 0] \cdot \frac{3 \cdot 3^3}{3}$$

$$= 27 \checkmark$$

$$W = \boxed{27}$$