

TW/AM 20753-242	TUTTOETS 7 / TUT TEST 7	2023
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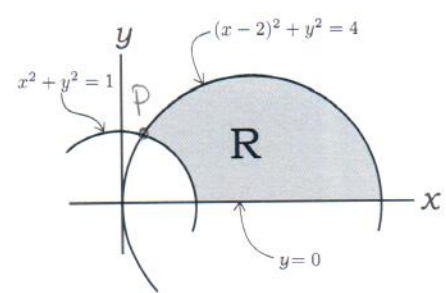
[9.11] Polar Coordinates - Double integrals

Toon eers aan dat die grens $(x-2)^2 + y^2 = 4$ in poolkoördinate gegee word deur $r = 4 \cos \theta$. Bereken dan T hieronder met poolkoördinate.

First show that the boundary $(x-2)^2 + y^2 = 4$ is given in polar coordinates as $r = 4 \cos \theta$. Then calculate T below, using polar coordinates.

$\frac{10}{10}$

$$\begin{aligned}
 T &= \int_{\theta=0}^{\arccos(\frac{1}{4})} \int_{r=1}^{4 \cos \theta} (r \sin \theta) r dr d\theta \\
 &= \int_{\theta=0}^{\arccos(\frac{1}{4})} \sin \theta \left[\frac{r^3}{3} \right]_1^{4 \cos \theta} d\theta \\
 &= \int_{\theta=0}^{\arccos(\frac{1}{4})} \left(\frac{64}{3} \cos^3 \theta \sin \theta - \frac{\sin \theta}{3} \right) d\theta \\
 &= - \left[\frac{64}{3} \frac{\cos^4 \theta}{4} \right]_0^{\arccos(\frac{1}{4})} + \left[\frac{\cos \theta}{3} \right]_0^{\arccos(\frac{1}{4})} \\
 &= - \frac{16}{3} \left[\left(\frac{1}{4} \right)^4 - 1 \right] + \frac{1}{3} \left[\frac{1}{4} - 1 \right] \\
 &= \frac{85}{16} - \frac{1}{4} = \frac{81}{16} \text{ or } 5.0625
 \end{aligned}$$



Boundary:

$$\begin{aligned}
 (x-2)^2 + y^2 &= x^2 - 4x + 4 + y^2 = 4 \\
 x^2 + y^2 - 4x &= 0 \\
 r^2 - 4r \cos \theta &= 0 \\
 r(r - 4 \cos \theta) &= 0 \quad \checkmark \\
 &\quad \uparrow \\
 &\quad r = 4 \cos \theta
 \end{aligned}$$

Point P:

$$\begin{aligned}
 r=1 \text{ and } r &= 4 \cos \theta \quad \checkmark \\
 1 &= 4 \cos \theta, \quad \theta = \arccos\left(\frac{1}{4}\right)
 \end{aligned}$$

$$T = \boxed{\frac{81}{16}} \quad 5.0625$$