

TW/AM 20753-242	TUTTOETS 5 / TUT TEST 5	2023
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Studentennummer / Student number:		

[9.8] Line integrals, [9.9] Path-independent Integrals

Bereken W . C bestaan uit twee stukke: 'n reguitlynstuk en 'n kwartsirkel.

Calculate W . C consists of two pieces: a straight line segment and quarter of a circle.

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}, \quad \mathbf{F} = -x \mathbf{i} + xy \mathbf{j},$$

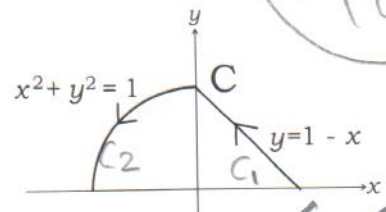
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$$W = \int_C \begin{bmatrix} -x \\ xy \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \int_C -x dx + xy dy$$

$$= \int_1^0 -t dt + \int_1^0 t(1-t)(-dt)$$

$$+ \int_{\pi/2}^{\pi} (-\cos t)(-\sin t) dt$$

$$+ (\cos t)(\sin t)(+\cos t dt)$$



$$C_1: \begin{cases} x = t \\ y = 1 - t \end{cases} \quad t \in [1, 0]$$

$$C_2: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad t \in [\pi/2, \pi]$$

$$= \int_1^0 (t^2 - 2t) dt + \int_{\pi/2}^{\pi} [(\cos t)(\sin t) + (\cos t)^2(\sin t)] dt$$

$$= \left[\frac{t^3}{3} - \frac{2t}{2} \right]_1^0 + \left[\frac{(\sin t)^2}{2} \right]_{\pi/2}^{\pi} + \left[\frac{(\cos t)^3}{3} (-1) \right]_{\pi/2}^{\pi}$$

$$= 0 - \left[\frac{1}{3} - 1 \right] + \frac{0 - 1^2}{2} - \frac{(-1)^3 - 0}{3}$$

$$= \frac{2}{3} - \frac{1}{2} + \frac{1}{3} = \frac{1}{2}$$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \boxed{\frac{1}{2}}$$

~~Daar is nog 'n vraag agterop.~~

There is another question overleaf.