$\frac{10}{10}$

TW/AM 20753-242	TUTTOETS 4 / *TUT TEST 4*	2023
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[9.6 Tangent planes, 9.7 Divergence and Curl]

1 Bereken ∇g (in terme van x, y en z). Beskou die oppervlak $g(x, y, z)$ =konstant, wat deur die punt $\mathbf{r}$ gaan. Vind die vergelyking van die raakvlak aan hierdie oppervlak. Skryf die vergelyking sodat daar geen breuke in die koëffisiënte voorkom nie.

Calculate ∇g (in terms of x, y and z). Consider the surface $g(x, y, z)$ =constant, going through the point $\mathbf{r}$. Find the equation of the tangent plane to this surface. Write the equations so that there are no fractions in the coefficients.

$$g(x, y, z) = xy - x^2 - z, \qquad \mathbf{r}_1 = \begin{bmatrix} -1/2 \\ 1 \\ -3/4 \end{bmatrix}$$

$$\underline{\nabla} g = \begin{bmatrix} y-2x \\ x \\ -1 \end{bmatrix} \qquad \underline{\nabla} g \Big|_{(-\frac{1}{2}, 1, -\frac{3}{4})} = \begin{bmatrix} 1-2(-\frac{1}{2}) \\ -\frac{1}{2} \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{1}{2} \\ -1 \end{bmatrix} \checkmark\checkmark$$

$$\underline{n} \cdot \underline{r} = \underline{n} \cdot \underline{r}_1$$

$$\underline{n} \cdot \underline{r}_1 = \begin{bmatrix} 2 \\ -\frac{1}{2} \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} \\ 1 \\ -\frac{3}{4} \end{bmatrix} = -1 - \frac{1}{2} + \frac{3}{4} = -\frac{3}{4}$$

Tangent plane: $\begin{bmatrix} 2 \\ -\frac{1}{2} \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{3}{4}$

$$2x - \tfrac{1}{2}y - z = -\frac{3}{4} \quad \text{or} \quad 8x - 2y - 4z = -3$$
$$\text{or} \quad -8x + 2y + 4z = 3$$

$\nabla g =$

$y-2x$
x
-1

✓

verg. van raakvlak:
eq. of tangent plane: $\boxed{-8x + 2y + 4z = 3}$ ✓

Daar is nog 'n vraag ~~twee vrae~~ agterop. *There is an ~~are two~~ other question~~s~~ overleaf.*

2 Bereken $\nabla \cdot \mathbf{F}$ sowel as $\nabla \times \mathbf{F}$ (albei in terme van x, y en z, en hou c 'n willekeurige konstante). Vind ook die waarde van c waarvoor $\nabla \times \mathbf{F} = \mathbf{0}$.

Calculate $\nabla \cdot \mathbf{F}$ as well as $\nabla \times \mathbf{F}$ (both in terms of x, y and z, and keep c an arbitrary constant). Also find the value of c for which $\nabla \times \mathbf{F} = \mathbf{0}$.

$$\mathbf{F}(x,y,z) = (cxze^y)\mathbf{i} + (zx^2e^y)\mathbf{j} + e^y(x^2 + cz - 2z)\mathbf{k}$$

$$\underline{\nabla} \cdot \underline{F} = \begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} \cdot \begin{bmatrix} cxze^y \\ zx^2e^y \\ e^y(x^2 + cz - 2z) \end{bmatrix} = cze^y + zx^2e^y + e^y(c-2)$$

$$= e^y(cz + zx^2 + c - 2)$$

$$\underline{\nabla} \times \underline{F} = \begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} \times \begin{bmatrix} cxze^y \\ zx^2e^y \\ e^yx^2 + e^ycz - 2e^yz \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{x^2e^y} + cze^y - 2e^yz - \cancel{x^2e^y} \\ cxe^y - 2xe^y \\ 2xze^y - cxze^y \end{bmatrix}$$

$$= \begin{bmatrix} (c-2)ze^y \\ (c-2)xe^y \\ -(c-2)xze^y \end{bmatrix}$$

$\nabla \cdot \mathbf{F} = \boxed{e^y(cz + zx^2 + c - 2)}$,

$\nabla \times \mathbf{F} = \begin{array}{|c|} \hline (c-2)ze^y \\ \hline (c-2)xe^y \\ \hline (c-2)(-xze^y) \\ \hline \end{array}$

$\nabla \times \mathbf{F} = \mathbf{0}$ as / if $c = \boxed{2}$