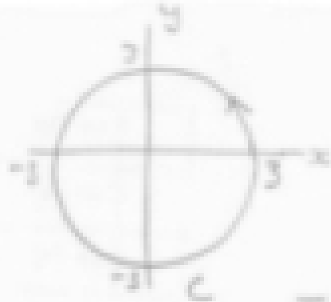
TUT 8: [9.12] 3, 7, 9, 13, 29.

[9.12] 03.

3.



$$\oint_C -y^2dx + x^2dy$$

$x = 3\cos t$

$dx = -3\sin t\,dt$

$y = 3\sin t$

$dy = 3\cos t\,dt$

$$= \int_{t=0}^{2\pi}\left[-(3\sin t)^2(-3\sin t) + (3\cos t)^2(3\cos t)\right]dt$$

$$= 27\int_{t=0}^{2\pi}(\sin^3 t + \cos^3 t)\,dt$$

$$= 27\int_{t=0}^{2\pi}\left[(1-\cos^2 t)\sin t + (1-\sin^2 t)\cos t\right]dt$$

$$= 27\left[\int_0^{2\pi}\sin t\,dt + \int_0^{2\pi}\cos^2 t\,(-\sin t)\,dt + \int_0^{2\pi}\cos t\,dt - \int_0^{2\pi}\sin^2 t\cos t\,dt\right]$$

$$= 27\left[0 + \left[\frac{\cos^3 t}{3}\right]_0^{2\pi} + 0 - \left[\frac{\sin^3 t}{3}\right]_0^{2\pi}\right]$$

$$= 27\left[0 + 0 + 0 - 0\right] = 0$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial x^2}{\partial x} - \frac{\partial(-y^2)}{\partial y} = 2x + 2y$$

$$\iint_R (2x+2y)\,dxdy = \int_{x=-3}^{3}\int_{y=-\sqrt{9-x^2}}^{\sqrt{9-x^2}}(2x+2y)\,dy\,dx$$

$$= \int_{x=-3}^{3}\left[2xy + \frac{2y^2}{2}\right]_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}dx$$

$$= \int_{-3}^{3} 2x\sqrt{9-x^2} + (9-x^2) - \left(-2x\sqrt{9-x^2} + (9-x^2)\right)dx$$

$$= \int_{-3}^{3} 4x\sqrt{9-x^2}\,dx = -2\int_{-3}^{3} -2x\sqrt{9-x^2}\,dx = -2\left[(9-x^2)^{\frac{3}{2}}\right]^3$$

[9.12] 07.

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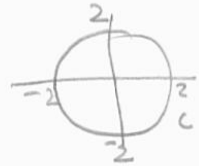
7. $P = x^4 - 2y^3, \quad Q = 2x^3 - y^4$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial}{\partial x}\left(2x^3 - y^4\right) - \frac{\partial}{\partial y}\left(x^4 - 2y^3\right) = 2\left(3x^2\right) + 2\left(3y^2\right)$$

$$= 6\left(x^2 + y^2\right)$$



$$\iint_R 6\left(x^2+y^2\right)\,dx\,dy = \int_{\theta=0}^{2\pi}\int_{r=0}^{2} 6r^2 \cdot r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi} d\theta \times 6\int_{r=0}^{2} r^3\,dr$$

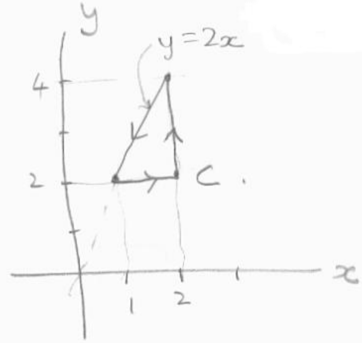
$$= \Big[\theta\Big]_0^{2\pi} \times 6\left[\frac{r^4}{4}\right]_0^2 = 2\pi \cdot 6 \cdot \left[\frac{16-0}{4}\right]$$

$$= 48\pi$$

[9.12] 09.

9. $P = 2xy, \quad Q = 3xy^2$

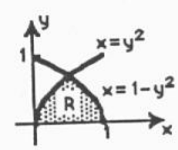
$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial}{\partial x}(3xy^2) - \frac{\partial}{\partial y}(2xy)$$
$$= 3y^2 - 2x$$



$$\oint_C (2xy\,dx + 3xy^2\,dy$$
$$= \int_{x=1}^{2}\int_{y=2}^{2x} (3y^2 - 2x)\,dy\,dx = \int_{x=1}^{2} \left[\frac{3y^3}{3} - 2xy\right]_2^{2x} dx$$
$$= \int_1^2 \left[(2x)^3 - 2x(2x) - (2^3 - 2x\cdot 2)\right] dx$$
$$= \int_1^2 (8x^3 - 4x^2 - 8 + 4x)\,dx = \left[\frac{8x^4}{4} - \frac{4x^3}{3} - 8x + \frac{4x^2}{2}\right]_1^2$$
$$= \frac{8\cdot 16}{4} - \frac{4\cdot 8}{3} - 8\cdot 2 + \frac{4\cdot 4}{2} - \left(\frac{8}{4} - \frac{4}{3} - 8 + \frac{4}{2}\right)$$
$$= \frac{56}{3} = 18.667$$

[9.12] 13.

13. $P = \frac{1}{3}y^3$, $P_y = y^2$, $Q = xy + xy^2$, $Q_x = y + y^2$



$$\oint_C \frac{1}{3}y^3\,dx + (xy + xy^2)\,dy = \iint_R y\,dA = \int_0^{1/\sqrt{2}}\int_{y^2}^{1-y^2} y\,dx\,dy$$
$$= \int_0^{1/\sqrt{2}} (xy)\Big|_{y^2}^{1-y^2} dy = \int_0^{1/\sqrt{2}} (y - y^3 - y^3)\,dy$$
$$= \left(\frac{1}{2}y^2 - \frac{1}{2}y^4\right)\Big|_0^{1/\sqrt{2}} = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

[9.12] 29.

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29. $P = x - y$, $Q = x + y$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 - (-1) = 2$$

$$W = \iint_R 2\,dA = \int_{\theta=0}^{\pi/2} \int_{r=1}^{2} 2\,r\,dr\,d\theta$$

$$= 2\left(\frac{\pi}{2}\right)\left[\frac{r^2}{2}\right]_1^2 = \pi\left[\frac{4}{2} - \frac{1}{2}\right] = \frac{3}{2}\pi$$