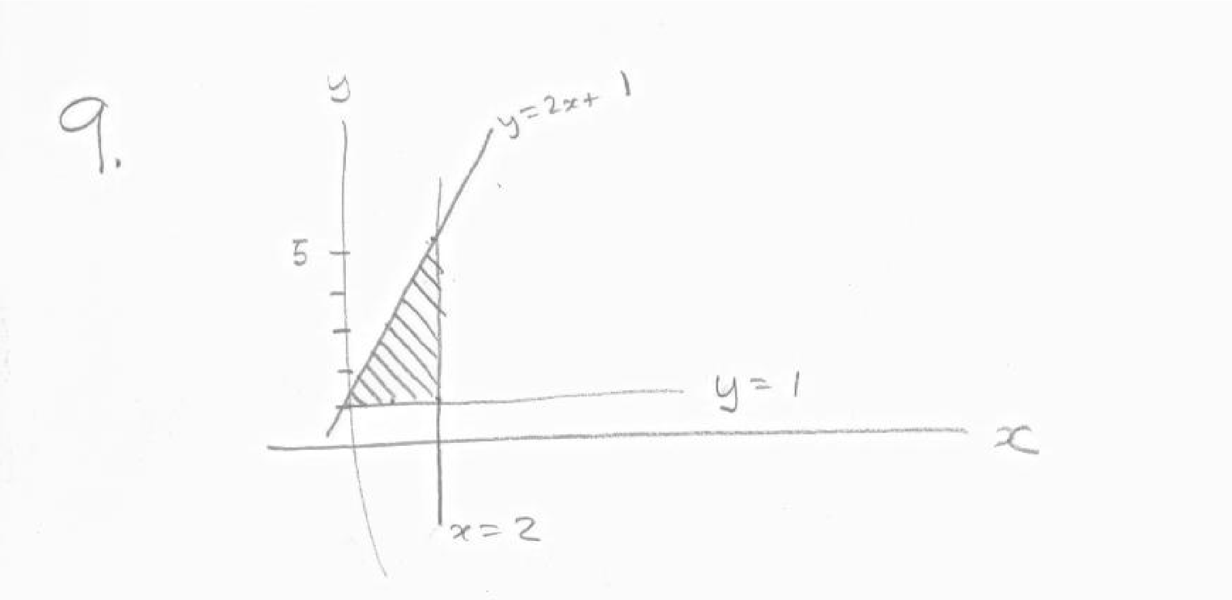
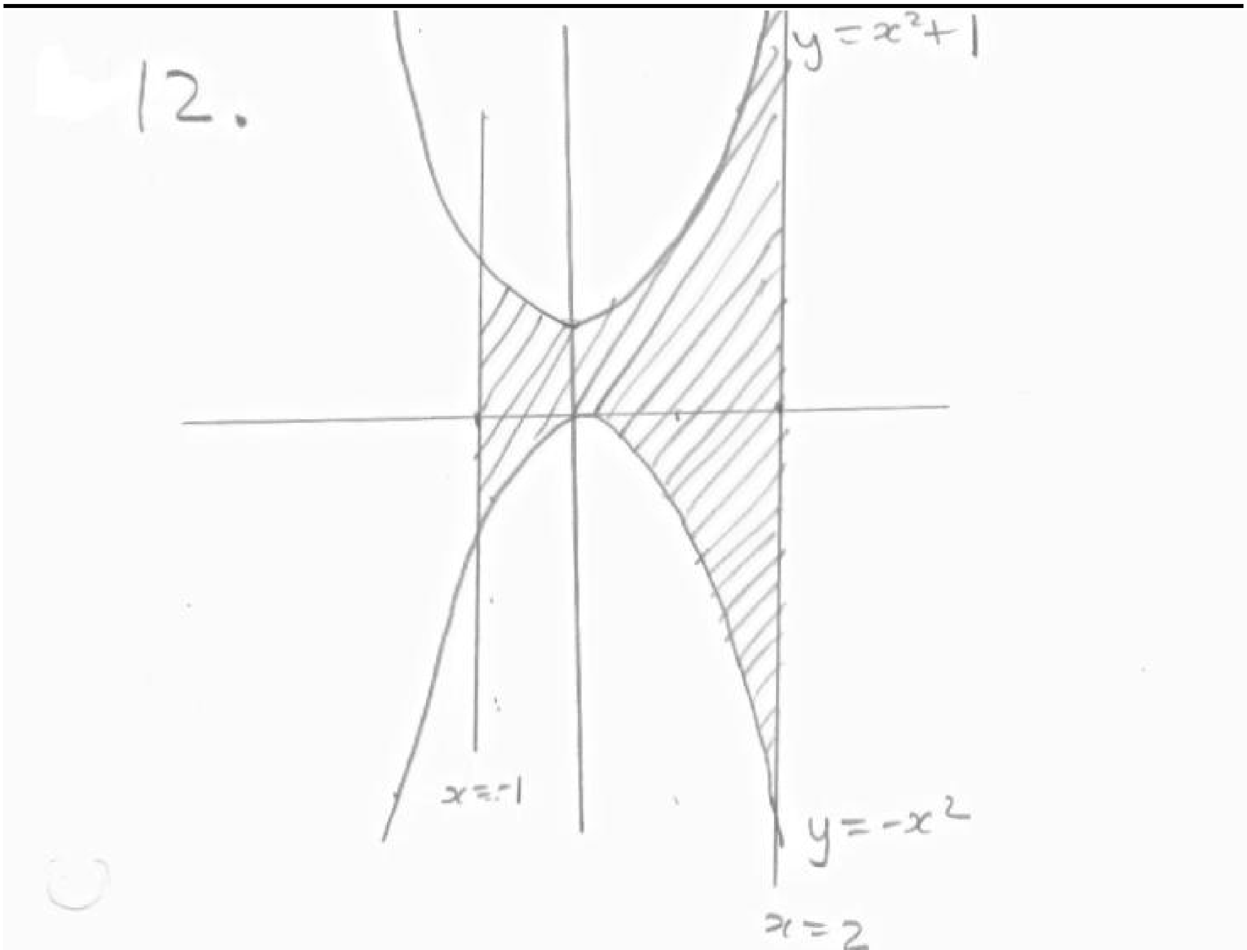
TUT 6: [9.10] 9, 12, 15, 21, 23, 27, 37, 49, 65.

[9.10] 09.

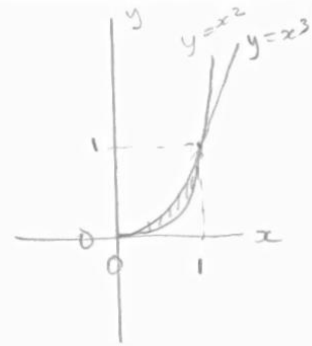


[9.10] 12.



[9.10] 15.

$$15.\ \int_{x=0}^{1}\int_{y=x^3}^{x^2}(2x+4y+1)\,dy\,dx$$



$$=\int_{x=0}^{1}\Big[2xy+2y^2+y\Big]_{x^3}^{x^2}dx$$

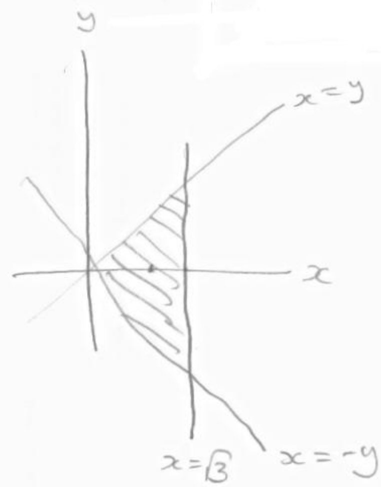
$$=\int^{1}\Big[2x^3+\cancel{2x^4}+x^2-\cancel{2x^4}-2x^6-x^3\Big]dx$$

$$=\left[\frac{2x^4}{4}+\frac{x^3}{3}-\frac{2x^7}{7}-\frac{x^4}{4}\right]_0^1=\frac{1}{2}+\frac{1}{3}-\frac{2}{7}-\frac{1}{4}=\frac{25}{84}=0.298$$

[9.10] 21.

21.

$$\int_{x=0}^{\sqrt{3}}\int_{y=-x}^{x}\sqrt{x^2+1}\,dy\,dx$$



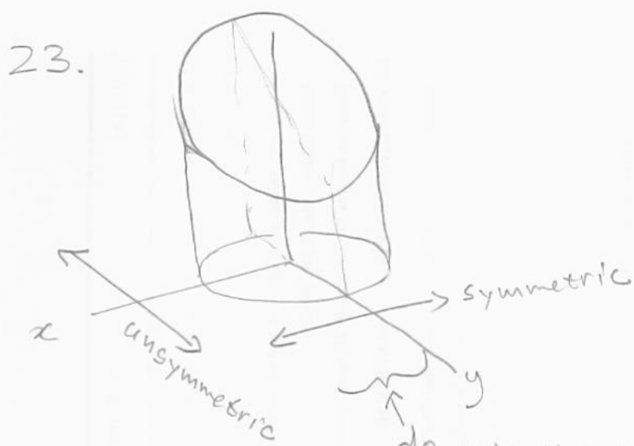
$$=\int_{x=0}^{\sqrt{3}}\Big[y\sqrt{x^2+1}\Big]_{-x}^{x}\,dx$$

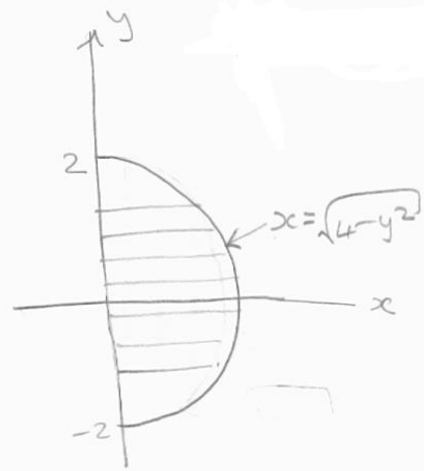
$$=\int_0^{\sqrt{3}}\left(x\sqrt{x^2+1}-(-x)\sqrt{x^2+1}\right)dx=\int_0^{\sqrt{3}}2x\sqrt{x^2+1}\,dx$$

$$=\left[\frac{(x^2+1)^{\frac{3}{2}}}{3/2}\right]_0^{\sqrt{3}}=\frac{2}{3}\Big[(3+1)^{3/2}-(0+1)^{3/2}\Big]=\frac{2}{3}(8-1)=\frac{14}{3}$$

[9.10] 23.

23.





do only this half, and multiply by 2.

$$\text{Volume} = 2\int_{y=-2}^{2}\int_{x=0}^{\sqrt{4-y^2}} (4-y)\,dx\,dy \longrightarrow (c)$$

$$= 2\int_{y=-2}^{2} \Big[(4-y)x\Big]_0^{\sqrt{4-y^2}}\,dy$$

$$= 2\int_{y=-2}^{2} 4\sqrt{4-y^2}\,dy - 2\int_{y=-2}^{2} y\sqrt{4-y^2}\,dy$$

$$= 8\underbrace{\int_{y=-2}^{2} \sqrt{4-y^2}\,dy}_{\text{Area of semi circle with radius 2.}} - 2\underbrace{\int_{y=-2}^{2} y\sqrt{4-y^2}\,dy}_{\text{Integral of odd function over symmetric interval} = 0.}$$

$$= 8\,\tfrac{1}{2}\left(\pi 2^2\right) - 0 = 16\pi.$$

[9.10] 27.

27.

$x=0, y=0:$

$0-0+2z=4$

$\Rightarrow z=2$

$x=2, y=0:$

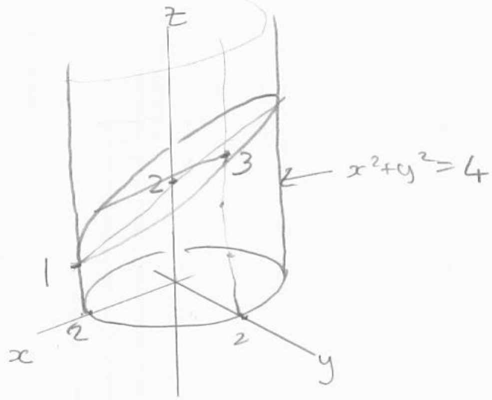
$2-0+2z=4$

$\Rightarrow z=1$

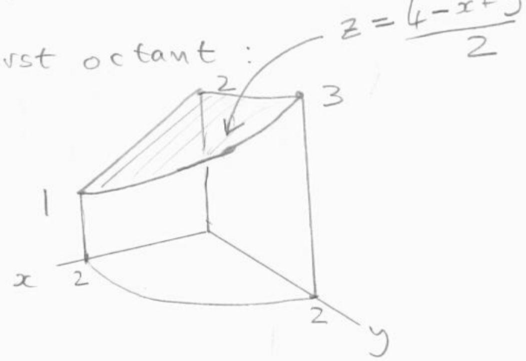
$x=0, y=2:$

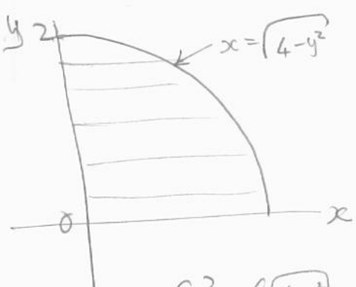
$0-2+2z=4$

$\Rightarrow z=3$



First octant:





$$\text{Volume} = \int_{y=0}^{2}\int_{x=0}^{\sqrt{4-y^2}} \left(2-\tfrac{1}{2}x+\tfrac{1}{2}y\right)dx\,dy$$

$$= \int_0^2 \left[2x - \frac{x^2}{4} + \frac{1}{2}xy\right]_0^{\sqrt{4-y^2}} dy$$

$$= \int_0^2 \left[2\sqrt{4-y^2} - \frac{4-y^2}{4} + \frac{1}{2}y\sqrt{4-y^2} - 0\right]dy$$

$$= 2\underbrace{\int_0^2 \sqrt{4-y^2}\,dy}_{\text{Area of quarter circle of radius 2.}} + \left[-y + \frac{1}{4}\frac{y^3}{3} + \frac{1}{4}\left(-\frac{(4-y^2)^{3/2}}{3/2}\right)\right]_0^2$$

$$= 2\left(\pi 2^2\right)\frac{1}{4} + \left[-2 + \frac{8}{12} + 0 - \left(-0 + 0 - \frac{2}{12}(4)^{3/2}\right)\right]$$

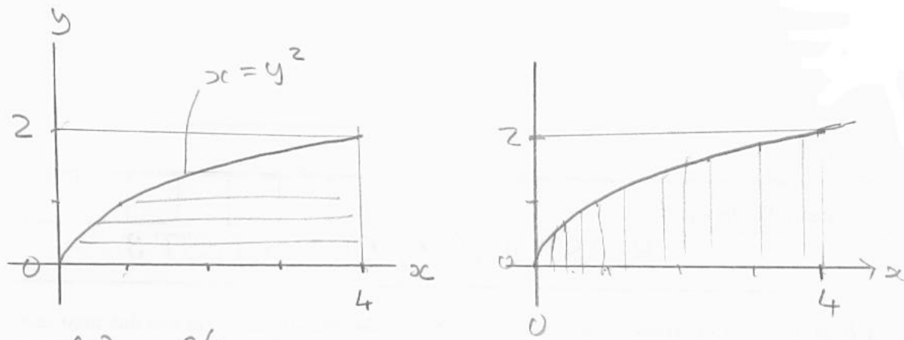
$$= 2\pi - 2 + \frac{2}{3} + \frac{16}{12}$$

$$= 2\pi + 0$$

$$= 2\pi$$

[9.10] 37.

37.



$$I = \int_{y=0}^{2} \int_{x=y^2}^{4} \cos x^{3/2} \, dx \, dy$$

$$= \int_{x=0}^{4} \int_{y=0}^{\sqrt{x}} \cos x^{3/2} \, dy \, dx$$

$$= \int_{x=0}^{4} \left[y \cos x^{3/2} \right]_0^{\sqrt{x}} dx$$

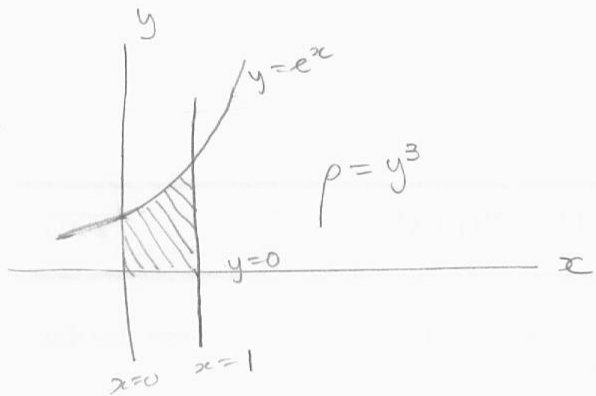
$$= \int_0^4 x^{\frac{1}{2}} \cos x^{3/2} dx = \frac{2}{3} \int_0^4 \frac{3}{2} x^{\frac{1}{2}} \cos x^{3/2} dx$$

$$= \frac{2}{3} \left[\sin x^{3/2} \right]_0^4 = \frac{2}{3} \sin(8) - 0$$

$$= \frac{2}{3} \sin(8) = 0.6596.$$

[9.10] 49.

49.



$$M = \int_{x=0}^{1}\int_{y=0}^{e^x} y^3\,dy\,dx = \int_0^1 \left[\frac{y^4}{4}\right]_0^{e^x} dx = \int_0^1 \left(\frac{e^{4x}}{4} - 0\right) dx$$

$$= \frac{1}{4}\left[\frac{e^{4x}}{4}\right]_0^1 = \frac{1}{16}\left[e^4 - 1\right] = 3.35$$

$$M\bar{y} = \int_{x=0}^{1}\int_{y=0}^{e^x} y \cdot y^3\,dy\,dx = \int_{x=0}^{1}\left[\frac{y^5}{5}\right]_0^{e^x} dx$$

$$= \int_0^1 \frac{e^{5x} - 0}{5}\,dx = \frac{1}{5}\left[\frac{e^{5x}}{5}\right]_0^1 = \frac{1}{25}\left[e^5 - 1\right]$$

$$\bar{y} = \frac{\frac{1}{25}(e^5-1)}{\frac{1}{16}(e^4-1)} = \frac{16}{25}\cdot\frac{(e^5-1)}{(e^4-1)}$$

$$M\bar{x} = \int_{x=0}^{1}\int_{y=0}^{e^x} x y^3\,dy\,dx = \int_{x=0}^{1}\left[\frac{x y^4}{4}\right]_0^{e^x} dx$$

$$= \int_0^1 \frac{1}{4}\left(x e^{4x}\right) dx = \frac{1}{4}\left[\frac{x e^{4x}}{4}\right]_0^1 - \frac{1}{4}\int_0^1 \frac{e^{4x}}{4}\,dx$$

$$= \frac{1}{16}\left[e^4 - 0\right] - \frac{1}{4}\left[\frac{e^{4x}}{16}\right]_0^1 = \frac{e^4}{16} - \frac{1}{64}\left[e^4 - 1\right]$$

$= \frac{1}{64}(3e^4 + 1)$, therefore $\bar{x} = \frac{16}{64}\frac{(3e^4+1)}{(e^4-1)} =$

$$\boxed{\frac{1}{4}\frac{(3e^4+1)}{(e^4-1)} = 0.769}$$

[9.10] 65.

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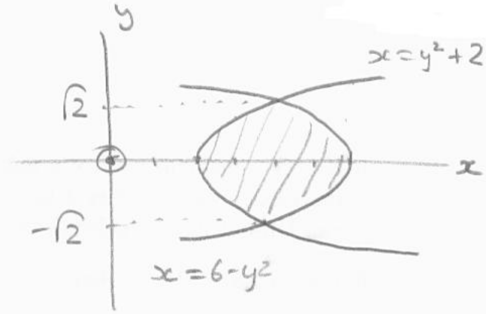
65 $\rho = \dfrac{k}{r^2} = \dfrac{k}{x^2+y^2}$



$$I_x = k\int_{y=-\sqrt{2}}^{\sqrt{2}}\int_{x=y^2+2}^{6-y^2} \frac{1\times y^2}{x^2+y^2}\,dx\,dy$$

$$I_y = k\int_{y=-\sqrt{2}}^{\sqrt{2}}\int_{x=y^2+2}^{6-y^2} \frac{x^2}{x^2+y^2}\,dx\,dy$$

$$I_z = I_x + I_y = k\int_{y=-\sqrt{2}}^{\sqrt{2}}\int_{x=y^2+2}^{6-y^2} 1\,dx\,dy$$

$$\frac{I_z}{k} = \int_{y=-\sqrt{2}}^{\sqrt{2}} \Big[\,x\,\Big]_{y^2+2}^{6-y^2} dy = \int_{-\sqrt{2}}^{\sqrt{2}} \Big[(6-y^2)-(y^2+2)\Big]\,dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} (-2y^2+4)\,dy = \left[-\frac{2y^3}{3} + 4y\right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \frac{-2\cdot 2\cdot\sqrt{2}}{3} + 4\sqrt{2} - \left(\frac{+2\cdot 2\sqrt{2}}{3} - 4\sqrt{2}\right)$$

$$= \sqrt{2}\left(-\frac{4}{3} - \frac{4}{3} + 4 + 4\right) = \frac{16\sqrt{2}}{3}$$

$$I_z = \frac{16\sqrt{2}}{3}\,k$$