TUT 1: [7.5] 1, 4, 12, 13, 22, 23, 26, 27, 41, 45, 54, 59, 67, 75.

[7.5] 01.

1. $\underline{a} = \underline{r}_2 - \underline{r}_1 = \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$

Vector eq:

$\underline{r} = \underline{r}_1 + t\underline{a} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$

[7.5] 04.

4. $\underline{a} = \underline{r}_2 - \underline{r}_1 = \begin{bmatrix} 10 \\ 2 \\ -10 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ -15 \end{bmatrix}, \quad \underline{r} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix} + t\begin{bmatrix} 5 \\ 5 \\ -15 \end{bmatrix}$

[7.5] 12.

12. $\underline{a} = \underline{r}_2 - \underline{r}_1 = \begin{bmatrix} -3 \\ 7 \\ 9 \end{bmatrix} - \begin{bmatrix} 4 \\ -8 \\ -1 \end{bmatrix} = \begin{bmatrix} -7 \\ 15 \\ 10 \end{bmatrix}, \quad \underline{r} = \begin{bmatrix} 4 \\ -8 \\ -1 \end{bmatrix} + t\begin{bmatrix} -7 \\ 15 \\ 10 \end{bmatrix}$

$$\left.\begin{aligned} x &= 4 - 7t \\ y &= -8 + 15t \\ z &= -1 + 10t \end{aligned}\right\} \text{parametric eq's.}$$

[7.5] 13.

13. $\underline{a} = \begin{bmatrix} 9 \\ 10 \\ 7 \end{bmatrix} \qquad \underline{r} = \begin{bmatrix} 1 \\ 4 \\ -9 \end{bmatrix} + t\begin{bmatrix} 9 \\ 10 \\ 7 \end{bmatrix}$

$x = 1 + 9t, \quad t = \dfrac{x-1}{9}$

$y = 4 + 10t, \quad t = \dfrac{y-4}{10}$

$z = -9 + 7t, \quad t = \dfrac{z+9}{7}$

$$\frac{x-1}{9} = \frac{y-4}{10} = \frac{z+9}{7} \Big\} \longrightarrow \text{Symmetric eq.}$$

[7.5] 22.

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22. $\underline{r} = \begin{bmatrix} 0 \\ -3 \\ 10 \end{bmatrix} + t \begin{bmatrix} 12 \\ -5 \\ -6 \end{bmatrix}$

$\left.\begin{aligned} x &= 12t \\ y &= -3-5t \\ z &= 10-6t \end{aligned}\right\}$ parametric eq.'s

$$\underbrace{\frac{x}{12} = \frac{y+3}{-5} = \frac{z-10}{-6}}_{\text{symmetric eq.'s}}$$

[7.5] 23.

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23. $\underline{a} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$ $\underline{r} = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$ $\left.\begin{aligned} x &= 6+2t \\ y &= 4-3t \\ z &= -2+6t \end{aligned}\right\}$ Parametric eq.

[7.5] 26.

26.(a) $\underline{r}_1 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$, $\underline{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\underline{r} = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

parall. to y-axis

$\left.\begin{aligned} x &= 1 \\ y &= 2+t \\ z &= 8 \end{aligned}\right\}$ parametric eq.'s

(b) $\underline{r}_1 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$, $\underline{a} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\underline{r} = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

perp. to xy-plane $\equiv$ paral. to z-axis

$\left.\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 8+t \end{aligned}\right\}$ parametric eq.'s

[7.5] 27.

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27. $\underline{r} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} + t \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}$

$\underline{r}(2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; i.e. the line goes through the origin.

Therefore $\underline{r} = t \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}$ is an equivalent version.

But $\underline{a}$ may be any length ($\neq 0$), therefore replace $\underline{a}$ by $\frac{1}{-3}\underline{a}$.

$$\underline{r} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t.$$

[7.5] 41.

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41. $\underline{n} = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}$, $\underline{r}_1 = \begin{bmatrix} 6 \\ 10 \\ -7 \end{bmatrix}$

$$\underline{n} \cdot \underline{r}_1 = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 10 \\ -7 \end{bmatrix} = -30 + 0 - 21 = -51$$

$$-5x + 3z = -51$$

or

$$5x - 3z = 51$$

[7.5] 45.

45. $\underline{a} = \underline{r}_2 - \underline{r}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$

$\underline{b} = \underline{r}_3 - \underline{r}_1 = \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \\ 2 \end{bmatrix}$

$\underline{r} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} + s \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} + t \begin{bmatrix} -4 \\ -6 \\ 2 \end{bmatrix}$ Not asked, but here is the vector equation

$\underline{n} = \underline{a} \times \underline{b} = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \times \begin{bmatrix} -4 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} (-2)(2) - (-1)(-6) \\ (-1)(-4) - (-1)(2) \\ (-1)(-6) - (-2)(-4) \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ -2 \end{bmatrix}$

$\underline{n} \cdot \underline{r}_1 = \begin{bmatrix} -10 \\ 6 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = -4$

Equation

$$-10x + 6y - 2z = -4$$

May be simplified by dividing by -2

$$5x - 3y + z = 2$$

[7.5] 54.

54. plane is perpendicular to $\underline{n} = y\text{-axis} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\underline{r}_1 = \begin{bmatrix} -7 \\ -5 \\ 18 \end{bmatrix}$ $\quad \underline{n} \cdot \underline{r}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -7 \\ -5 \\ 18 \end{bmatrix} = -5$

$\underline{n} \cdot \underline{r} = -5$ i.e. $\underline{y = -5}$

[7.5] 59.

59. $\underline{r}_1 = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$ $\underline{n} = \begin{bmatrix} -3 \\ 1 \\ -\frac{1}{2} \end{bmatrix}$ $\underline{n} \cdot \underline{r}_1 = \begin{bmatrix} -3 \\ 1 \\ -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = -6$

$-3x + y - \frac{1}{2}z = -6$

May be simplified by multiplying by -2:

$6x - 2y + z = 12$

[7.5] 67.

67. $4x - 2y - z = 1$ Let $z = t$

$x + y + 2z = 1$

$4x - 2y = 1 + t$ In matrix form:

$x + y = 1 - 2t$ $\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1+t \\ 1-2t \end{bmatrix}$

Solution:

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1+t \\ 1-2t \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 1+t+2-4t \\ -1-t+4-8t \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 3-3t \\ 3-9t \end{bmatrix}$

$\left.\begin{aligned} x &= \tfrac{1}{2} - \tfrac{1}{2}t \\ y &= \tfrac{1}{2} - \tfrac{3}{2}t \\ z &= t \end{aligned}\right\}$ Parametric eq.

[7.5] 75.

75. line: $\underline{r} = \begin{bmatrix}4\\0\\1\end{bmatrix} + t\begin{bmatrix}3\\-1\\5\end{bmatrix}$ other plane: $\begin{bmatrix}1\\1\\1\end{bmatrix}\cdot\underline{r} = 7$

The plane must be perpendicular to the other plane, therefore $\begin{bmatrix}1\\1\\1\end{bmatrix}$ must lie in the plane.

The plane is therefore

$$\underline{r} = \begin{bmatrix}4\\0\\1\end{bmatrix} + t\begin{bmatrix}3\\-1\\5\end{bmatrix} + s\begin{bmatrix}1\\1\\1\end{bmatrix} = \underline{r}_1 + t\underline{a} + s\underline{b}$$

$$\underline{n} = \underline{a}\times\underline{b} = \begin{bmatrix}3\\-1\\5\end{bmatrix}\times\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}-6\\2\\4\end{bmatrix} \qquad \underline{n}\cdot\underline{r}_1 = \begin{bmatrix}-6\\2\\4\end{bmatrix}\cdot\begin{bmatrix}4\\0\\1\end{bmatrix} = -20$$

Eq. of the plane $-6x + 2y + 4z = -20$

Simplified: $3x - y - 2z = 10$