20753-242 2023

PREPARATION FOR ASSESSMENT A2

Format:

The duration of Assessment A2 is 120 minutes and it consists of

Section A: consists of multiple choice questions (five choices), it is filled in on an additional answer sheet,

Section B: consists of longer questions and is answered on the question paper.

When filling in the multiple choice answer sheet of Section A, colour in the small ellipse of your choice in either pen or pencil. Do not use heat erasable pens (since the scanner causes the marks to disappear). If you have made a mistake, erase well, or draw a clear cross through the incorrect choice, and fill in the correct one.

Formulae:

Only the following formulae are given in the test:

Green:

$$\oint_C P dx + Q dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Stokes:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS$$

Gauss:

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_D (\nabla \cdot \mathbf{F}) \, dV$$

Cylindrical coordinates:

$$x = r\cos\theta \qquad dV = r \, dz \, dr \, d\theta$$
$$y = r\sin\theta$$
$$z = z$$

Spherical coordinates:

$$x = \rho\cos\theta\sin\phi \qquad dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$
$$y = \rho\sin\theta\sin\phi$$
$$z = \rho\cos\phi$$

Other relevant formulae must be known by heart.

Contents:

Assessment A2 mainly covers the work done during the last term, although the concepts and skills you have learnt in the third term will still be tested both directly and indirectly (for example you must still be able to calculate grad, div, and curl, do line integrals, draw vector fields, etc.).

More precisely, Assessment A2 covers the following sections in ZILL: 9.11, 9.12, 9.13, 9.14, 9.15, 9.16.

For this assessment you must be able to do the following:

9.11
- Must be able to convert Cartesian coordinates to polar coordinates and to convert polar coordinates back to Cartesian coordinates.
- Must know the area element in polar coordinates ($dA = r dr d\theta$).
- Must be able to calculate double integrals in polar coordinates and must be able to calculate areas, volumes, masses, and mass centres.

9.12
- Must know and be able to prove Green's theorem for a simple region that may be con-

sidered to be simultaneously of type I and type II.

- Must be able to calculate both sides of Green's theorem (and hence verify the theorem). Line integrals must be parameterized correctly. If required, surface integrals must also be calculated in polar coordinates.

— *<u>Omit:</u> Regions containing holes as well as regions where the function is not sufficiently differentiable (e.g. Examples 4, 5, and 6.)*

9.13

- Must know the expression for the surface element ($dS = \left(\sqrt{1 + (f_x)^2 + (f_y)^2}\right) dA$). (*Not necessary to be able to derive it.*).
- Must be able to calculate the surface area of a given surface.
- Must be able to calculate surface integrals of the form $\iint_S G(x, y, z)dS$.
- Must be able to calculate the mass of a curved surface that has variable density.
- Must be able to calculate the flux of a given vector field $\mathbf{F}$ through a given surface S ($\iint_S \mathbf{F} \cdot \mathbf{n} dS$.)

9.14

- Must know Stokes' theorem. (*Not necessary to be able to prove it.*).
- Must be able to calculate both sides of Stokes' theorem (and hence verify the theorem). Line integrals must be parameterized correctly. For surface integrals only projection onto the xy-plane will be required. (*Therefore, leave out projection onto the xz-, or yz-planes.*)

9.15

- Must be able to calculate triple integrals where either (a) the boundary is given in a figure, or (b) the boundary is given by means of equations.
- Must be able to interchange the order of integration correctly where necessary or when asked.
- Must be able to calculate the volume of bodies from $V = \iiint_D dV$.
- Must be able to calculate masses and coordinates of mass centers of bodies with variable density using: $m = \iiint_D \rho(x, y, z)dV$, $m\bar{x} = \iiint_D x\rho(x, y, z)dV$, $m\bar{y} = \iiint_D y\rho(x, y, z)dV$, $m\bar{z} = \iiint_D z\rho(x, y, z)dV$.

— *<u>Omit</u>: Moments of inertia that require triple integration.*

- Must be able to convert cylindrical coordinates to Cartesian and back.
- Must know the expression of the volume element ($dV = r dr d\theta dz$) of cylindrical coordinate and be able to use it to calculate triple integrals in cylindrical coordinates.
- Must be able to convert spherical coordinates to Cartesian and back.
- Must know the expression of the volume element ($dV = r^2 \sin\phi dr d\phi d\theta$) of spherical coordinates and be able to use it to calculate triple integrals in spherical coordinates.

9.16

- Must know the divergence theorem (Gauss' theorem). (*Not necessary to be able to prove it.*)
- Must be able to calculate both sides of the divergence theorem (and hence verify the theorem).
- Must be able to use the divergence theorem to calculate the flux out of a body in a simpler way.

General

Must still be able to draw vector fields if given the expression or vice versa. Must be able to determine if a given expression involving ∇ is a vector or a scalar (or not well defined).

Must know the following terms (and be able to express in symbols where appropriate, e.g. '$\mathbf{F}$ is incompressible' means $\nabla \cdot \mathbf{F} = 0$): 'vector field', 'tangent line', 'tangent plane', 'gradient', 'rotation', 'curl', 'divergence', 'irrotational', 'incompressible', 'exact differential', 'path independent', 'normal on a surface', 'arc length', 'surface element', 'circulation around a closed curve', 'volume element', 'closed path', 'flux through a surface', 'flux out of a volume', 'conservative field', 'work done'.
