

SECTION A

1. A: $\underline{r} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ B: $x = t \quad \left. \begin{array}{l} x = t \\ y - 1 = -2t \\ z - 1 = t \end{array} \right\} \begin{array}{l} x = t \\ y = 1 - 2t \\ z = 1 + t \end{array}$

B: $\underline{r} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

(a) $1+t=1 \Rightarrow t=0$
 $-1+t=2 \Rightarrow t=3$ No

(b) $1+t=3 \quad t=2$
 $-1+t=1 \quad t=2$ Yes
 $t=2 \quad t=2$

→ (b) 2

2. $\underline{w} = \underline{a} \times \underline{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 0-1 \\ -2-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$ Use \underline{r}_1

$\underline{w} \cdot \underline{r}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = 1+1-6 = -4$

$x - y - 3z = -4$ or $-x + y + 3z = 4$

→ (d) 4

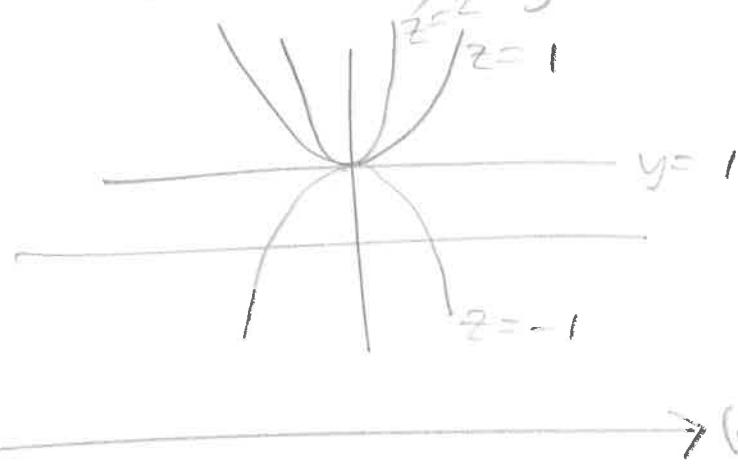
3. D: $\underline{r} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \quad \begin{array}{l} x = 1+t \\ y = 1-t \\ z = 1-3t \end{array}$

or $\underline{r} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \quad \begin{array}{l} x = 1-t \\ y = 1+t \\ z = 1+3t \end{array}$

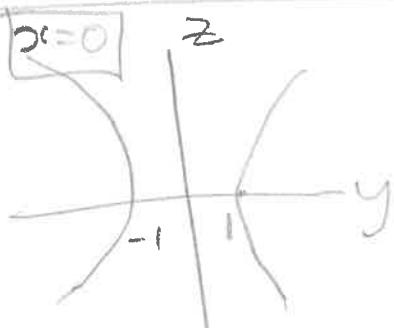
→ (b) 2

2

$$4. \quad z = \frac{y-1}{x^2}, \quad y-1 = zx^2, \quad y = 1 + zx^2$$



5.



$$(a) \quad y^2 + z^2 = 1, \quad \text{No}$$

$$(b) \quad -y + z^2 = 0, \quad \text{No}$$

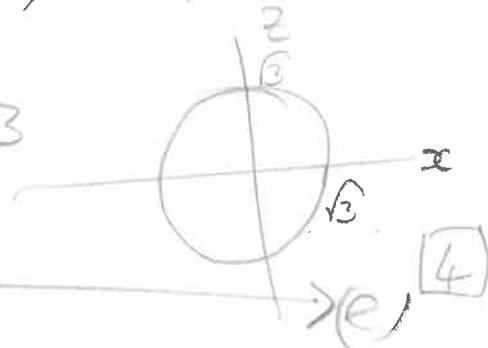
$$(c) \quad y + z^2 = 0, \quad \text{No}$$

$$(d) \quad -y^2 + z = 1, \quad \text{No}$$

$$(e) \quad y^2 - z^2 = 1, \quad \text{Yes, possibly.}$$

$y=2$ Check (e):

$$4 - x^2 - z^2 = 1, \quad x^2 + z^2 = 3$$



$$6. \quad 0 = z - 3 + x^2 + y^2 = G$$

$$\nabla G = \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix} \quad \text{point} = \begin{bmatrix} 2 \\ 1 \\ 3 - 1^2 - 2^2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\nabla G \begin{cases} x=2 \\ y=1 \end{cases} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = \underline{u}$$

$$\underline{v}_1 \cdot \underline{v}_2 = \underline{v}_1 \cdot \underline{v}_1 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = 8 + 2 - 2 = 8 \quad 4$$

$$4x + 2y + z = 8$$

(c)

3

$$7. \underline{u}_{\text{normalized}} = \frac{1}{13} \begin{bmatrix} 5 \\ -12 \end{bmatrix}$$

$$D_u f \Big|_{\substack{x=\frac{1}{3} \\ y=1}} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \Big|_{\substack{x=\frac{1}{3} \\ y=1}} \cdot \frac{1}{13} \begin{bmatrix} 5 \\ -12 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 9x^2 + 6y^2 \\ 12xy \end{bmatrix} \Big|_{\substack{x=\frac{1}{3} \\ y=1}} \cdot \begin{bmatrix} 5 \\ -12 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 1+6 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -12 \end{bmatrix}$$

$$= \frac{1}{13} (25 - 48) = \frac{-13}{13} = -1$$

4

→ (e)

$$8. \nabla \cdot ((\underline{a}, \underline{r}) \underline{a}) = \nabla \cdot (\underline{a}_1 x + \underline{a}_2 y + \underline{a}_3 z) \begin{pmatrix} \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_3 \end{pmatrix}$$

$$= \nabla \cdot \begin{bmatrix} \underline{a}_1^2 x + \underline{a}_1 \underline{a}_2 y + \underline{a}_1 \underline{a}_3 z \\ \underline{a}_1 \underline{a}_2 x + \underline{a}_2^2 y + \underline{a}_2 \underline{a}_3 z \\ \underline{a}_1 \underline{a}_3 x + \underline{a}_2 \underline{a}_3 y + \underline{a}_3^2 z \end{bmatrix}$$

$$= \underline{a}_1^2 + \underline{a}_2^2 + \underline{a}_3^2 = \underline{a} \cdot \underline{a}$$

2

→ (b)

$$9. f \cdot \underline{v} = \begin{bmatrix} z \\ x \\ y \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xz - y^2 \\ xy - z^2 \\ yz - x^2 \end{bmatrix}$$

$$\nabla \cdot (f \cdot \underline{v}) = z + x + y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

2

$$= \underline{e} \cdot \underline{r}$$

→ (b)

4

$$10. \nabla \times (\rho \mathbf{F})$$

$$= \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} x^2 - y^2 \\ xy - z^2 \\ zy - x^2 \end{bmatrix} = \begin{bmatrix} z - (-2z) \\ x - (-2x) \\ y - (-2y) \end{bmatrix} = 3 \begin{bmatrix} z \\ x \\ y \end{bmatrix} = 3 \mathbf{f}$$

2 → (e)

$$11. (a) v \quad (b) v \quad (c) s \quad (d) v \quad (e) \text{undefined}$$

2 → (c)

$$12. (a) s \quad (b) s \quad (c) s \quad (d) v \quad (e) s$$

2 → (c)

$$13. \nabla \cdot \mathbf{F} = 4zy^2 + x + (-4zy^2) = x$$

2 → (e)

$$14.$$

$$\nabla \times \mathbf{J} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} 2y^2 \\ 1+2xy \\ 1-x \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -(-1) \\ 2y - 4y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2y \end{bmatrix}$$

2 → (b)

15.

15. $y = 0$

5

(a) $\underline{E} = \underline{x^2}$

(b) $\underline{E} = \underline{x^2} + \underline{x^2}$

(c) $\underline{E} = \underline{0}$

(d) $\underline{E} = -\underline{x^2}$

(e) $\underline{E} = \underline{0}$

$x = 0$

Check (c) and (e)

(c) $\underline{E} = \underline{y^2}$ [possibly]

(e) $\underline{E} = \underline{y^2}$, No.

$\underline{x=1}, \underline{y=1}$

Check (c)

$\underline{E} = \underline{i} + \underline{j}$

$\underline{bc=-1}, \underline{y=1}$

$\underline{E} = -\underline{i} + \underline{j}$

$\rightarrow (c)$

4

16. $\underline{E} = \begin{pmatrix} y^3 \\ 3xy^2 \end{pmatrix}$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3y^2 - 3y^2 = 0$$

Do path-independently

$$= 1 - (-1)$$

$$\phi = \int y^3 dx$$

$$= xy^3 + \dots$$

$$\phi = xy^3$$

$$\left. \begin{aligned} \phi &= \int 3xy^2 dy \\ &= \frac{3xy^3}{3} + \dots \end{aligned} \right\}$$

$$= 2 \quad \rightarrow (b)$$

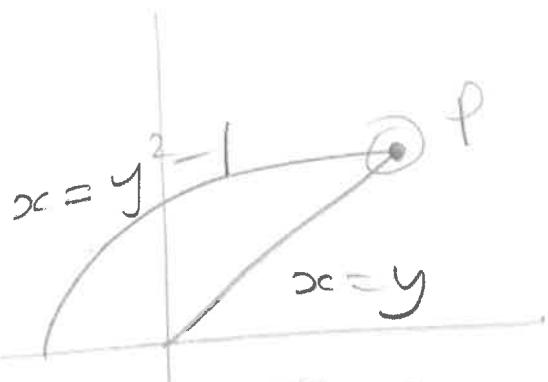
$$W = \int_{(-1,1)}^{(1,1)} d\phi = \begin{bmatrix} xy^3 \end{bmatrix}_{(-1,1)}^{(1,1)}$$

6

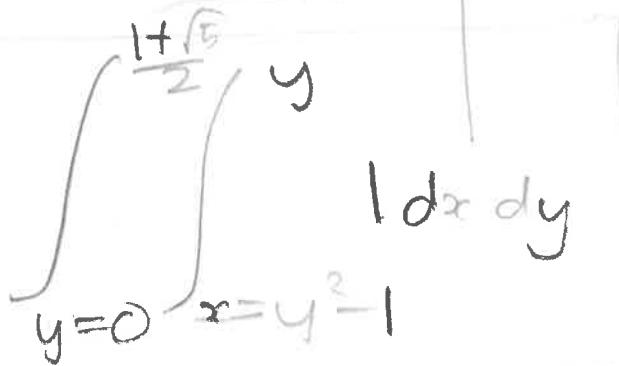
17. Type 2:

$$y^2 = x + 1$$

$$y = \sqrt{1+x}$$



$$\text{Area} =$$



P:

$$y = y^2 - 1$$

$$y^2 - y - 1 = 0$$

$$y = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\begin{aligned}
 &= \frac{1 \pm \sqrt{5}}{2} \text{ positive} \\
 &= \frac{1 + \sqrt{5}}{2}
 \end{aligned}$$

(dy)

4

1. 2. 3. 4. 5. 6. 7. 8

B D B D E C E B

9. 10. 11. 12. 13. 14. 15.

B E C D E B C

16. 17.

B D

Afdeling B / Section B (30 punte / 30 marks)

Beantwoord vrae B1 tot B3 op die vraestel.

Answer questions B1 to B3 on the question paper.

B1 Laat $\mathbf{F}(x, y, z)$ vektorveld wees, $g(x, y, z)$ 'n skalaarfunksie, beide genoegsaam kontinu. Laat $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Brei die volgende uit en vereenvoudig.

Let $\mathbf{F}(x, y, z)$ be a vector field, and let $g(x, y, z)$ be a scalar function, both sufficiently continuous. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Expand the following and simplify. [8]

(a) $\nabla \cdot (\nabla \times \mathbf{F})$

(b) $\nabla \cdot (g\mathbf{r})$

Wenk: Skryf \mathbf{F} as $P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$.Hint: Write \mathbf{F} as $P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$.

$$\begin{aligned}
 (a) \nabla \cdot (\nabla \times \mathbf{F}) &= \nabla \cdot \left(\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \right) \\
 &= \nabla \cdot \begin{bmatrix} R_y - Q_z \\ P_z - R_x \\ Q_x - P_y \end{bmatrix} = \frac{\partial}{\partial x}(R_y - Q_z) + \frac{\partial}{\partial y}(P_z - R_x) \\
 &\quad + \frac{\partial}{\partial z}(Q_x - P_y) \\
 &= R_{yz} - Q_{zx} + P_{zy} - R_{xy} + Q_{xz} - P_{yz} = 0
 \end{aligned}$$

$$\begin{aligned}
 (b) \nabla \cdot (g\mathbf{r}) &= \nabla \cdot \left(\begin{bmatrix} gx \\ gy \\ gz \end{bmatrix} \right) \\
 &= \frac{\partial}{\partial x}(gx) + \frac{\partial}{\partial y}(gy) + \frac{\partial}{\partial z}(gz) \\
 &= g_x x + g \cdot 1 + g_{yy} y + g \cdot 1 + g_z z + g \cdot 1 \\
 &= 3g + \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underline{\underline{3g + \nabla g \cdot \mathbf{r}}}
 \end{aligned}$$

B2 Bereken die integraal U hieronder. Let op dat \mathbf{F} nie konserwatif is nie. C bestaan uit 'n kwartsirkel in die yz -vlak gevvolg deur 'n reguit lynstuk. Begin- en eindpunt-koördinate word gegee sowel as die koördinate van die aansluitingspunt tussen die twee stukke van C .

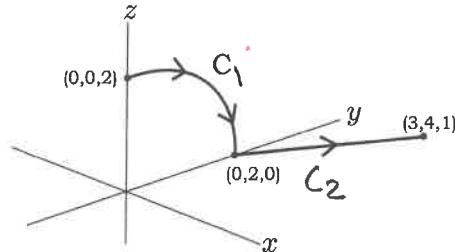
$$U = \int_C \mathbf{F} \cdot d\mathbf{r},$$

$$U = \int_C x dx + zy dy + zx dz$$

$$\begin{aligned} &= \int_{t=\frac{\pi}{2}}^0 0 + (2 \sin t)(2 \cos t) (-2 \sin t) dt \\ &\quad + 0 \\ &\quad + \int_0^1 (3t)(3) dt \\ &\quad + (t)(2+2t)(2) dt \\ &\quad + (3t) dt \end{aligned}$$

Calculate the integral U below. Note that \mathbf{F} is not conservative. C consists of a quarter of a circle in the yz -plane, followed by a straight line segment. The coordinates of the starting point, the end point as well as the point where the two sections of C meet, are given.

$$\mathbf{F} = xi + zyj + xk$$



$$\left[\begin{array}{l} C_1: \begin{cases} x=0 \\ y=2 \cos t \\ z=2 \sin t \end{cases} \quad t \in \left[\frac{\pi}{2}, 0\right] \\ C_2: \begin{cases} \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \\ x=3t \\ y=2+2t \\ z=t \end{cases} \quad t \in [0, 1] \end{array} \right]$$

$$\begin{aligned} &= -8 \int_{\pi/2}^0 (\sin^2 t)(\cos t) dt + \int_0^1 (16t + 4t^2) dt \\ &= -8 \left[\frac{(\sin t)^3}{3} \right]_{\pi/2}^0 + \left[\frac{16t^2}{2} + \frac{4t^3}{3} \right]_0^1 \\ &= -8 \left[0 - \frac{1}{3} \right] + \left[8 + \frac{4}{3} - 0 \right] \\ &= 12 \end{aligned}$$

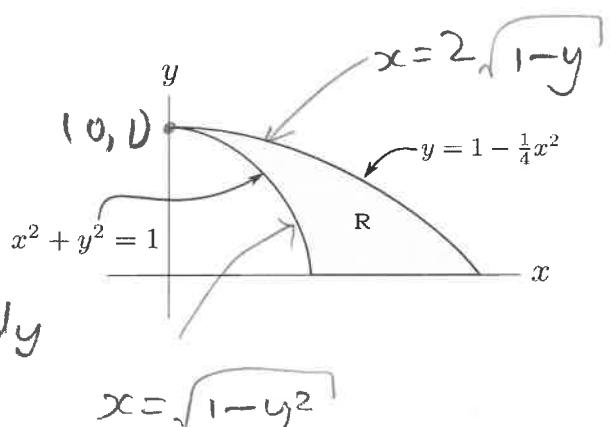
B3 Bereken T hieronder.Calculate T below.

10

$$T = \iint_R 30xy^2 \, dA$$

As type 2:

$$T = \int_{y=0}^1 \int_{x=\sqrt{1-y^2}}^{2\sqrt{1-y}} (30xy^2) \, dx \, dy$$



$$= \int_{y=0}^1 \left[\frac{30x^2y^2}{2} \right]_{\sqrt{1-y^2}}^{2\sqrt{1-y}} \, dy$$

$$= \int_0^1 \left(15(2\sqrt{1-y})^2 y^2 - 15(\sqrt{1-y^2})^2 y^2 \right) \, dy$$

$$= \int_0^1 (60(1-y)y^2 - 15(1-y^2)y^2) \, dy$$

$$= \int_0^1 (60y^2 - 60y^3 - 15y^2 + 15y^4) \, dy$$

$$= \int_0^1 (15y^4 - 60y^3 + 45y^2) \, dy \quad T = \boxed{3}$$

$$= \left[\frac{15y^5}{5} - \frac{60y^4}{4} + \frac{45y^3}{3} \right]_0^1 = 3$$