[9.11] 4, 7, 9, 12, 25, 26, 27, 34.

[9.11] 04.

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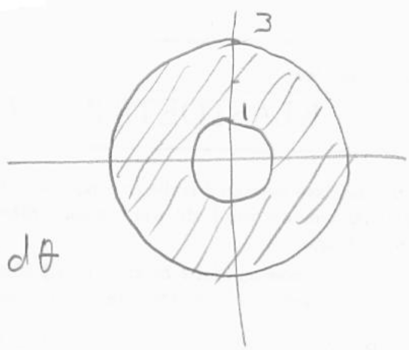
4. $A = \int_0^{\pi/4} \int_0^{8\sin 4\theta} r\,dr\,d\theta = \int_0^{\pi/4} \frac{1}{2}r^2 \Big|_0^{8\sin 4\theta} d\theta = \frac{1}{2}\int_0^{\pi/4} 64\sin^2 4\theta\,d\theta$

$= 32\left(\frac{1}{2}\theta - \frac{1}{16}\sin 8\theta\right)\Big|_0^{\pi/4} = 4\pi$

[9.11] 07.

7.



$$V = \int_{\theta=0}^{2\pi}\int_{r=1}^{3} \sqrt{16-r^2}\; r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi} d\theta \cdot \frac{1}{2}\int_{r=1}^{3} 2r\sqrt{16-r^2}\,dr = \left[\theta\right]_0^{2\pi} \cdot \frac{1}{2}\left[\frac{(16-r^2)^{3/2}}{(-1)\,3/2}\right]_1^3$$

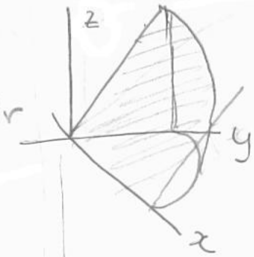
$$= 2\pi \cdot -\frac{1}{2}\left((16-9)^{3/2} - (16-1)^{3/2}\right) \times \frac{2}{3}$$

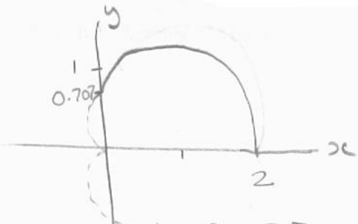
$$= \frac{2\pi}{3}\left((15)^{3/2} - (7)^{3/2}\right) = 82.88$$

[9.11] 09.

9.

$$V = \int_{\theta=0}^{\pi/2} \int_{r=0}^{1+\cos\theta} r\sin\theta \, r d\theta dr$$





$\theta \in [0, \frac{\pi}{2}]$

$$= \int_{\theta=0}^{\pi/2} \int_{r=0}^{1+\cos\theta} r^2 dr \cdot \sin\theta d\theta$$

$y = r\sin\theta$

$$= \int_{\theta=0}^{\pi/2} \left[\frac{r^3}{3}\right]_0^{1+\cos\theta} \sin\theta d\theta = \frac{1}{3}\int_{\theta=0}^{\pi/2} (1+\cos\theta)^3 \sin\theta d\theta$$

$$= \frac{1}{3}\left[\frac{(1+\cos\theta)^4}{4}(-1)\right]_0^{\pi/2} = \frac{-1}{12}\left((1+0)^4 - (1+1)^4\right)$$

$$= \frac{16-1}{12} = \frac{15}{12} = \frac{5}{4}$$

[9.11] 12.

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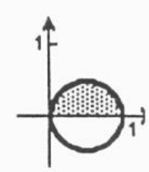
12. The interior of the upper-half circle is traced from $\theta = 0$ to $\pi/2$. The density is kr. Since both the region and the density are symmetric about the polar axis, $\bar{y} = 0$.



$$m = \int_0^{\pi/2}\int_0^{\cos\theta} kr^2\,dr\,d\theta = k\int_0^{\pi/2} \frac{1}{3}r^3\Big|_0^{\cos\theta} d\theta = \frac{k}{3}\int_0^{\pi/2}\cos^3\theta\,d\theta$$

$$= \frac{k}{3}\left(\frac{2}{3} + \frac{1}{3}\cos^2\theta\right)\sin\theta\Big|_0^{\pi/2} = \frac{2k}{9}$$

$$M_y = k\int_0^{\pi/2}\int_0^{\cos\theta}(r\cos\theta)(r)(r\,dr\,d\theta) = k\int_0^{\pi/2}\int_0^{\cos\theta} r^3\cos\theta\,dr\,d\theta = k\int_0^{\pi/2}\frac{1}{4}r^4\cos\theta\Big|_0^{\cos\theta} d\theta$$

$$= \frac{k}{4}\int_0^{\pi/2}\cos^5\theta\,d\theta = \frac{k}{4}\left(\sin\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta\right)\Big|_0^{\pi/2} = \frac{2k}{15}$$

Thus, $\bar{x} = \dfrac{2k/15}{2k/9} = 3/5$ and the center of mass is $(3/5, 0)$.

[9.11] 25.

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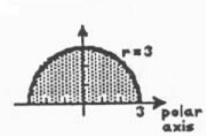
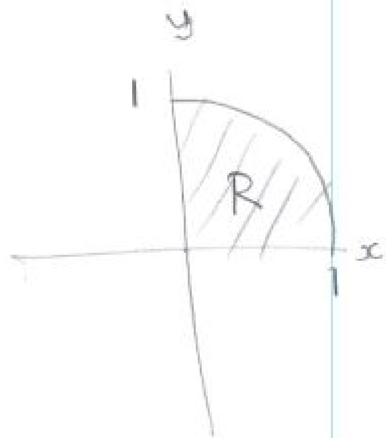
25. $\displaystyle\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sqrt{x^2+y^2}\,dy\,dx = \int_0^{\pi}\int_0^3 |r|r\,dr\,d\theta = \int_0^{\pi}\frac{1}{3}r^3\Big|_0^3 d\theta = 9\int_0^{\pi} d\theta = 9\pi$



[9.11] 26.

9.11, 27



$$\int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy$$

$$= \int_{\theta=0}^{\pi/2} \int_{r=0}^{1} e^{r^2}\, r\, dr\, d\theta$$

$$= \int_{\theta=0}^{\pi/2} d\theta \times \frac{1}{2} \int_{r=0}^{1} 2r e^{r^2}\, dr$$

$$= \left[\theta\right]_0^{\pi/2} \cdot \frac{1}{2}\left[e^{r^2}\right]_0^1 = \frac{\pi}{2} \cdot \frac{1}{2}\left[e^1 - e^0\right]$$

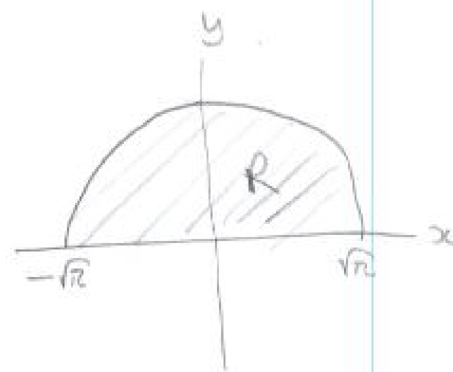
$$= \frac{\pi}{4}(e-1) = 1.3495$$

[9.11] 27.

$\boxed{9.11, 28}$

$$\int_{x=-\sqrt{\pi}}^{\sqrt{\pi}} \int_{y=0}^{\sqrt{\pi - x^2}} \sin\left(x^2+y^2\right) dy\, dx$$

$$= \int_{\theta=0}^{\pi} \int_{r=0}^{\sqrt{\pi}} \sin(r^2)\, r\, dr\, d\theta$$



$$= \int_{\theta=0}^{\pi} 1\, d\theta \cdot \int_{r=0}^{\sqrt{\pi}} \sin(r^2)\, r\, dr$$

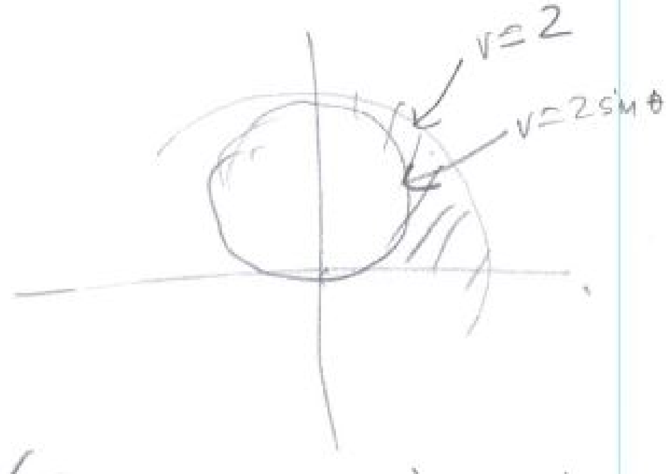
$$= \Big[\theta\Big]_0^{\pi} \times \left[-\frac{1}{2}\cos(r^2)\right]_0^{\sqrt{\pi}} = -\frac{1}{2}\pi \Big[-1-(1)\Big]$$

$$= -\frac{1}{2}\pi\,(-2) = \pi$$

[9.11] 34a.

9.11, 34

$$\iint_R (x+y)\,dA$$



$$= \int_{\theta=0}^{\pi/2} \int_{r=2\sin\theta}^{2} (r\cos\theta + r\sin\theta)\, r\,dr\,d\theta.$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\Big[\cos\theta + \sin\theta\Big]_{2\sin\theta}^{2} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2} (\cos\theta + \sin\theta)\left(2^3 - (2\sin\theta)^3\right) d\theta$$

$$= \frac{8}{3}\int_0^{\pi/2} \left(\cos\theta + \sin\theta - \cos\theta\sin^3\theta - \sin^4\theta\right) d\theta$$

$$= \frac{8}{3}\left[\sin\theta - \cos\theta - \frac{\sin^4\theta}{4}\right]_0^{\pi/2} - \frac{8}{3}X$$

$$= \frac{8}{3}\left[1 + 0 - \frac{1}{4} - 0 + 1 - 0\right] - \frac{8}{3}X$$

[9.11] 34b.

9.11, 34 continued

$$X = \int_0^{\pi/2} \sin^4\theta \, d\theta$$

but
$$\sin^4\theta = \left(\sin^2\theta\right)^2$$
$$= \left(\frac{1-\cos 2\theta}{2}\right)^2.$$
$$= \frac{1}{4}\left(1 - 2\cos 2\theta + (\cos 2\theta)^2\right)$$
$$= \frac{1}{4}\left(1 - 2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)\right).$$
$$= \frac{1}{4}\left(1 - 2\cos 2\theta + \frac{1}{2} + \frac{1}{2}\cos 4\theta\right)$$
$$= \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$

$$X = \int_0^{\pi/2}\left(\frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta\right)d\theta$$
$$= \left[\frac{3}{8}\theta - \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)\right]_0^{\pi/2} = \frac{3}{8}\cdot\frac{\pi}{2} - 0 + 0 + 0 - 0$$
$$= \frac{3\pi}{16}$$

and
$$\iint_R (x+y)\,dA = \frac{14}{3} - \frac{8}{3}\left(\frac{3\pi}{16}\right) = \frac{14}{3} - \frac{\pi}{2}$$
$$= 3.0959$$