Appl.Maths. B242-2023: LECTURE 24

LECTURE 24 SUMMARY

$\boxed{1}$

VECTOR DERIVATIVES

GRAD $\quad \underline{\nabla} f \longrightarrow$ vector
- points to dir. of maximum increase
- length is the rate of increase

contour surf.

$G(x,y,z) = w.$

DIV $\quad \underline{\nabla} \cdot \underline{F} \longrightarrow$ scalar

$\frac{\text{flux}}{\text{volume}}$ out of a point.

$\boxed{2}$

CURL $\quad \underline{\nabla} \times \underline{E} \longrightarrow$ vector
- points perpendicular to loop with maximum circulation
- length is the (maximum circulation)/area

VECTOR INTEGRALS

$$\underbrace{\oint_C \underline{E}\cdot d\underline{r}}_{\text{circulation of } \underline{E} \text{ around } C}$$

$$\underbrace{\iint_S \underline{E}\cdot\underline{n}\, dS}_{\text{flux of } \underline{E} \text{ through } S}$$

THREE THEOREMS

$\boxed{3}$

STOKES

$$\underbrace{\oint_C \underline{F}\cdot d\underline{r}}\;=\;\iint_S \overset{\text{der. of } \underline{F}}{(\underline{\nabla}\times\underline{F})}\cdot\underline{n}\, dS$$

$$\left(\begin{array}{l}\text{The circulation of } \underline{F}\\ \text{around } C\end{array}\right) = \left(\begin{array}{l}\text{Flux of curl of } \underline{F}\\ \text{through } S\end{array}\right)$$

GREEN

$$\oint_C \begin{bmatrix}P\\Q\\0\end{bmatrix}\cdot\begin{bmatrix}dx\\dy\\\cancel{dz}\end{bmatrix} = \iint_R \left[\cancel{\begin{bmatrix}\partial_x\\\partial_y\\\partial_z\end{bmatrix}\times\begin{bmatrix}P\\Q\\0\end{bmatrix}}\;\begin{bmatrix}0\\0\\\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\end{bmatrix}\right]\cdot\begin{bmatrix}0\\0\\1\end{bmatrix} dA$$

$\boxed{4}$

~~DIVERG~~ TH

$$\iint_S \underline{F}\cdot\underline{n}\, dS = \iiint_B \overset{\text{der. of } \underline{F}}{\underline{\nabla}\cdot\underline{F}}\, dV$$

$$\left(\begin{array}{l}\text{The flux of } \underline{F} \text{ out of } B\\ (\text{through } S)\end{array}\right) = \left(\begin{array}{l}\text{total (sum) of the}\\ \text{divergence of } \underline{F}\\ \text{throughout } B\end{array}\right)$$

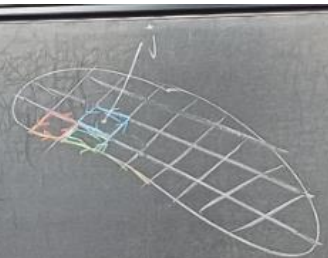
EXPLAIN STOKES

$$\oint_C \underline{F}\cdot d\underline{r} \overset{?}{=} \oint_{C_1} \underline{F}\cdot d\underline{r} + \oint_{C_2} \underline{F}\cdot d\underline{r}$$

5

$$= \sum_j \frac{\text{circ}_j}{\text{area}_j} \times \text{area}_j$$

$$= \iint_S (\underline{\nabla} \times \underline{F}) \cdot \underline{n} \, dS .$$

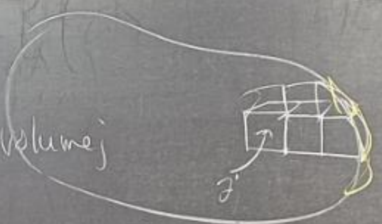


EXPLAIN DIVERGENCE TH.

$$\iint_S \underline{F} \cdot \underline{n} \, dS = \iint_{S_1} \underline{F} \cdot \underline{n}_1 \, dS_1 + \iint_{S_2} \underline{F} \cdot \underline{n}_2 \, dS_2$$

6

$$\sum_j \frac{(\text{flux out})_j}{\text{Volume}_j} \times \text{Volume}_j$$



$$\iint_S \underline{F} \cdot \underline{n} \, dS = \iiint_B \underline{\nabla} \cdot \underline{F} \, dV$$

$$\iint_R 1\,dA = \text{area}$$

$$\iiint_B 1\,dV = \text{volume}$$

Disc

$P = 2\pi r$

$A = \pi r^2$

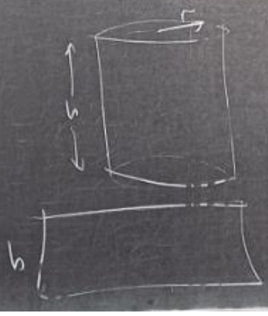
Sphere

Surface $= 4\pi r^2$ $\iint_S 1\,dS$

Volume $= \frac{4}{3}\pi r^3$

$S: z = \sqrt{1-r^2}$

Cylinder:



Surface (round part) $= 2\pi r h$

Volume $= \pi r^2 h$.