

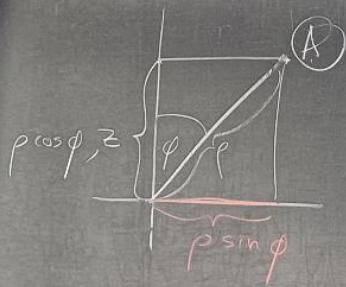
Appl. Maths. B242-2023: LECTURE 22

LECTURE 22 [9.15] TRIPLE INT'S, SPHERICAL COORD'S

Cart $dV = dx dy dz$ (x, y, z)
 Cyl. $dV = r dr d\theta dz$ (r, θ, z)
 Spher. $dV = \underbrace{\rho^2}_{m^2} \underbrace{\sin \rho}_{m} \underbrace{d\rho}_{m} \underbrace{d\phi}_{m} \underbrace{d\theta}_{m}$ (ρ, ϕ, θ)
↑ angles
↑ distance

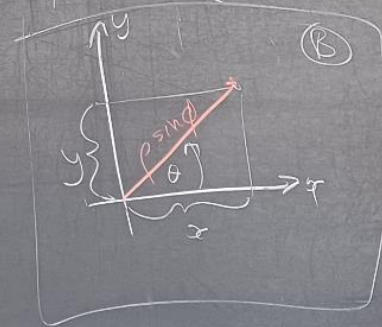
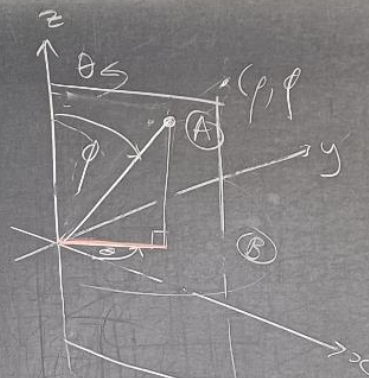
Spherical coord's

$\rho \in [0, \infty)$
 $\phi \in [0, \pi]$
 $\theta \in [0, 2\pi)$



Spher \rightarrow Cart

$x = \rho \sin \phi \cos \theta$ ①
 $y = \rho \sin \phi \sin \theta$ ②
 $z = \rho \cos \phi$ ③



Cat \rightarrow Spher.

3

$$\begin{aligned}x^2 + y^2 + z^2 &= \rho^2 \sin^2 \theta \cos^2 \phi + \rho^2 \sin^2 \theta \sin^2 \phi + \rho^2 \cos^2 \theta \\&= \rho^2 \sin^2 \theta (\cos^2 \phi + \sin^2 \phi) + \rho^2 \cos^2 \theta \\&= \rho^2 (\sin^2 \theta + \cos^2 \theta)\end{aligned}$$

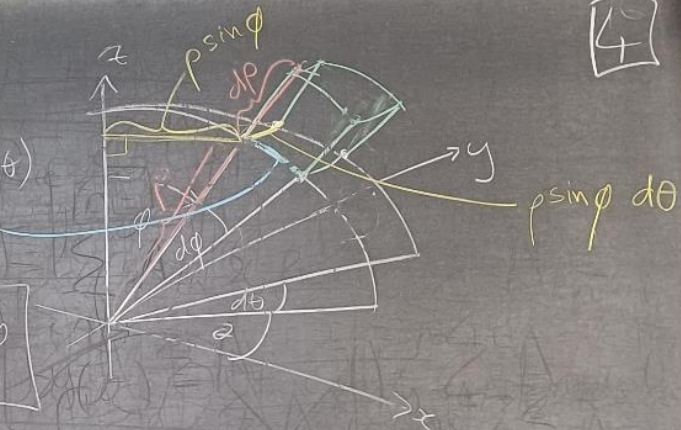
$$\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arctan\left(\frac{y}{x}\right) \quad (\text{check quadrant}) \\ \phi = \arccos\left(\frac{z}{\rho}\right) \quad (\text{check quadrant}) \end{cases}$$

Volume element

4

$$dV = (\rho d\rho) (d\phi) (\rho \sin\theta d\theta)$$

$$dV = \rho^2 \sin\theta d\rho d\theta d\phi$$

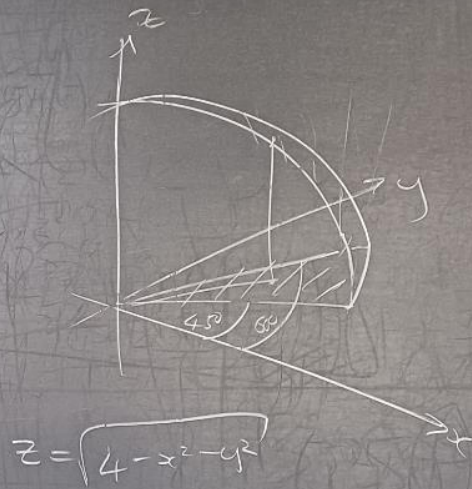
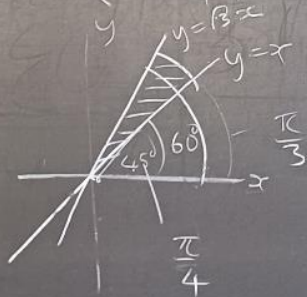


Example (76): Find volume of B.

5

$$B: \begin{cases} x^2 + y^2 + z^2 = 4 \\ y = x \\ y = \sqrt{3}x \\ z = 0 \end{cases}$$

only pos. octant



$$\text{Volume} = \iiint_B 1 \, dV$$

Cylindrical

$$V = \int_{\theta=\pi/4}^{\pi/3} \int_{r=0}^2 \int_{z=0}^{\sqrt{4-r^2}} 1 \, dz \, r \, dr \, d\theta$$

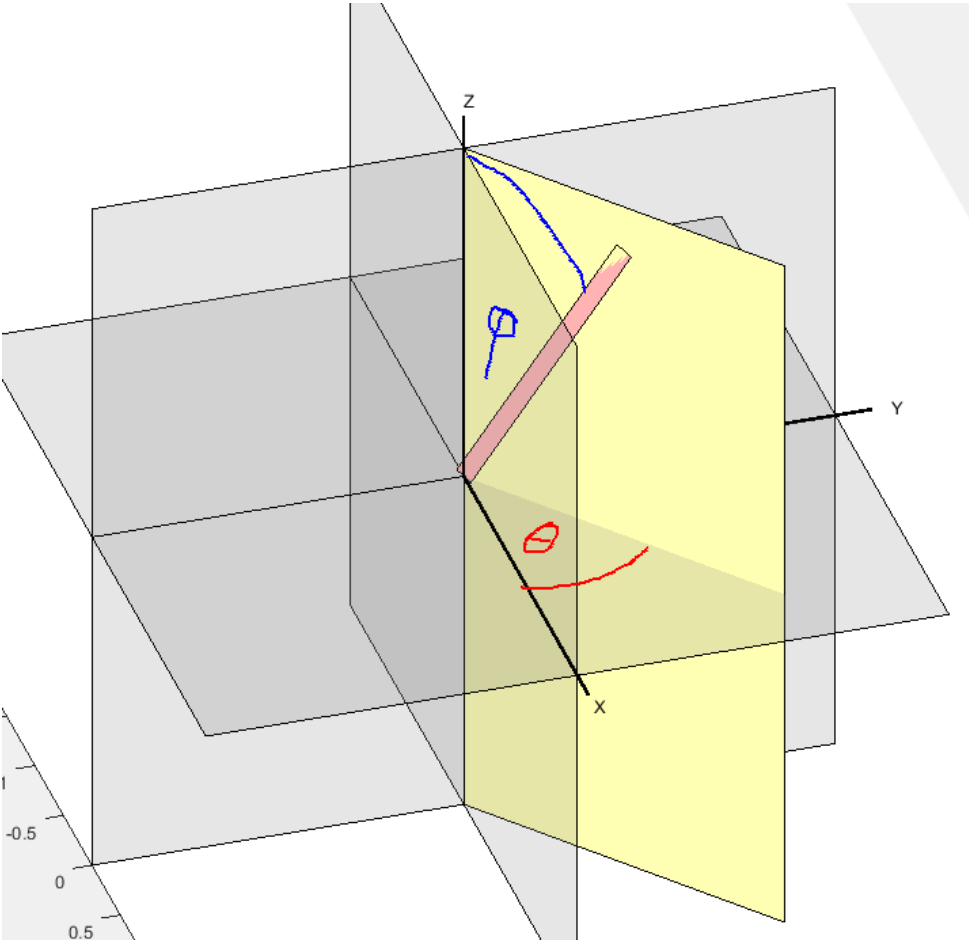
Spherical

$$V = \int_{\theta=\pi/4}^{\pi/3} \int_{\phi=0}^{\pi/2} \int_{\rho=0}^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

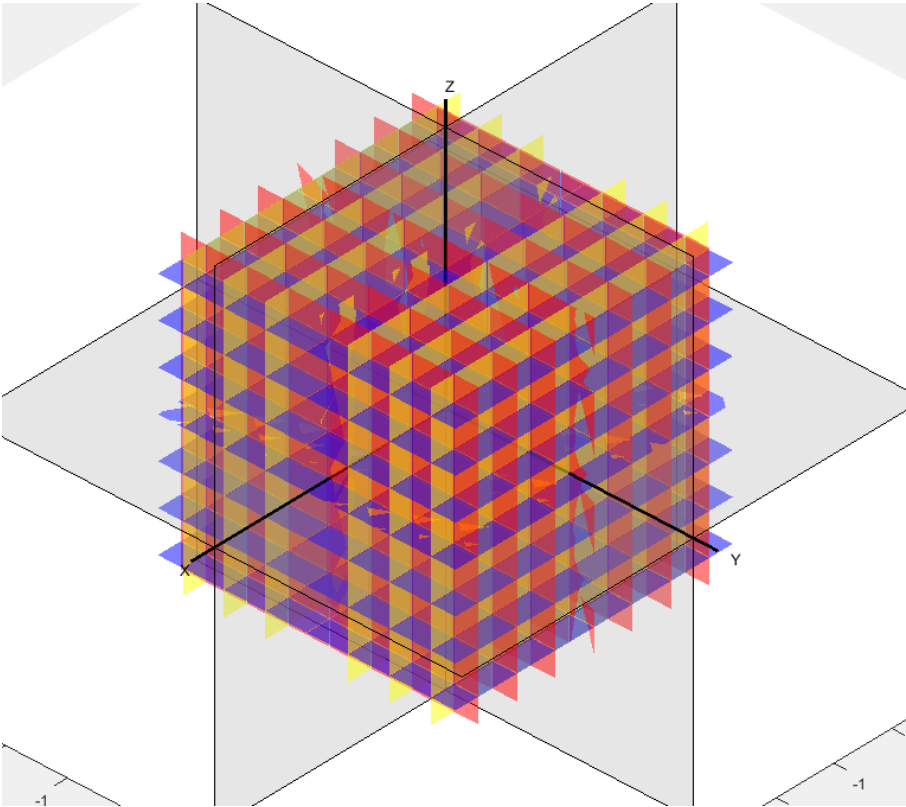
$$= \frac{2\pi}{9}$$

6

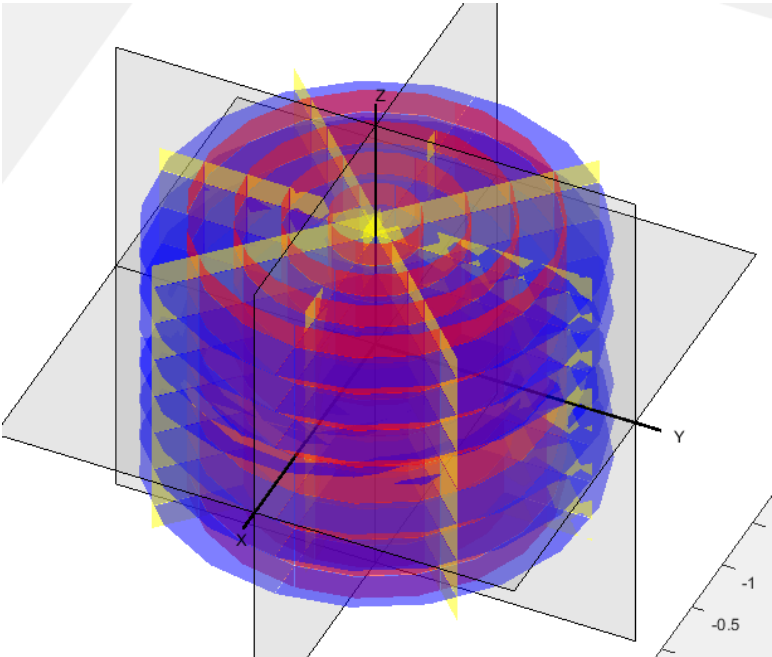
THE MEANING OF PHI AND THETA IN SPHERICAL COORDINATES



A CARTESIAN GRID



A CYLINDRICAL GRID



A SPHERICAL GRID

