

Appl. Maths. B242-2023: LECTURE 21

LECTURE 21 [9.15] TRIPPLE INTEGRALS

$$I = \iiint_B F(x,y,z) dV$$

volume elem.

$$dV = dz dx dy$$

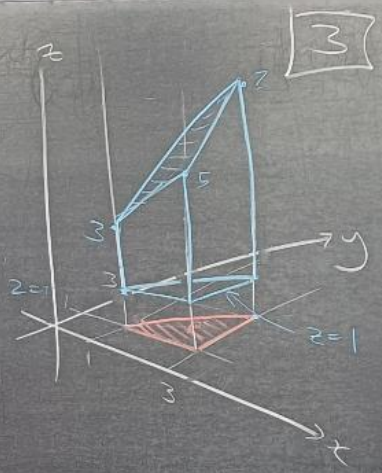
$r dr d\theta$

$$I = \int_{x=a}^b \left[\int_{y=g_1(x)}^{g_2(x)} \left[\int_{z=f_1(x,y)}^{f_2(x,y)} F(x,y,z) dz \right] dy \right] dx$$

Example: Find mass of B,
density = xy

B: $\begin{cases} z=1 \leftarrow \text{floor} \\ z=1+x+y \leftarrow \text{ceiling, plane} \\ y=1 \\ y=x \\ x=1 \leftarrow \text{Region R} \\ x=3 \end{cases}$

$$\begin{aligned}
 &x=1, y=1, z=3 \\
 &x=3, y=1, z=5 \\
 &x=3, y=3, z=7
 \end{aligned}$$



$$\begin{aligned}
 \text{mass} &= \iiint_B \rho(x,y,z) dV \\
 &= \int_{y=1}^3 \int_{x=y}^3 \left[\int_{z=1}^{1+x+y} xy dz \right] dx dy \\
 &= \int_{y=1}^3 \int_{x=y}^3 [xyz]_1^{1+x+y} dx dy \\
 &= \int_{y=1}^3 \int_{x=y}^3 (xy(1+x+y) - xy) dx dy = \dots \text{some work.}
 \end{aligned}$$

$$\text{Volume} = \iiint_B 1 dV \qquad \text{mass} = \iiint_B \rho dV$$

$$\text{centroid } m\bar{x} = \iiint_B x \rho(x,y,z) dV, \quad m\bar{y} = \dots, \quad m\bar{z} = \iiint_B z \rho dV$$

mom. of inertia

$$I_{xx} = \iiint_B \frac{(y^2+z^2)}{\sqrt{y^2+z^2}} \rho(x,y,z) dV \qquad \iiint r^2 dM.$$

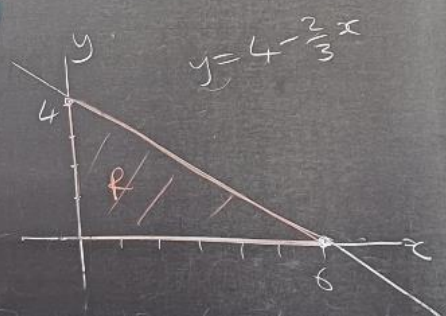
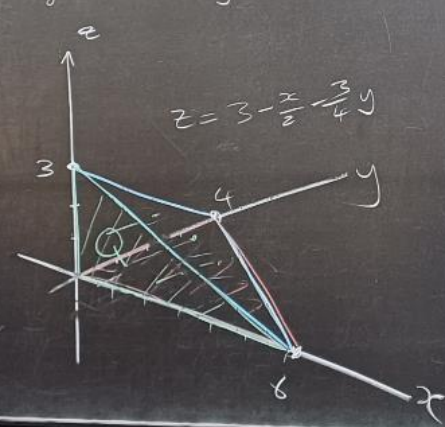


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Changing the order

$$I = \int_{x=0}^6 \int_{y=0}^{4-\frac{2}{3}x} \int_{z=0}^{3-\frac{x}{3}-\frac{3}{4}y} F(x,y,z) dz dy dx$$

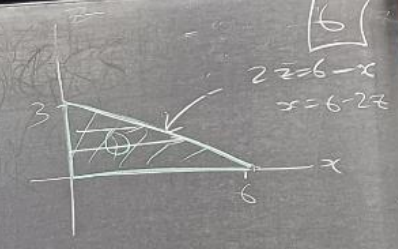
Change to $dy dx dz$ floor + ceil



$$2x + 3y + 4z = 12$$

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$$I = \int_{z=0}^3 \int_{x=0}^{6-2z} \int_{y=0}^{4-\frac{2}{3}x-\frac{4}{3}z} F(x,y,z) dy dx dz$$



$$2x + 3y + 4z = 12$$

$$3y = 12 - 2x - 4z$$

$$y = 4 - \frac{2}{3}x - \frac{4}{3}z$$

CYLINDRICAL COORDS

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Cartesian

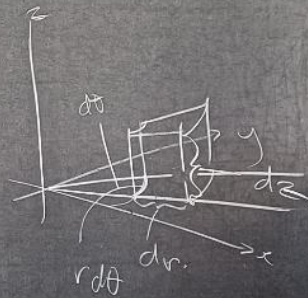
$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$

Cylindrical

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan\left(\frac{y}{x}\right) \text{ check quadrants} \\ z = z \end{cases}$$

$$dV = dr \cdot r d\theta \cdot dz$$

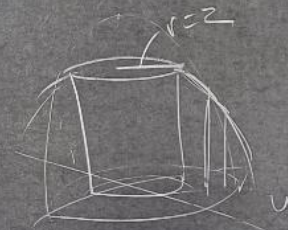
$$dV = r dr d\theta dz$$



Example: Find mass D , $\rho = 1.2z$

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$$D: \begin{cases} z \leq 9 - x^2 - y^2 \\ z \geq 0 \\ x^2 + y^2 \geq 4 \end{cases}$$



$$\begin{aligned} \text{mass} &= \iiint_D \rho \, dV \\ &= \int_{\theta=0}^{2\pi} \int_{r=2}^3 \int_{z=0}^{9-r^2} \frac{6}{5} z \, dz \, r \, dr \, d\theta \end{aligned}$$

$$= \dots = 25\pi$$

