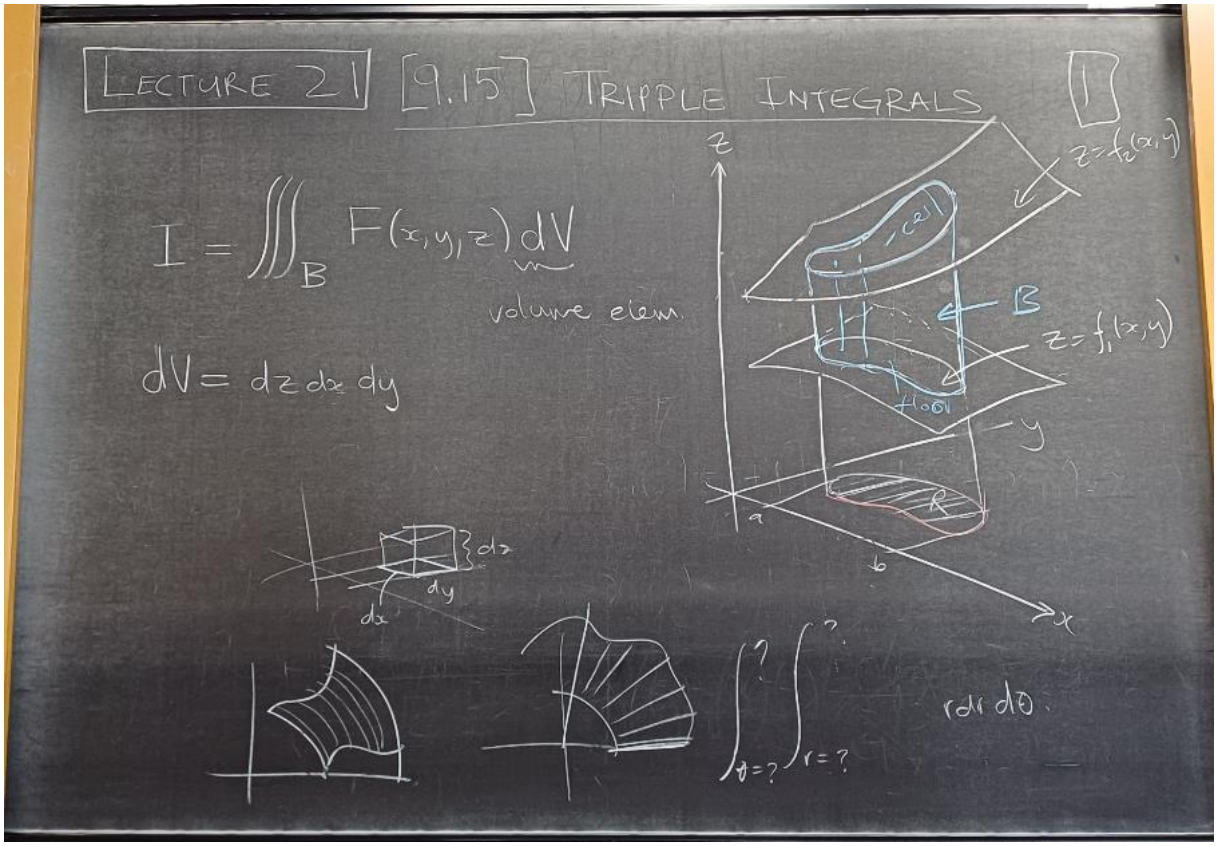
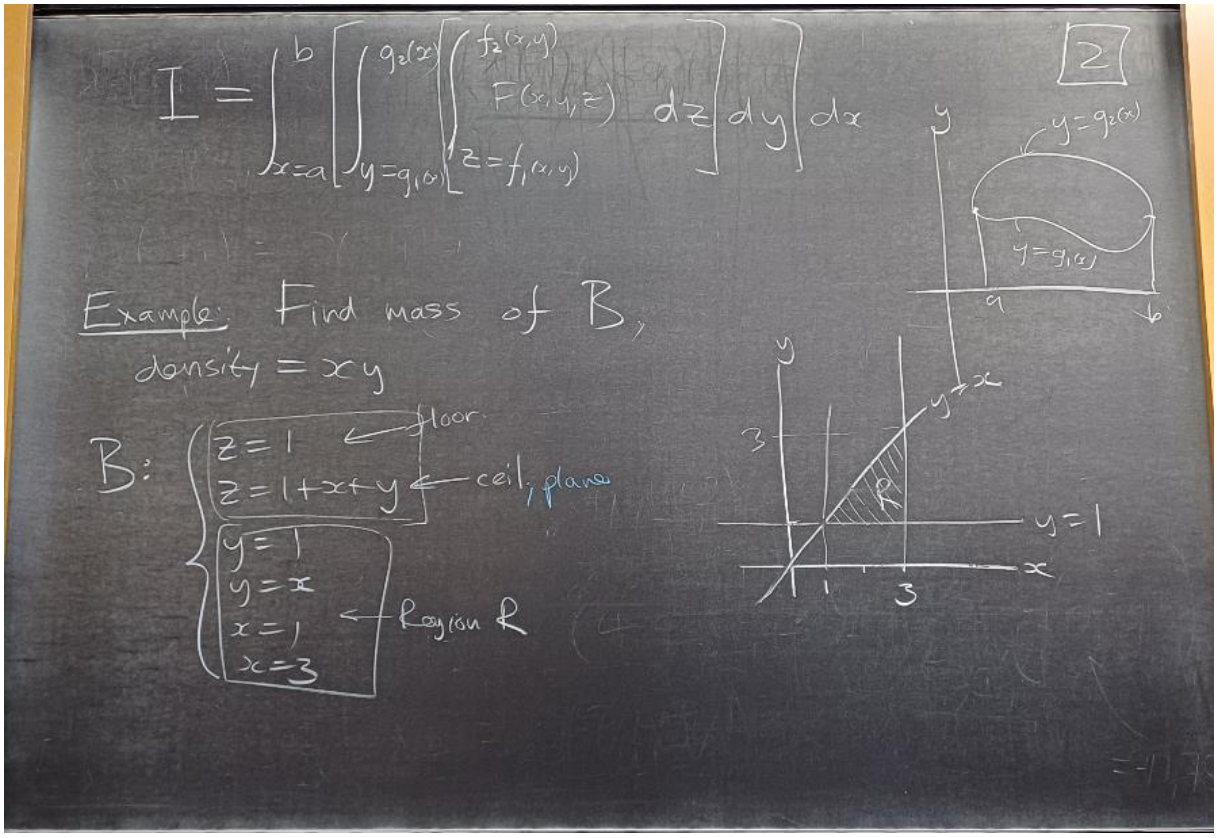
Appl.Maths. B242-2023: LECTURE 21



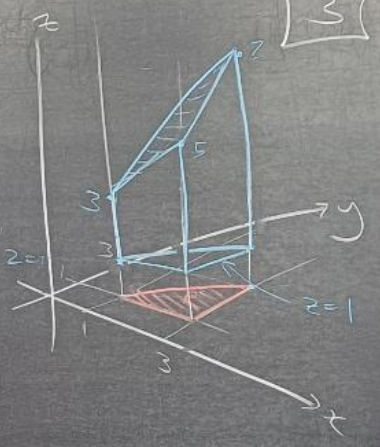


3

$x=1, y=1, z=3$

$x=3, y=1, z=5$

$x=3, y=3, z=7$



$$\text{mass} = \iiint_B \rho(x,y,z)\,dV$$

$$= \int_{y=1}^{3}\left[\int_{x=y}^{3}\left[\int_{z=1}^{1+x+y} xy\,dz\right]dx\right]dy$$

$$= \int_{y=1}^{3}\int_{x=y}^{3}\left[xyz\right]_{1}^{1+x+y}\,dx\,dy$$

$$= \int_{y=1}^{3}\int_{x=y}^{3}\left(xy(1+x+y)-xy\right)dx\,dy = \cdots \text{ same work.}$$

4

$$\text{volume} = \iiint_B 1\,dV \qquad \text{mass} = \iiint_B \rho\,dV$$

centroid

$$m\bar{x} = \iiint_B x\rho(x,y,z)\,dV, \quad m\bar{y} = \cdots, \quad m\bar{z} = \iiint_B z\rho\,dV$$

mom. of inertia

$$I_{xx} = \iiint_B (y^2+z^2)\rho(x,y,z)\,dV \qquad \iiint r^2\,dm.$$

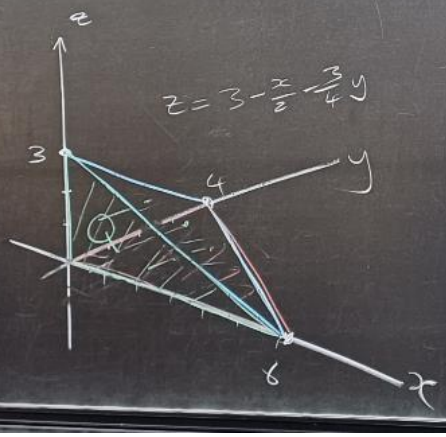
5

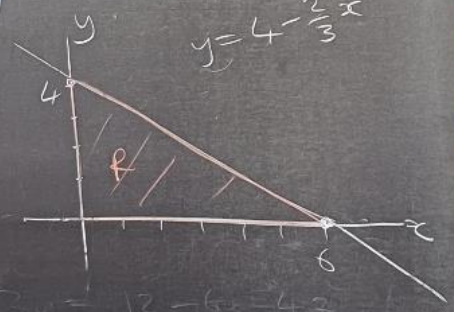
Changing the order

$$I = \int_{x=0}^{6} \int_{y=0}^{4-\frac{2}{3}x} \int_{z=0}^{3-\frac{x}{2}-\frac{3}{4}y} F(x,y,z)\, dz\, dy\, dx$$

Region — floor + ceil

Change to $dy\, dx\, dz$

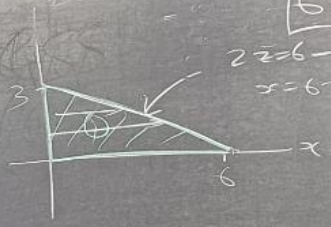




$2x + 3y + 4z = 12$

6

$$I = \int_{z=0}^{3} \int_{x=0}^{6-2z} \int_{y=0}^{4-\frac{2}{3}x-\frac{4}{3}z} F(x,y,z)\, dy$$



$2x + 3y + 4z = 12$

$3y = 12 - 2x - 4z$

$y = 4 - \frac{2}{3}x - \frac{4}{3}z$

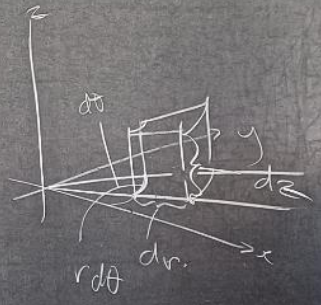
CYLINDRICAL COORDS

7

Cartesian		Cylindrical	
$x = r\cos\theta$	$\longleftrightarrow$	$r = \sqrt{x^2+y^2}$	
$y = r\sin\theta$		$\theta = \arctan\left(\frac{y}{x}\right)$	check quadrant
$z = z$		$z = z$	

$$dV = dr\ r\,d\theta\,dz$$

$$\boxed{dV = r\,dr\,d\theta\,dz}$$



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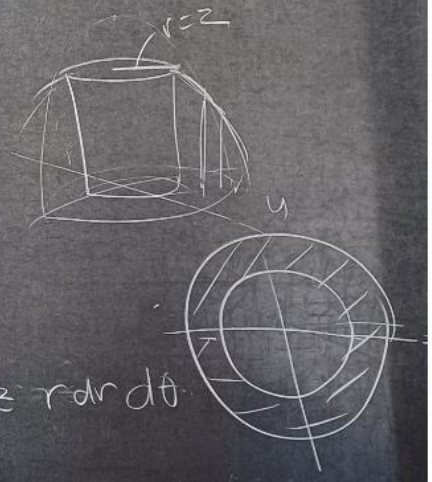
Example: Find mass of D, $\rho = 1.2z$

$$D: \begin{cases} z \le 9 - x^2 - y^2 \\ z \ge 0 \\ x^2 + y^2 \ge 4 \end{cases}$$



$$\text{mass} = \iiint_D \rho\, dV$$

$$= \int_{\theta=0}^{2\pi} \int_{r=2}^{3} \int_{z=0}^{9-r^2} \frac{6}{5} z\, dz\, r\, dr\, d\theta$$

$$= \ldots = 25\pi$$