

Appl. Maths. B242-2023: LECTURE 20

LECTURE 20 VECTOR DERIVATIVES + [9.7, 9.14] 1

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<u>GRAD</u>	<u>DIV</u>	<u>CURL</u>
$\nabla f = \text{vector}$	$\nabla \cdot \mathbf{E} = \text{scalar}$	$\nabla \times \mathbf{E} = \text{vector}$
$\updownarrow$ $D_u f \Big _{(x_0, y_0, z_0)}$	$\updownarrow$ FLUX	$\updownarrow$ CIRCULATION
Grad is a vector < direction in maximal rate of change < magnitude is the rate of change		Curl is a vector < direction in which circ/area is maximal < magnitude

GRAD  $\leftrightarrow$  DIR. PER. 2

$w = g(x, y, z)$

$D_u g = \nabla g \cdot \underline{u} = 0$

$\uparrow$   
tangent to surface

$\underline{u}$  tangent to surface

$\nabla g \cdot \underline{u} = 0$

$\nabla g$  is perpendicular to contour surface.

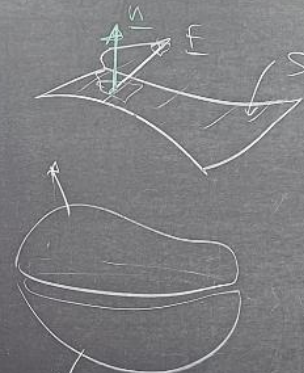
# Div $\leftrightarrow$ Flux

3

$$\underline{E}(x, y, z)$$

$$\text{flux}^{\text{up}} = \iint_S \underline{E} \cdot \underline{n} \, dS$$

$$\text{flux out} = \iint_{S_1} \underline{E} \cdot \underline{n}_1 \, dS_1 + \iint_{S_2} \underline{E} \cdot \underline{n}_2 \, dS_2$$



$$\nabla \cdot \underline{E} = \lim_{\Delta V \rightarrow 0} \frac{\iint_{\Delta S} \underline{E} \cdot \underline{n} \, dS}{\Delta V}$$



# CIRCULATION

4

$$\underline{E}(x, y, z)$$

circulation of  $\underline{E}$  around  $C$

$$= \oint_C \underline{E} \cdot d\underline{r} = \int_C \underline{E}(\underline{r}(t)) \cdot \underline{r}'(t) \, dt$$



$$\underline{r}(t)$$

$$\underline{r}'(t)$$



$$dt \underline{r}'(t) = \frac{d\underline{r}}{dt} dt$$

# Curl + Circulation

5

$$\underline{F}(x, y, z)$$



STOKES:

$$\oint_{\Delta C} \underline{F} \cdot d\underline{r} = \iint_{\Delta S} (\nabla \times \underline{F}) \cdot \underline{n} \, dS$$

circulation of  $\underline{F}$   
around  $\Delta C$

flux of  $\nabla \times \underline{F}$  through  $\Delta S$

Shrink  $\Delta S$

$$\approx (\nabla \times \underline{F}) \cdot \underline{n} \iint_{\Delta S} 1 \, dS$$

$$= (\nabla \times \underline{F}) \cdot \underline{n} \text{ Area}$$

Circulation  
area

$$\approx (\nabla \times \underline{F}) \cdot \underline{n}$$

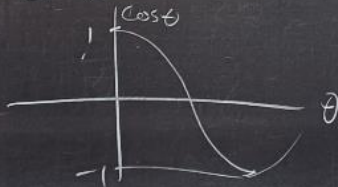
6

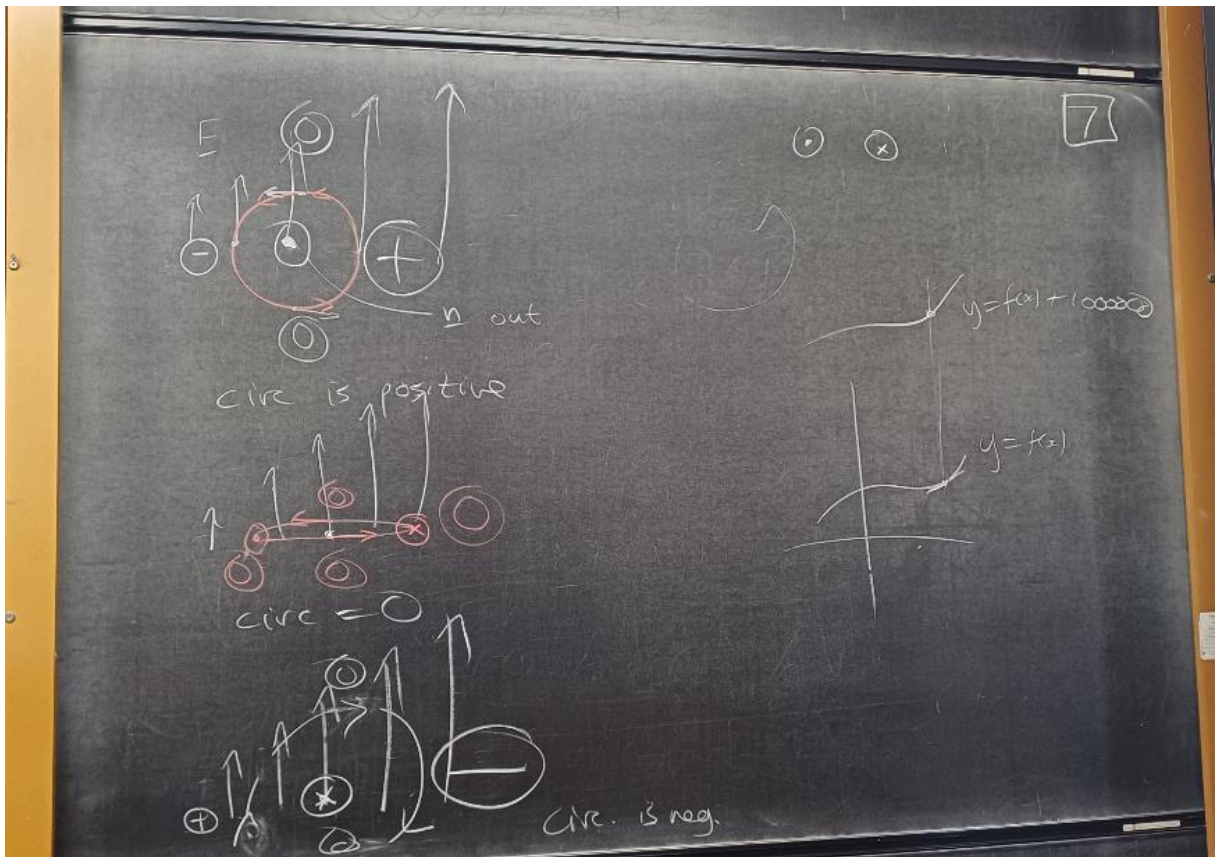
$$\Delta S \rightarrow 0, \Delta C \rightarrow 0$$

$$\underbrace{(\nabla \times \underline{F}) \cdot \underline{n}}_{\text{max}} = \lim_{\Delta S \rightarrow 0} \frac{\text{circ of } \underline{F} \text{ around disc}}{\text{area of disc}}$$

maximum when  $\underline{n}$  points  
along  $\nabla \times \underline{F}$

$$\underline{a} \cdot \underline{b} = a b \cos \theta$$





## CIRCULATION DEMO

