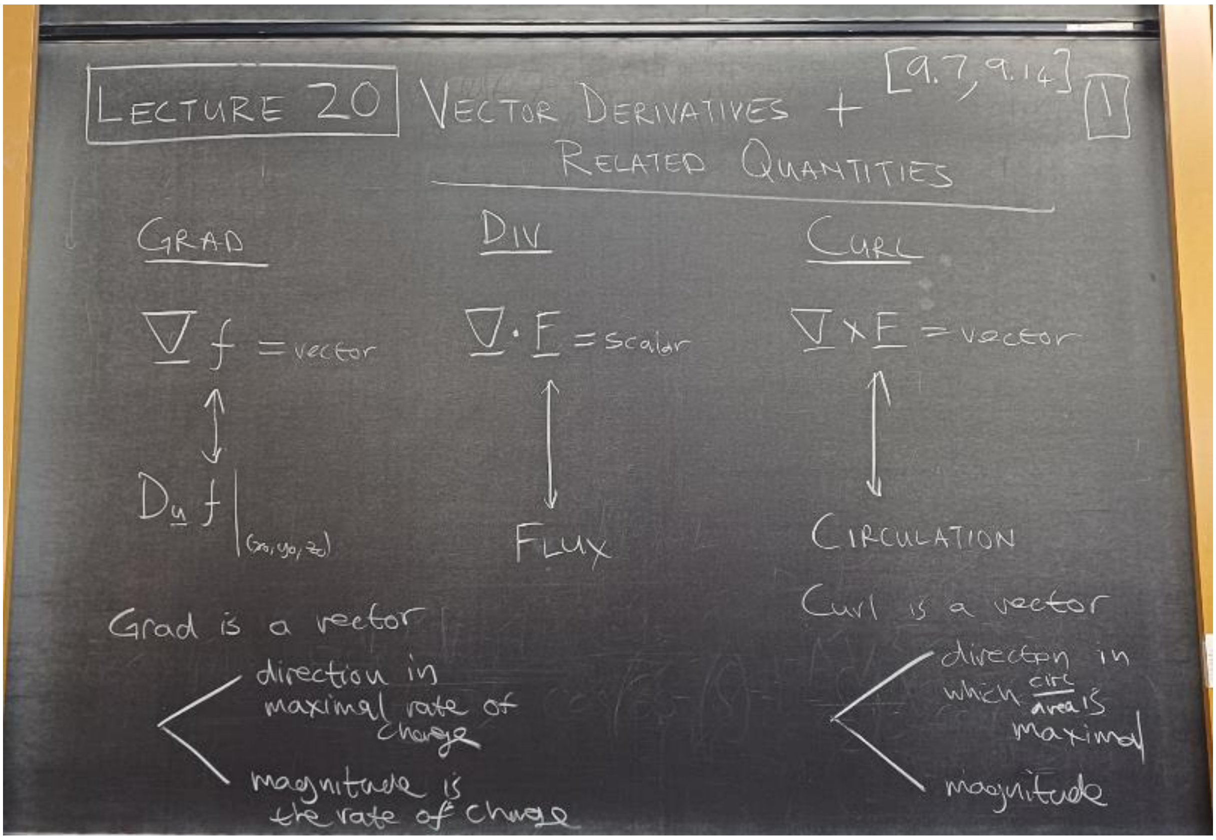
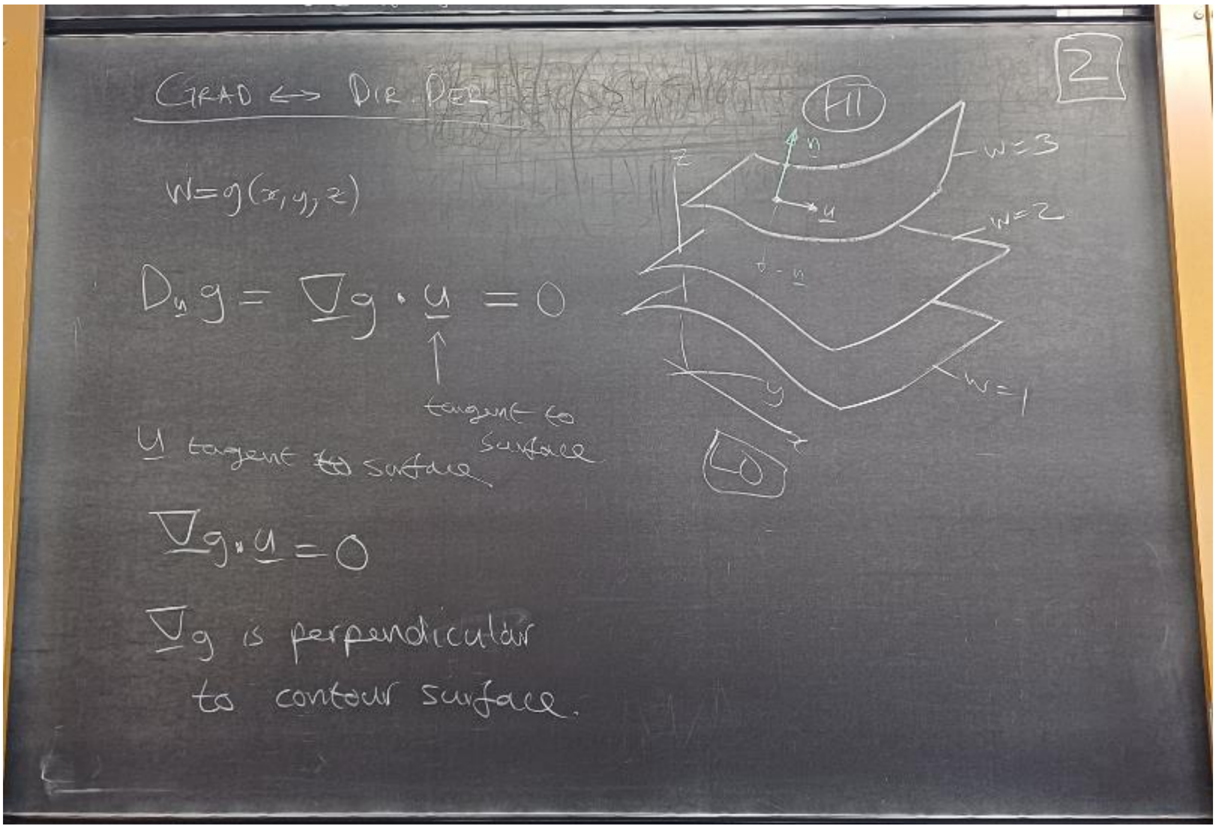
Appl.Maths. B242-2023: LECTURE 20



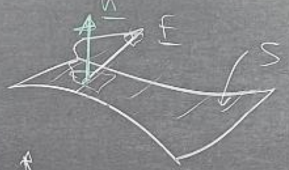


$\boxed{3}$

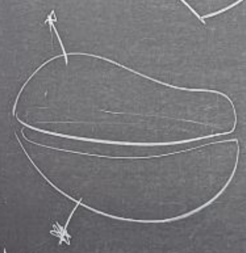
Div $\longleftrightarrow$ Flux

$\underline{E}(x,y,z)$

$$\text{flux}^{\text{up}} = \iint_S \underline{E}\cdot\underline{n}\, dS$$



$$\text{flux out} = \iint_{S_1} \underline{E}\cdot\underline{n}_1\, dS_1 + \iint_{S_2} \underline{E}\cdot\underline{n}_2\, dS_2$$



$$\underline{\nabla}\cdot\underline{E} = \lim_{\Delta V\to 0} \frac{\iint_{\Delta S} \underline{E}\cdot\underline{n}\, dS}{\Delta V}$$

$\boxed{4}$

Circulation

$\underline{E}(x,y,z)$

circulation of $\underline{E}$ around C $= \oint_C \underline{E}\cdot d\underline{r}$ $\quad \underline{r}'(t)\,dt$

$\underline{r}(t)$

$\underline{r}'(t)$

$$dt\,\underline{r}'(t) = \frac{d\underline{r}}{\cancel{dt}}\cancel{dt}$$

Curl + Circulation

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$\underline{F}(x, y, z)$

Stokes:

$$\underbrace{\oint_{\Delta C} \underline{F} \cdot d\underline{r}}_{\text{circulation of } \underline{F} \text{ around } \Delta C} = \underbrace{\iint_{\Delta S} (\underline{\nabla} \times \underline{F}) \cdot \underline{n} \, dS}_{\text{flux of } \underline{\nabla} \times \underline{F} \text{ through } \Delta S}$$

Shrink ΔS

$$\simeq (\underline{\nabla} \times \underline{F}) \cdot \underline{n} \iint_{\Delta S} 1 \, dS$$

$$= (\underline{\nabla} \times \underline{F}) \cdot \underline{n} \ \text{Area}$$

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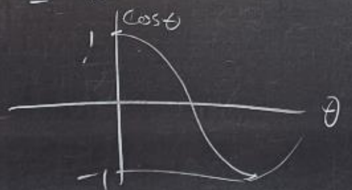
$$\frac{\text{circulation}}{\text{area}} \simeq (\underline{\nabla} \times \underline{F}) \cdot \underline{n}$$

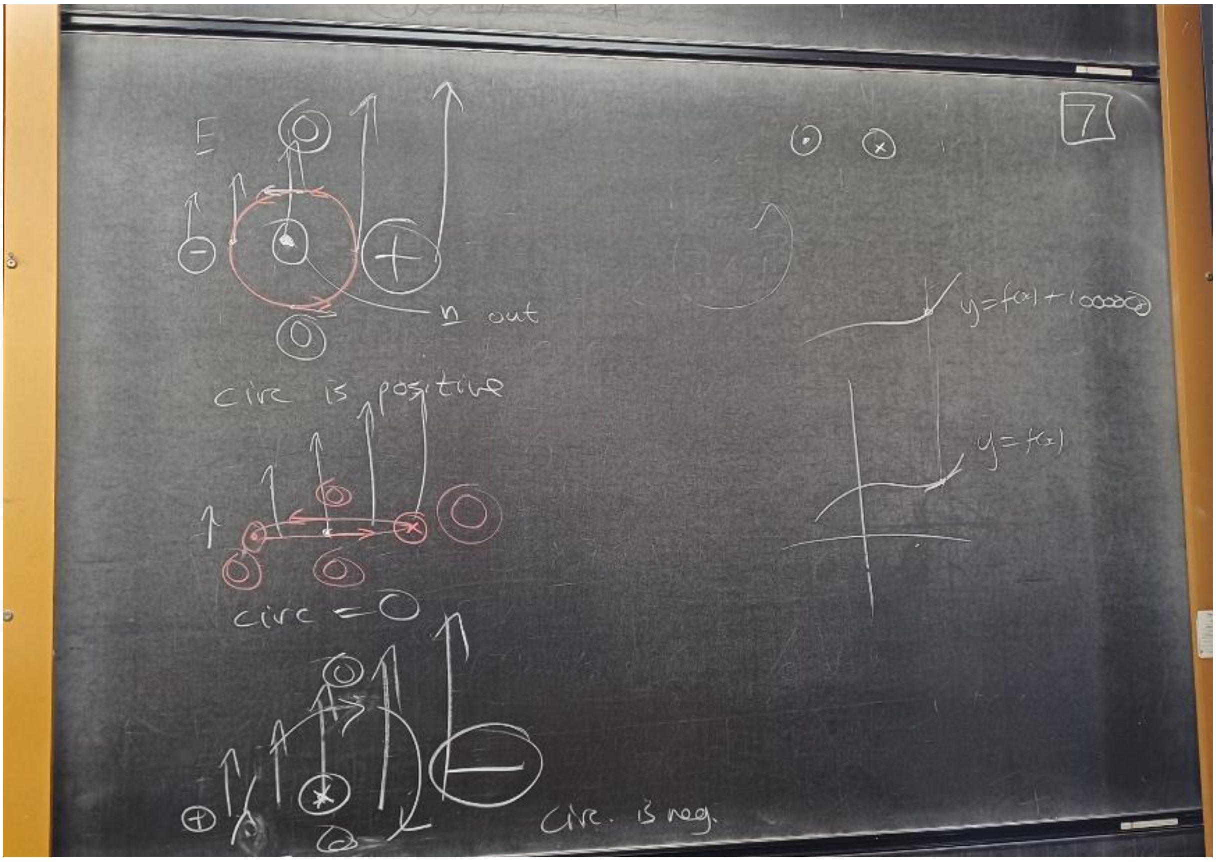
$\Delta S \to 0, \ \Delta C \to 0$

$$(\underline{\nabla} \times \underline{F}) \cdot \underline{n} = \lim_{\Delta S \to 0} \frac{\text{circ of } \underline{F} \text{ around disk}}{\text{area of disc.}}$$

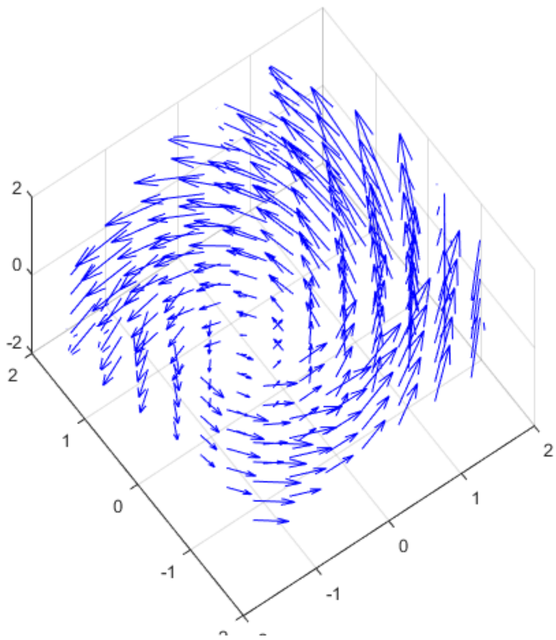
maximum when $\underline{n}$ points along $\underline{\nabla} \times \underline{F}$

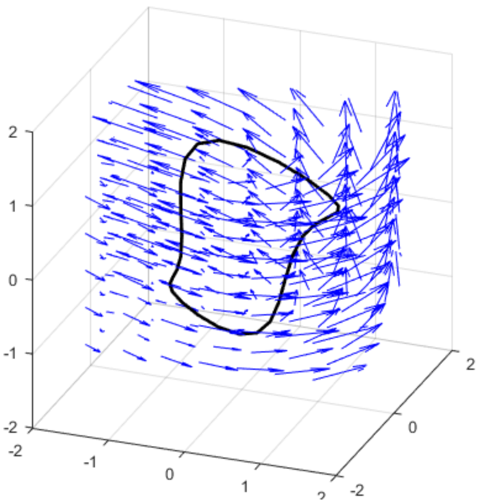
$$\underline{a} \cdot \underline{b} = a \, b \cos\theta$$

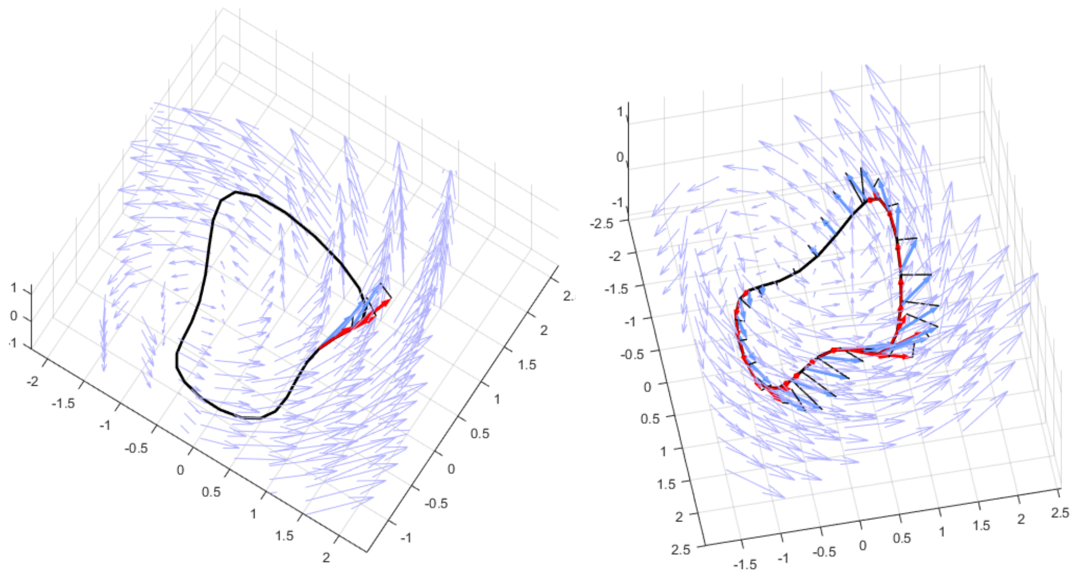




CIRCULATION DEMO







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