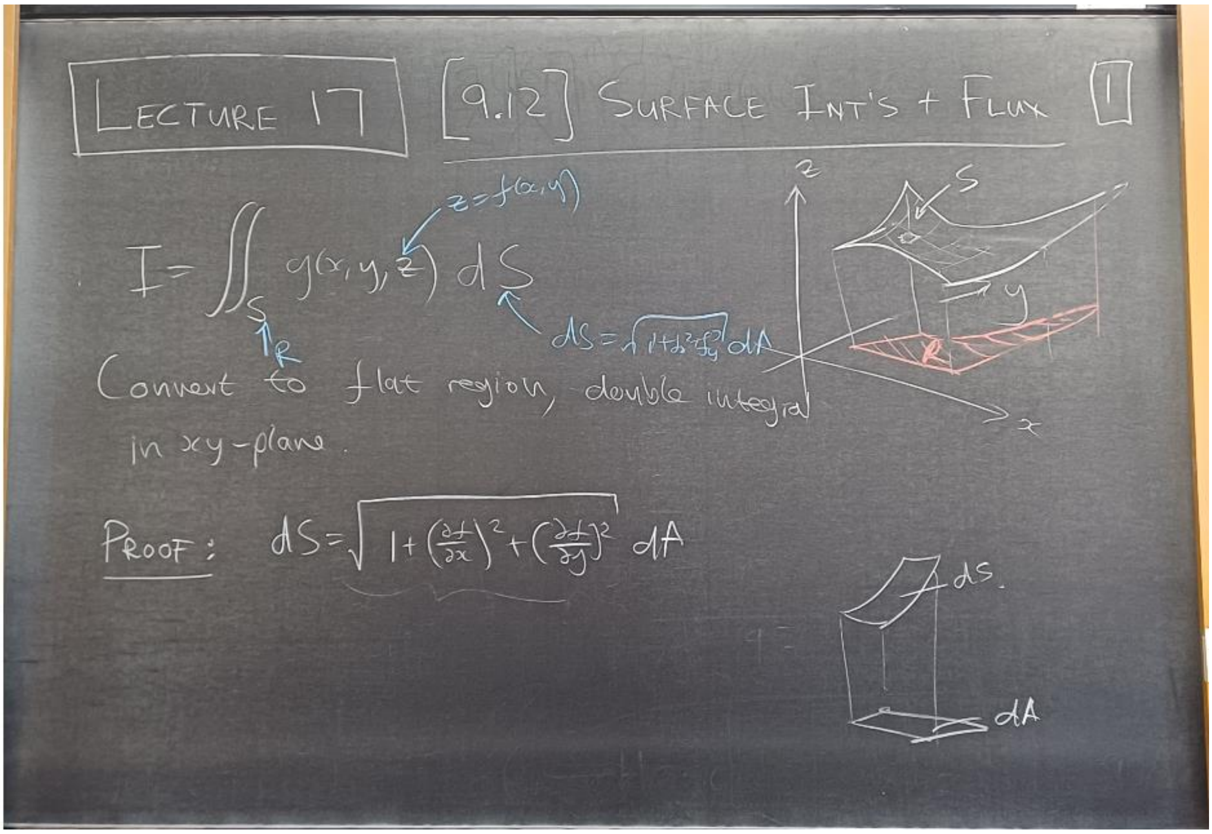
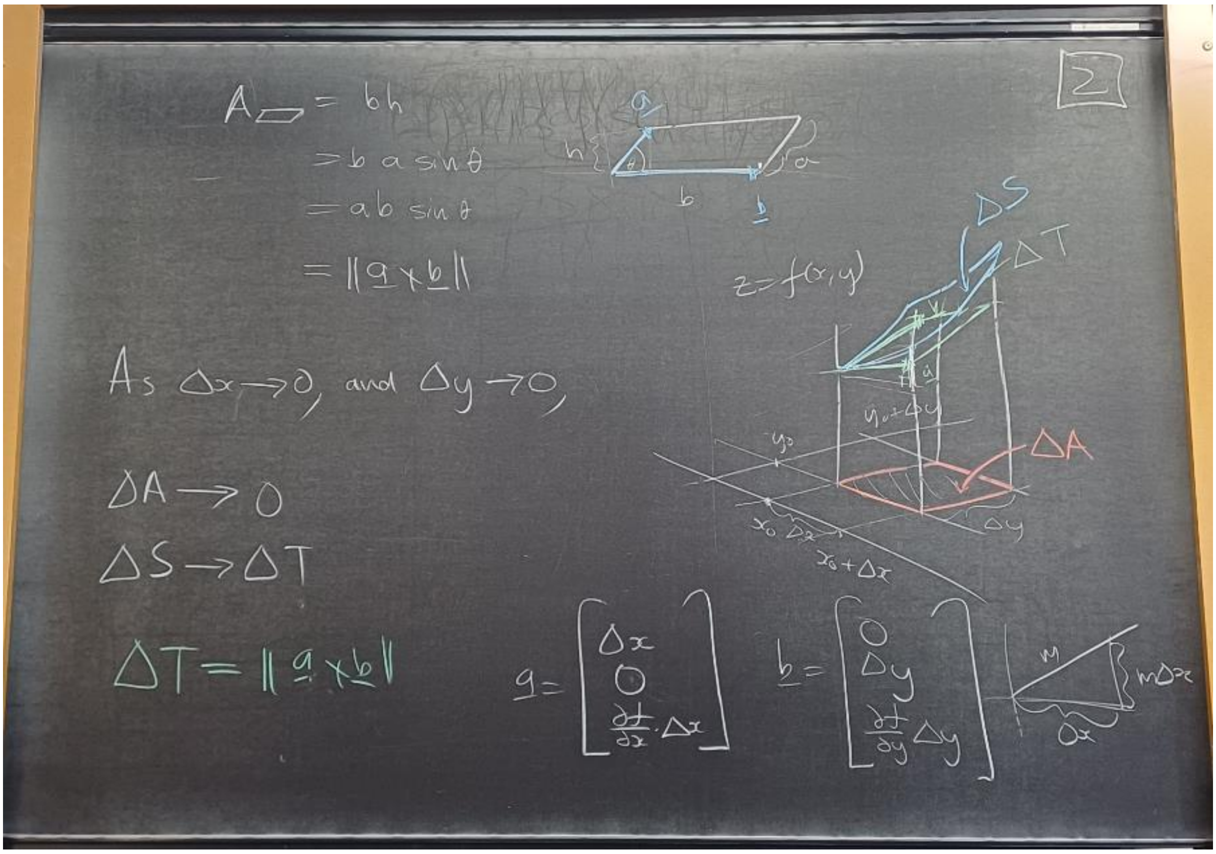
Appl.Maths. B242-2023: LECTURE 17





3

$$\Delta T = \left\| \begin{bmatrix} \Delta x \\ 0 \\ \Delta x\, f_x \end{bmatrix} \times \begin{bmatrix} 0 \\ \Delta y \\ \Delta y\, f_y \end{bmatrix} \right\|$$

$$= \left\| \begin{bmatrix} 0 - \Delta x \Delta y\, f_x \\ 0 - \Delta x \Delta y\, f_y \\ \Delta x \Delta y - 0 \end{bmatrix} \right\|$$

$$\|\lambda \underline{a}\| = |\lambda|\, \|\underline{a}\|$$

$$= \underbrace{\Delta x \Delta y}\, \left\| \begin{bmatrix} -f_x \\ -f_y \\ 1 \end{bmatrix} \right\|$$

$$= \Delta A \sqrt{1^2 + (-f_x)^2 + (-f_y)^2}$$

$$\Delta S \to \Delta T \to \Delta A \sqrt{1 + f_x^2 + f_y^2}$$

Take limit. $\boxed{dS = \sqrt{1 + f_x^2 + f_y^2}\, dA}$

4

Uses:

- area $\quad A = \iint_S 1\, dS$
- mass $\quad m = \iint_S \rho(x,y,z)\, dS$
- coords of centroid

$$\begin{cases} m\bar{x} = \iint_S x \rho\, dS \\ m\bar{y} = \iint_S y \rho\, dS \\ m\bar{z} = \iint_S z \rho\, dS \end{cases}$$

- moments of inertia

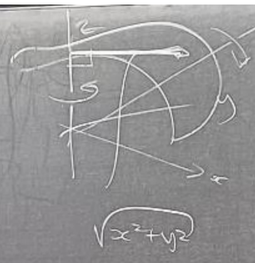
$$M = I \alpha$$

$$I_{zz} = \iint_S \rho \sqrt{x^2+y^2}^{\,2}\, dS$$

I_{xx} $\quad I_{yy}$

⊙ flux through S.

⊙ flux out of a body B.



FLUX:

upward flux of $\underline{E}$ through S

$z = f(x,y)$

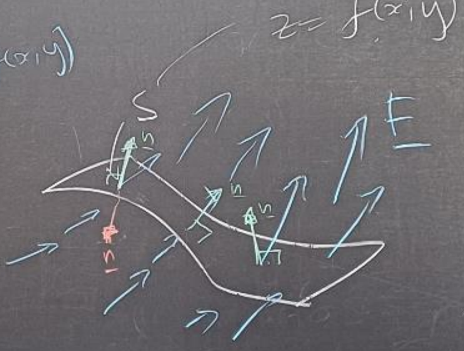
subset

normal

$$= \iint_S \underline{E}\cdot\underline{n}\, dS$$

$S \leftarrow R$

$dS \leftarrow ds$

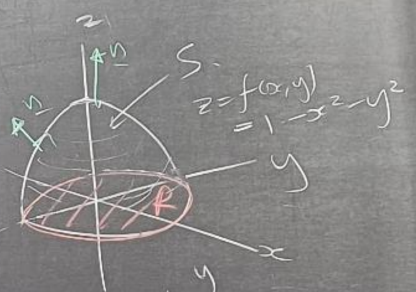


$\underline{n}$ is "upwards" if its z-component is positive.

Example: Find upward flux of $\underline{E}$ through S.

S: $z = 1 - x^2 - y^2$, $z \geq 0$

$$\underline{E} = \begin{bmatrix} y \\ x \\ 3 \end{bmatrix}$$

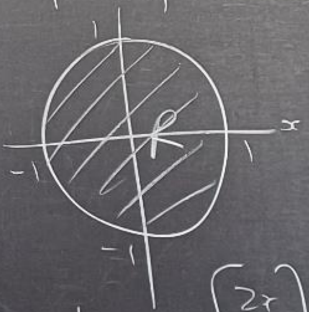


$S \to R$ ✓

Normal: ✓

$$G = 1 - x^2 - y^2 - z$$

$$-\underline{\nabla} G = \begin{bmatrix} +2x \\ +2y \\ +1 \end{bmatrix} \frac{1}{\sqrt{1^2 + (2x)^2 + (2y)^2}}$$



$$\underline{n} = \frac{1}{\sqrt{1 + 4x^2 + 4y^2}} \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix}$$

$\boxed{7}$

Surface element:

$z = 1 - x^2 - y^2$

$$dS = \sqrt{1 + (-2x)^2 + (-2y)^2}\, dA$$

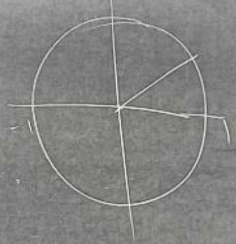
$$= \sqrt{1 + 4x^2 + 4y^2}\, dA$$

$$\text{flux} = \iint_S \underline{F} \cdot \underline{n}\, dS$$

$$= \iint_R \begin{bmatrix} y \\ x \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix} \frac{1}{\cancel{\sqrt{1+4x^2+4y^2}}} \cancel{\sqrt{1+4x^2+4y^2}}\, dA$$

$\boxed{8}$

$$= \iint_R (4xy + 3)\, dA$$



$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{1} (4 r\cos\theta\, r\sin\theta + 3)\, r\, dr \cdot d\theta$$

$\vdots$ work

$\vdots$

$$= 3\pi$$