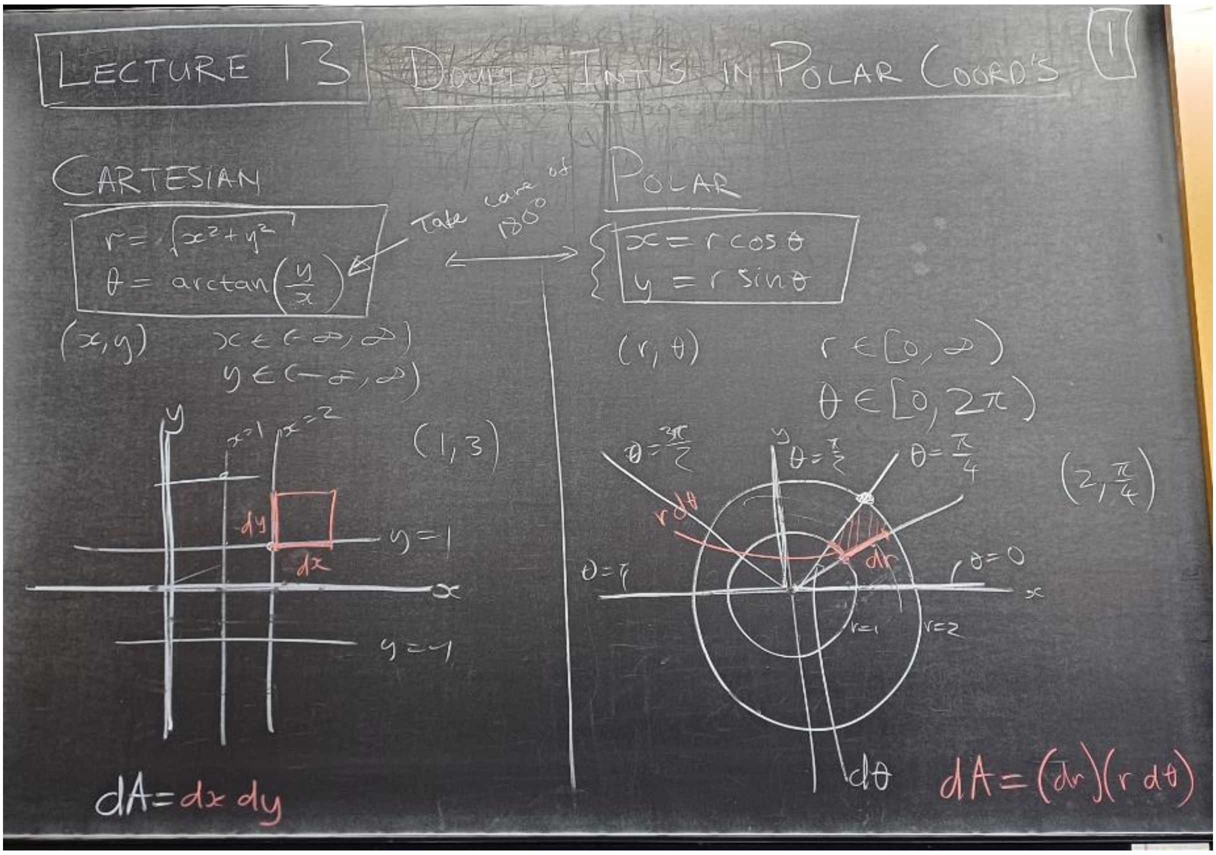
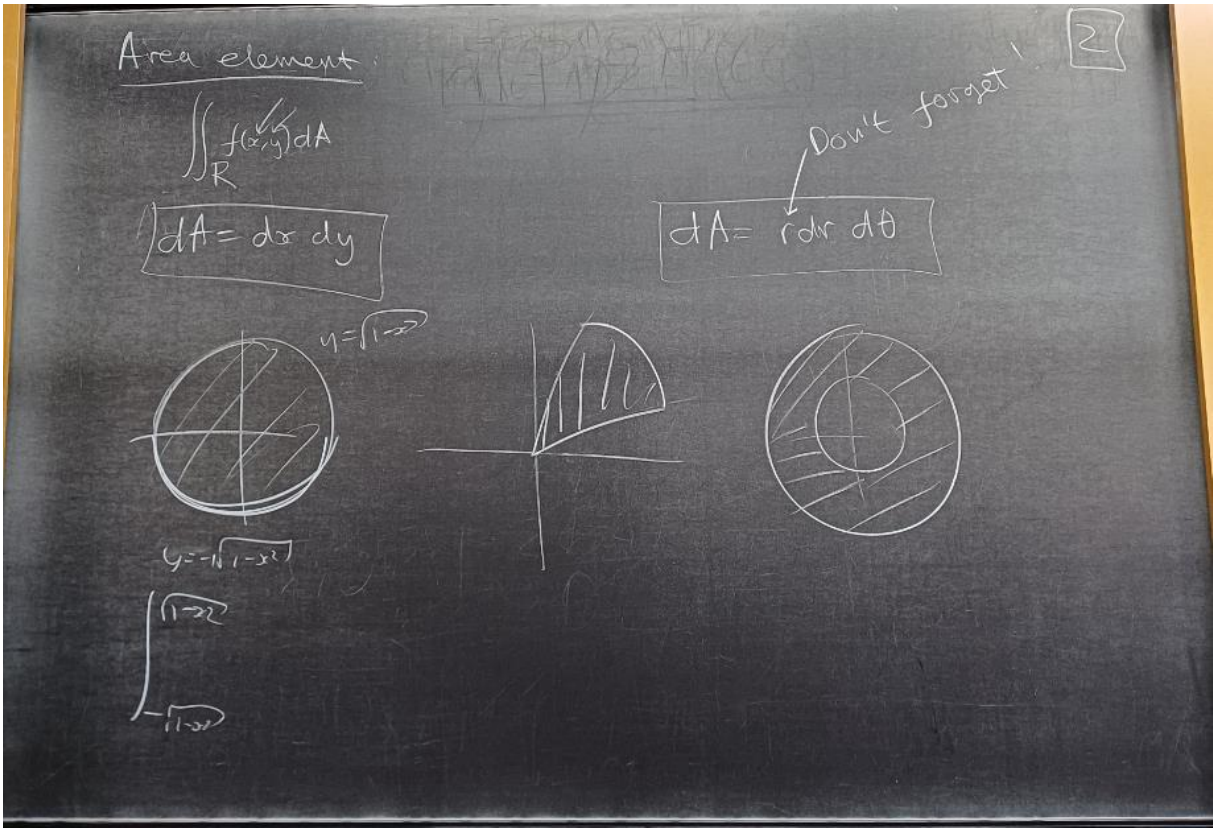
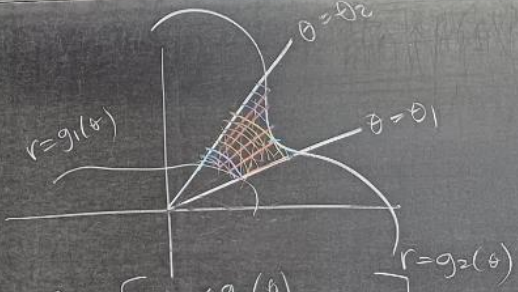
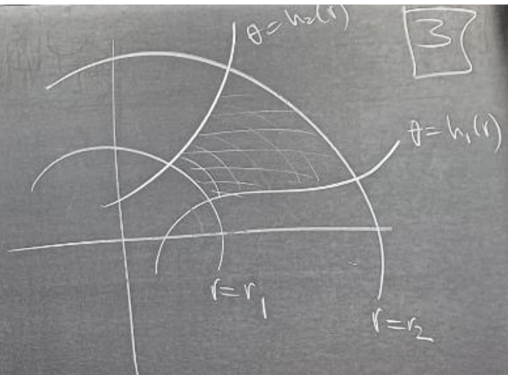
Appl.Maths. B242-2023: LECTURE 13





3





$$\int_{\theta=\theta_1}^{\theta_2}\left[\int_{r=g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\,dr\right] d\theta$$

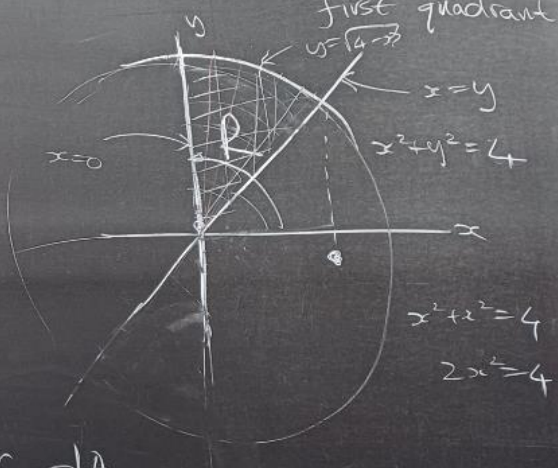
4

EXAMPLE:

$$U=\iint_R \frac{x}{1+(x^2+y^2)^{3/2}}\,dA$$

$$R:\begin{cases} x=y \\ x=0 \\ x^2+y^2=4 \end{cases}$$

first quadrant



$x^2+x^2=4$

$2x^2=4$

In Cartesian

$$U=\int_{x=0}^{\sqrt{2}}\int_{y=x}^{\sqrt{4-x^2}} (\text{expression})\,dy\,dx.$$

In Polar

$$U=\int_{\theta=\frac{\pi}{4}}^{\pi/2}\int_{r=0}^{2} \frac{r\cos\theta}{1+r^3}\,r\,dr\,d\theta$$

$\boxed{5}$

$$= \int_{\theta=\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\int_{r=0}^{2} \frac{r^2}{1+r^3} \cdot \cos\theta \, dr \right] d\theta$$

$$= \int_{\theta=\pi/4}^{\pi/2} \cos\theta \underbrace{\left[\int_{0}^{2} \frac{r^2}{1+r^3} \, dr \right]} d\theta$$

$$= \left[\int_{\theta=\pi/4}^{\pi/2} \cos\theta \, d\theta \right] \times \left[\frac{1}{3} \int_{0}^{2} \frac{3r^2}{1+r^3} \, dr \right]$$

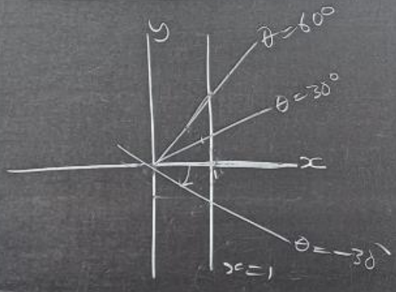
$$= \Big[\sin\theta \Big]_{\pi/4}^{\pi/2} \cdot \left[\frac{1}{3} \ln(1+r^3) \right]_0^2$$

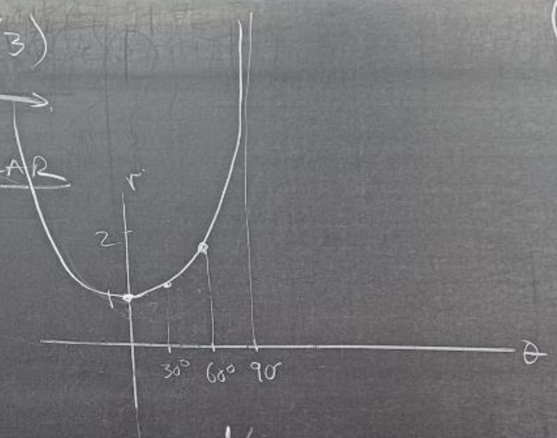
$$= \left(1 - \frac{1}{\sqrt{2}}\right) \frac{1}{3} \left[\ln(9) - \ln(1) \right]$$

$\boxed{6}$

$$= \frac{1}{3}\left(1 - \frac{1}{\sqrt{2}}\right) 2\ln(3)$$

PLOTTING IN POLAR

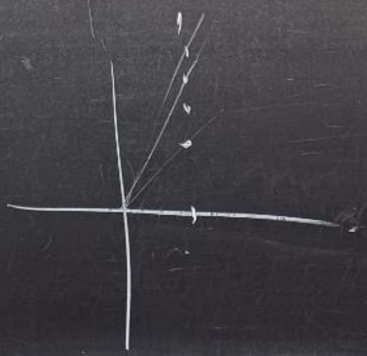




$\boxed{x = 1}$

$r\cos\theta = 1$

$$\boxed{r = \frac{1}{\cos\theta}}$$



OFF-CENTRE CIRCLES

7

$$y^2 = 2x - x^2$$

$$\boxed{x^2 - 2x + y^2 = 0}$$

$$x^2 - 2x + 1 + y^2 = 1$$

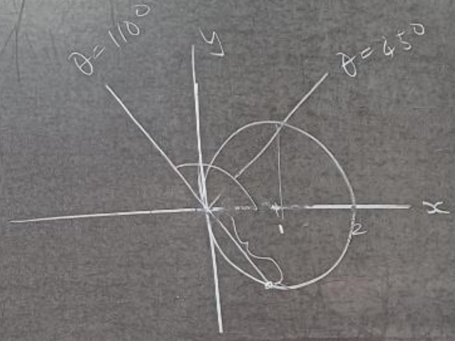
$$(x-1)^2 + y^2 = 1$$

$$x^2 + y^2 - 2x = 0$$

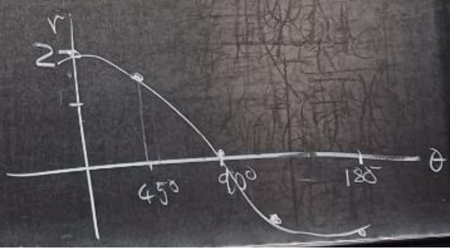
$$r^2 - 2r\cos\theta = 0$$

$$r(r - 2\cos\theta) = 0$$

$$\boxed{r = 2\cos\theta}$$



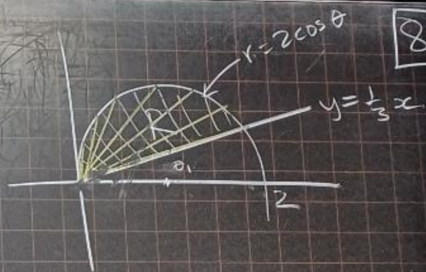
$r = f(\theta)$



EXAMPLE

8

Find area of R



$\tan\theta_1 = \frac{1}{3}$

In Polar

$$\text{Area} = \int_{\theta=\operatorname{atan}(\frac{1}{3})}^{\pi/2} \left[\int_{r=0}^{2\cos\theta} r\,dr \right] d\theta$$

$$= \int_{\operatorname{atan}(\frac{1}{3})}^{\pi/2} \left[\frac{r^2}{2} \right]_0^{2\cos\theta} d\theta = \int_{\operatorname{atan}(\frac{1}{3})}^{\pi/2} \frac{4\cos^2\theta}{2}\, d\theta$$

$$= 2\int_{\operatorname{atan}(\frac{1}{3})}^{\pi/2} \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \left[\theta + \frac{\sin(2\theta)}{2}\right]_{\operatorname{atan}\left(\frac{1}{3}\right)}^{\pi/2} \qquad \boxed{9}$$

$$= \frac{\pi}{2} - \frac{3}{10} - \arctan\left(\frac{1}{3}\right) = 0.949$$