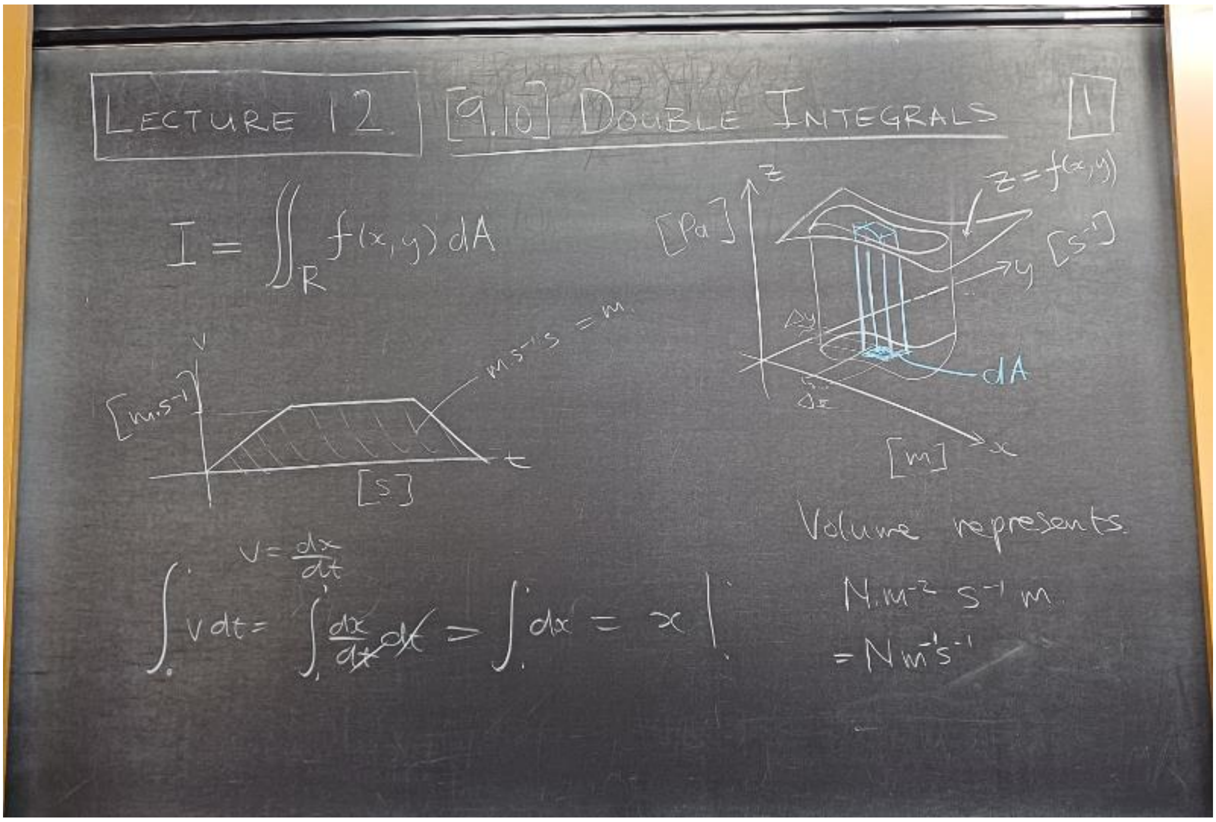
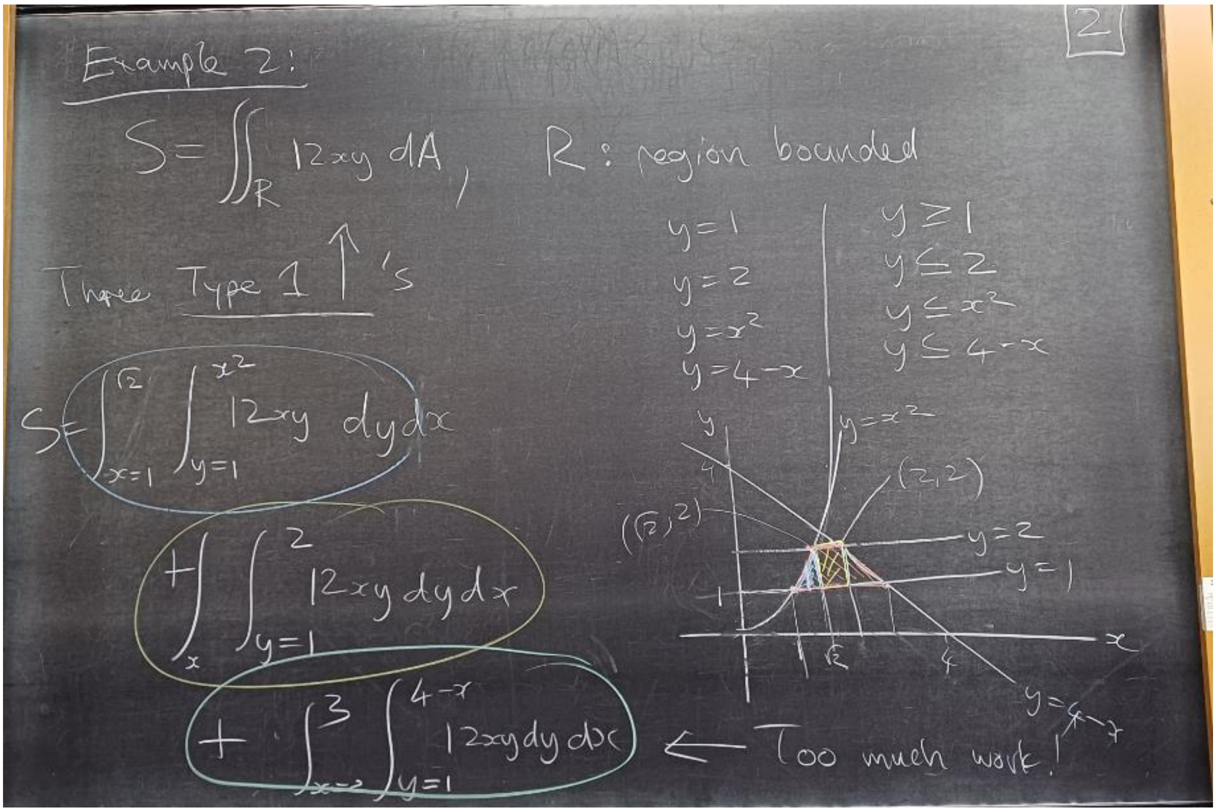
Appl.Maths. B242-2023: LECTURE 12

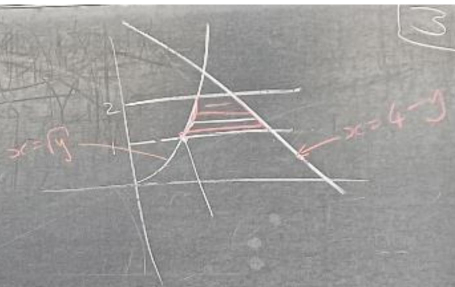




3

As Type 2

$$S = \int_{y=1}^{2}\int_{x=\sqrt{y}}^{4-y} 12xy\,dx\,dy$$



REVERSE ORDER

$$I = \int_{x=0}^{2}\int_{y=x^2}^{4} x e^{y^2}\,dy\,dx$$

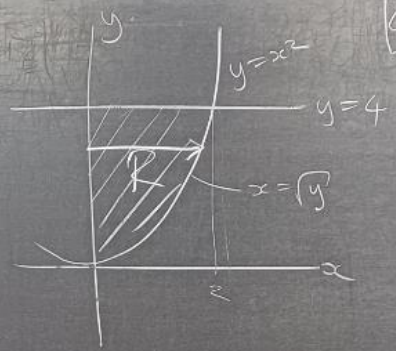
What is $\int e^{y^2}dy = \cancel{\int \frac{e^u}{2\sqrt{u}}\,du}$ $\quad y^2=u$

$dy = \frac{du}{2y}$

$= -\frac{\sqrt{\pi}}{2}\,\mathrm{erfi}(y)$ imaginary error function.

4

As Type 2:



$$\int_{y=0}^{4}\left[\int_{x=0}^{\sqrt{y}} x e^{y^2}\,dx\right]dy$$

$$= \int_{y=0}^{4}\left[\frac{x^2}{2}e^{y^2}\right]_0^{\sqrt{y}}dy$$

$$= \int_0^4\left(\frac{y}{2}e^{y^2} - 0\right)dy = \frac{1}{4}\int_0^4 2y\,e^{y^2}\,dy$$

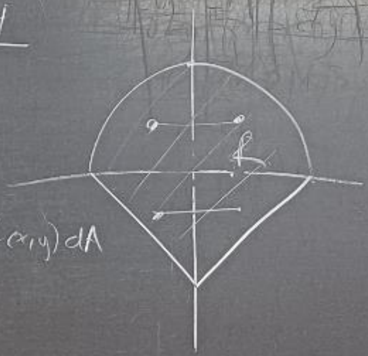
$$= \frac{1}{4}\left[e^{y^2}\right]_0^4 = \frac{1}{4}\left(e^{16}-1\right)$$

$$\int f(g(x))\,g'(x)\,dx = F(g(x))$$

SYMMETRY

5

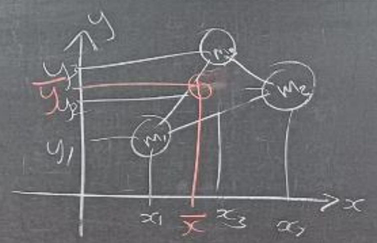
$$\iint_R f(x,y)\,dA \overset{?}{=} 2\iint_P f(x,y)\,dA$$



MASS + CENTROIDS

"lamina"

6

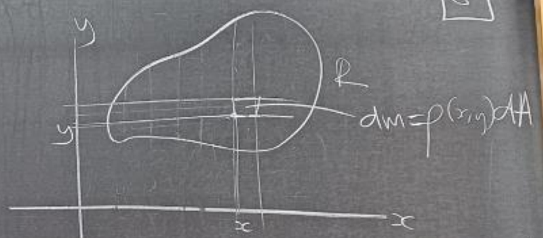


Point masses.

$$m = m_1 + m_2 + m_3 = \sum_{k=1}^{N} m_k$$

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m}$$

$$m\bar{x} = \sum_{k=1}^{N} m_k x_k$$



area density $= \rho(x,y)$

Flat object

$$m = \iint_R \rho\,dA$$

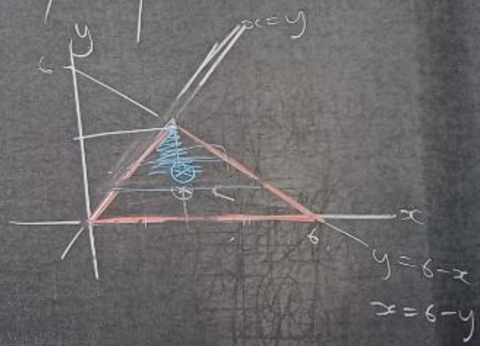
$$m\bar{x} = \iint_R x\rho\,dA$$

$$m\bar{y} = \iint_R y\rho\,dA$$

$\boxed{7}$

Example. Find the coordinates of the centroid of R.

$R: x=y,\ y=6-x,\ y=0$, $\rho = 2y$



$\bar{x} = 3$ (from symmetry)

$$m = \iint_R 2y\, dA$$

$$= \int_{y=0}^{3} \int_{x=y}^{6-y} 2y\, dx$$

$$= \cdots\cdots\ 18$$

$\boxed{8}$

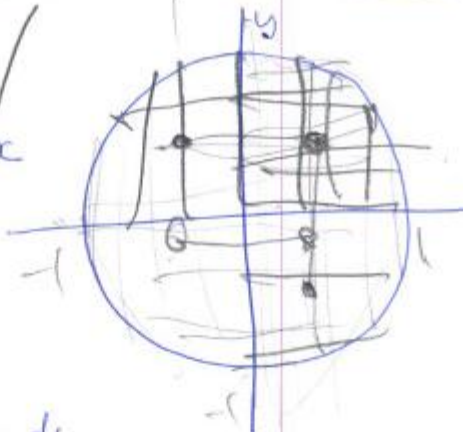
$$m\bar{y} = \iint_R y\rho\, dA = \int_{y=0}^{3} \int_{x=y}^{6-y} (2y)(y)\, dx\, dy$$

$$= \cdots\cdots\ 27$$

$$\bar{y} = \frac{27}{18} = 1.5$$

USING SYMMETRY

$$K = \iint_R 2xy^2 \, dA \qquad R: x^2+y^2 \le 1$$



[A] $$K = \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2xy^2 \, dy \, dx$$

[B] $$K = \int_{y=-1}^{1} \int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2xy^2 \, dx \, dy$$

[C] $$K = 2\int_{x=-1}^{1} \int_{y=0}^{\sqrt{1-x^2}} 2xy^2 \, dy \, dx$$

(D) $$K = 2\int_{y=-1}^{1} \int_{x=0}^{\sqrt{1-y^2}} 2xy^2 \, dx \, dy$$

(E) $$K = 4\int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} 2xy^2 \, dx \, dy$$