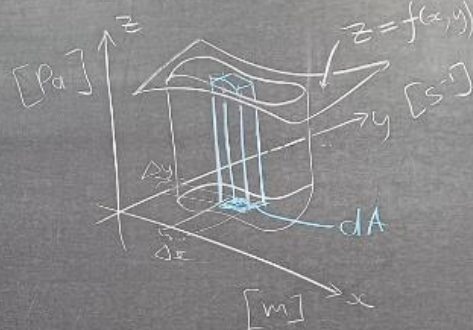
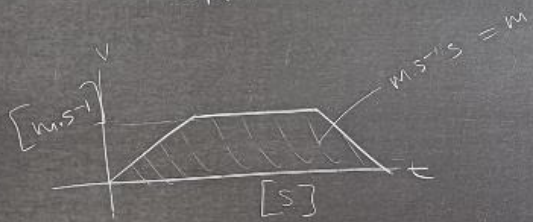


Appl. Maths. B242-2023: LECTURE 12

LECTURE 12 [9.10] DOUBLE INTEGRALS [1]

$$I = \iint_R f(x,y) dA$$



$$\int v dt = \int \frac{dx}{dt} dt = \int dx = x$$

Volume represents
 $N \cdot m^{-2} s^{-1} m$
 $= N m^{-1} s^{-1}$

Example 2: [2]

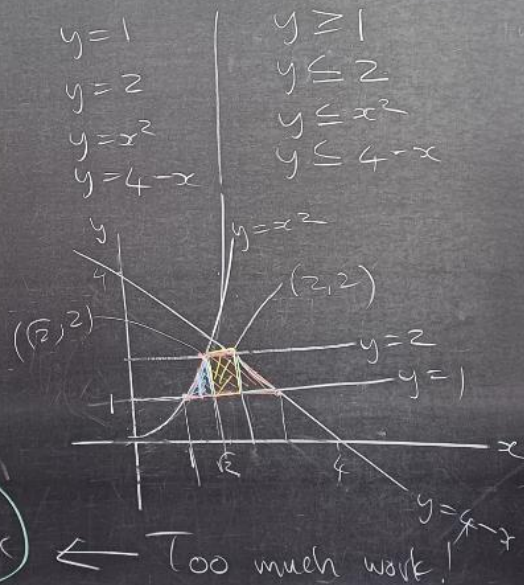
$$S = \iint_R 12xy dA, \quad R: \text{region bounded}$$

Three Type 1 ↑'s

$$S = \int_{x=1}^{\sqrt{2}} \int_{y=1}^{x^2} 12xy dy dx$$

$$+ \int_x^2 \int_{y=1}^2 12xy dy dx$$

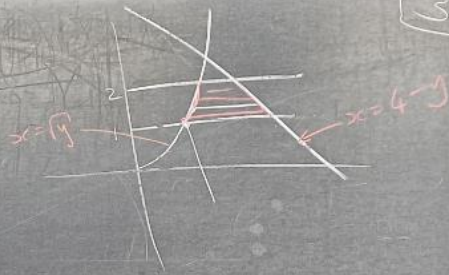
$$+ \int_{x=2}^3 \int_{y=1}^{4-x} 12xy dy dx$$



← Too much work!

As Type 2

$$S = \int_{y=1}^2 \int_{x=\sqrt{y}}^{4-y} 12xy \, dx \, dy$$



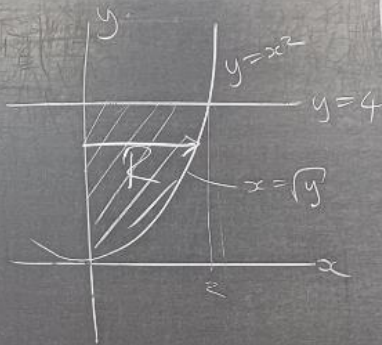
REVERSE ORDER

$$I = \int_{x=0}^2 \int_{y=x^2}^4 x e^{y^2} \, dy \, dx$$

What is $\int e^{y^2} \, dy = \int \frac{e^u \, du}{2y}$ $y^2 = u$
 $dy = \frac{du}{2y}$
 $= -\frac{\sqrt{\pi}}{2} \operatorname{erfi}(y)$ *imaginary error function.*

As Type 2

$$\int_{y=0}^4 \left[\int_{x=0}^{\sqrt{y}} x e^{y^2} \, dx \right] dy$$
$$= \int_{y=0}^4 \left[\frac{x^2}{2} e^{y^2} \right]_0^{\sqrt{y}} dy$$



$$= \int_0^4 \left(\frac{y}{2} e^{y^2} - 0 \right) dy = \frac{1}{4} \int_0^4 2y e^{y^2} dy$$

$$= \frac{1}{4} \left[e^{y^2} \right]_0^4 = \frac{1}{4} (e^{16} - 1)$$

$$\int f(g(x)) g'(x) \, dx = F(g(x))$$

SYMMETRY

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$$\iint_R f(x,y) dA$$

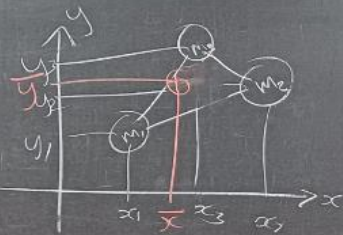
$$\stackrel{?}{=} 2 \iint_P f(x,y) dA$$



MASS + CENTROIDS

"lamina"

6



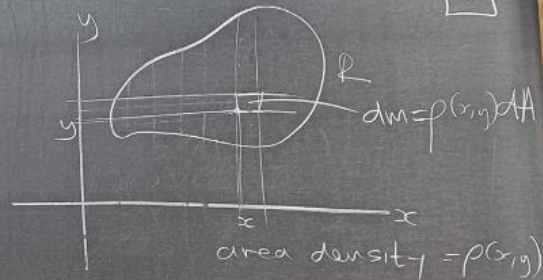
Point masses:

$$m = m_1 + m_2 + m_3$$

$$= \sum_{k=1}^N m_k$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m}$$

$$m \bar{x} = \sum_{k=1}^N m_k x_k$$



Flat object

$$m = \iint_R \rho dA$$

$$m \bar{x} = \iint_R x \rho dA$$

$$m \bar{y} = \iint_R y \rho dA$$

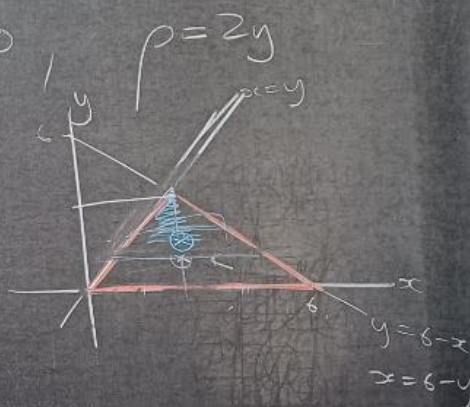
Example. Find the coordinates of the centroid of R .

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$$R: x=y, y=6-x, y=0$$

$$\bar{x} = 3 \quad (\text{from symmetry})$$

$$\begin{aligned} m &= \iint_R 2y \, dA \\ &= \int_{y=0}^3 \int_{x=y}^{6-y} 2y \, dx \\ &= \dots \dots \dots 18 \end{aligned}$$



$$\begin{aligned} m\bar{y} &= \iint_R y \rho \, dA = \int_{y=0}^3 \int_{x=y}^{6-y} (2y)(y) \, dx \, dy \\ &= \dots \dots \dots 27 \end{aligned}$$

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$$\bar{y} = \frac{27}{18} = 1.5$$

USING SYMMETRY

$$K = \iint_R 2xy^2 dA \quad R: x^2 + y^2 \leq 1$$

$$[A] \quad K = \int_{x=-1}^1 \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2xy^2 dy dx \quad \checkmark$$

$$[B] \quad K = \int_{y=-1}^1 \int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2xy^2 dx dy \quad \checkmark$$

$$[C] \quad K = 2 \int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} 2xy^2 dy dx \quad \checkmark$$

$$[D] \quad K = 2 \int_{y=-1}^1 \int_{x=0}^{\sqrt{1-y^2}} 2xy^2 dx dy \quad \times$$

$$[E] \quad K = 4 \int_{y=0}^1 \int_{x=0}^{\sqrt{1-y^2}} 2xy^2 dx dy \quad \times$$

