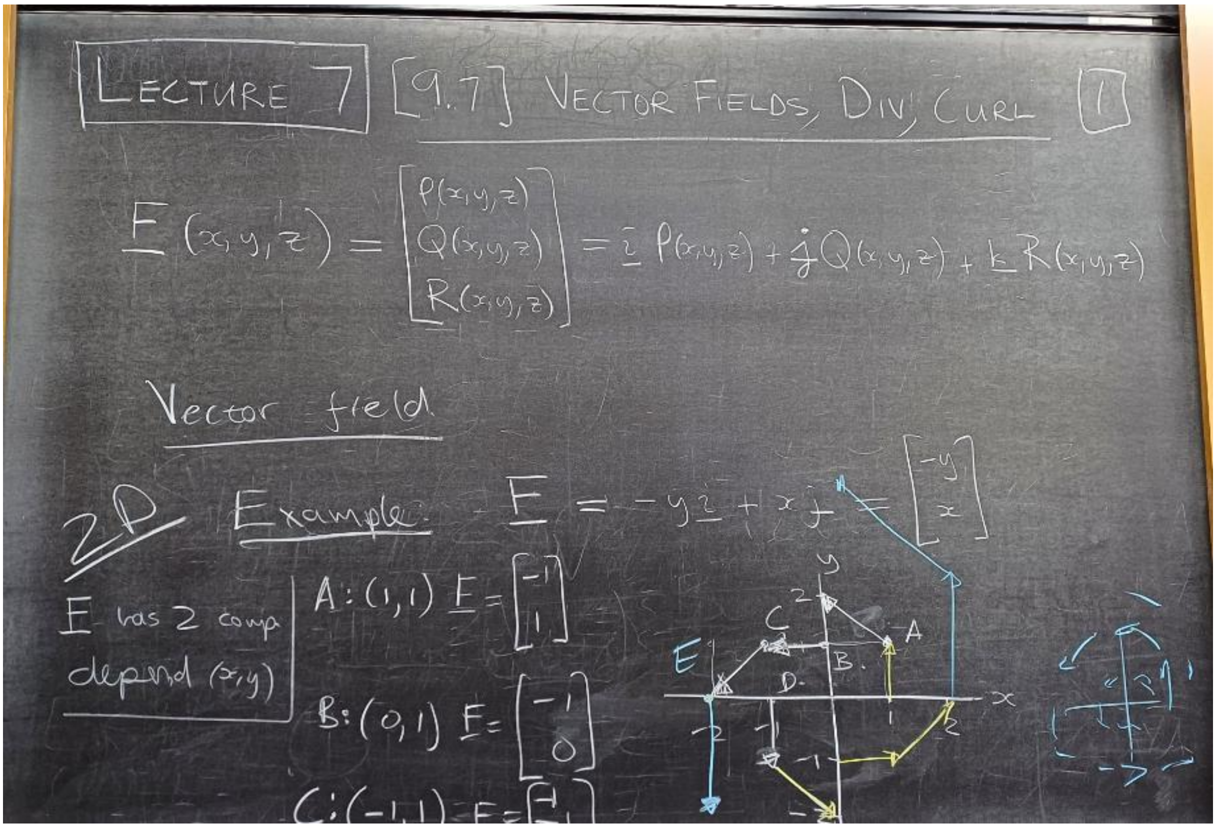
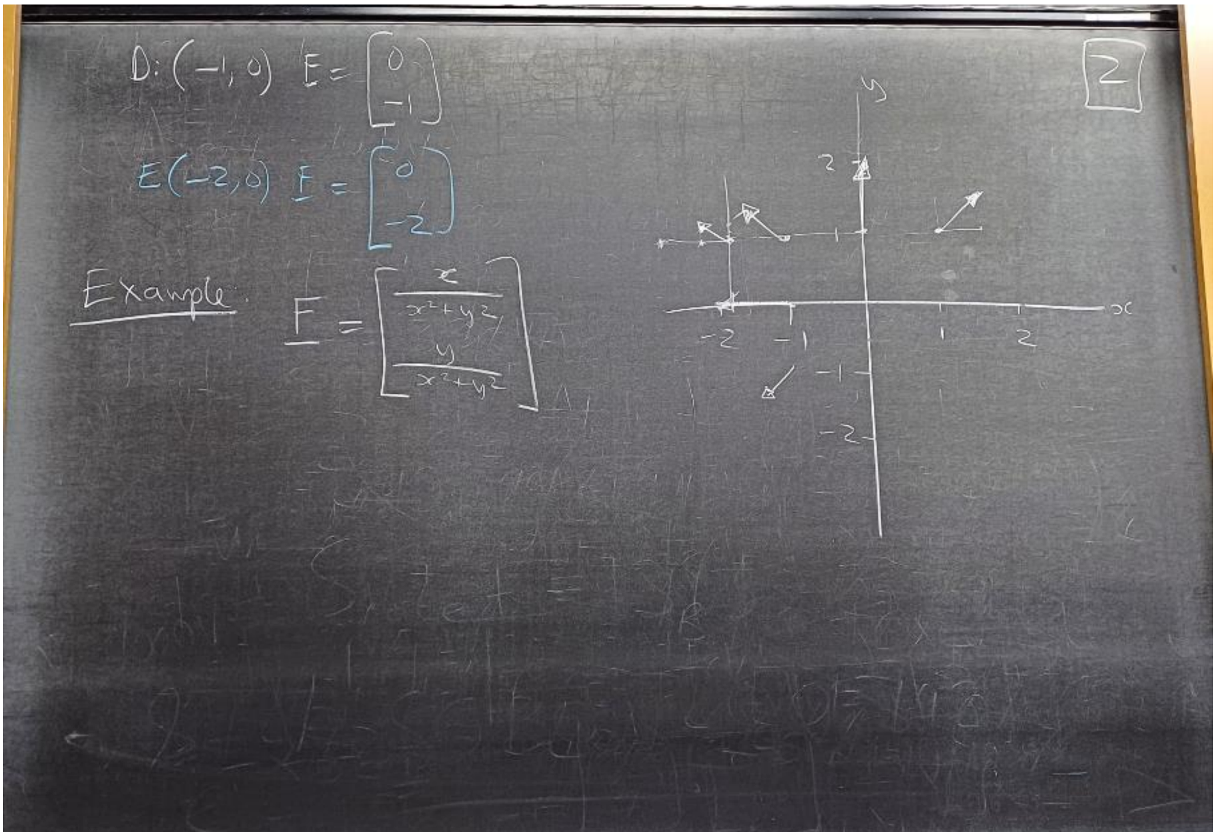
Appl.Maths. B242-2023: LECTURE 7





SUMMARY

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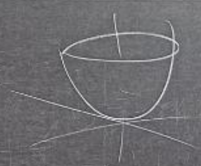
$\underline{r}(t)$

space curve

$\underline{r}'(t)$ — tangent vector

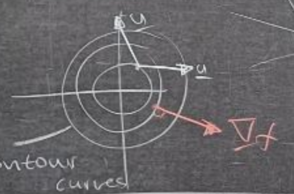
$z = f(x,y)$



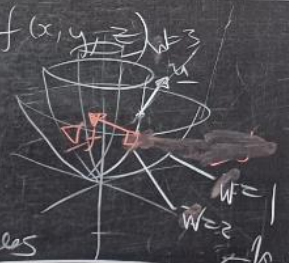
surface

$D_{\underline{u}} f \Big|_{(x_0,y_0)}$, $\underline{\nabla} f \Big|_{(x_0,y_0)}$

GRADIENT.



$w = f(x,y,z)$



volume function.

$D_{\underline{u}} f \Big|_{(x_0,y_0,z_0)}$, $\underline{\nabla} f \Big|_{(x_0,y_0,z_0)}$

~~$\underline{\nabla} f (x_0, y_0, z_0)$~~

$\underline{F}(x,y,z)$?

DIVERGENCE

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$$\underline{\nabla} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$$

$$\underline{\nabla} \cdot \underline{F} = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \cdot \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \left(\frac{\partial}{\partial x}\right)P + \left(\frac{\partial}{\partial y}\right)Q + \left(\frac{\partial}{\partial z}\right)R$$

$$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$= P_x + Q_y + R_z$$

Subscript notation

$$\frac{\partial f}{\partial x} = f_x \quad , \quad \frac{\partial^2 f}{\partial x^2} = f_{xx} \quad , \quad \frac{\partial^2 f}{\partial x \partial y} = f_{xy}$$

Example: Calculate $\underline{\nabla} \cdot \underline{F}$ for $\underline{F} = \begin{bmatrix} xyz \\ y^2 z e^x \\ 2x + z^3 \end{bmatrix}$

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$$\underline{\nabla}\cdot\underline{F} = \begin{bmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{bmatrix}\cdot\begin{bmatrix} xyz \\ y^2ze^x \\ 2x+z^3\end{bmatrix} = yz + 2yze^x + 3z^2$$

CURL:

$$\underline{\nabla}\times\underline{F} = \begin{bmatrix}\partial/\partial x\\ \partial/\partial y\\ \partial/\partial z\end{bmatrix}\times\begin{bmatrix} P \\ Q \\ R\end{bmatrix} = \begin{bmatrix}\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\\ \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\\ \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\end{bmatrix}$$

Example: Find curl of $\underline{F} = xyz\,\underline{i} + y^2ze^x\,\underline{j} + (2x+z^3)\,\underline{k}$

$$\underline{\nabla}\times\underline{F} = \begin{bmatrix}\partial_x\\ \partial_y\\ \partial_z\end{bmatrix}\times\begin{bmatrix} xyz \\ y^2ze^x \\ 2x+z^3\end{bmatrix} = \begin{bmatrix} 0 - y^2e^x \\ xy - 2 \\ y^2ze^x - xz\end{bmatrix}$$

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THREE DIFFERENTIAL OPERATORS

Gradient

$\underline{\nabla} f \longrightarrow$ vector field

f ↑ scalar func

$\underline{\nabla} f$ point in dir of maximum increase

$\underline{\nabla} f$, "del f", "gradient of f", "grad f"

Divergence

$\underline{\nabla}\cdot\underline{F} \longrightarrow$ scalar function

$\underline{F}$ ↑ vector field

$\underline{\nabla}\cdot\underline{F}$ "del dot F", "divergence of F", "div F"

Curl

$\underline{\nabla}\times\underline{F} \longrightarrow$ vector field.

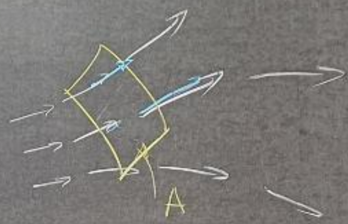
$\underline{F}$ ↑ vector field

$\underline{\nabla}\times\underline{F}$ "del cross F" "curl F"

Flux of $\underline{F}$ through a surface

$\boxed{7}$

Fluid velocity



$\underline{F}$ in $m.s^{-1}$

Flux of $\underline{F}$ in $(m.s^{-1})(m^2)$

$m^3.s^{-1}$

volume per time.

$$\text{Flux} = \|\underline{F}_\perp\| \cdot A$$

$$\text{Flux} = \underline{F} \cdot \underline{n}\ A$$

$$= \underbrace{\|\underline{F}\| \cdot 1 (\cos\theta)}\,A$$

$$= F_\perp A$$

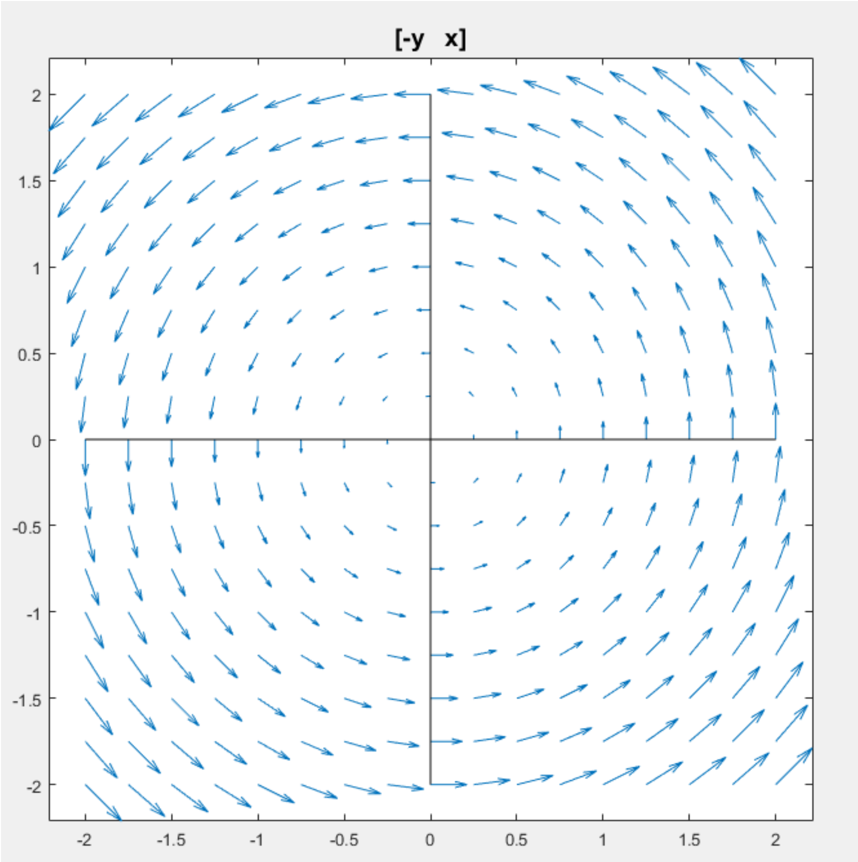
$$\text{flux } \int_{\text{surface}} \underline{F} \cdot \underline{n}\, dA$$

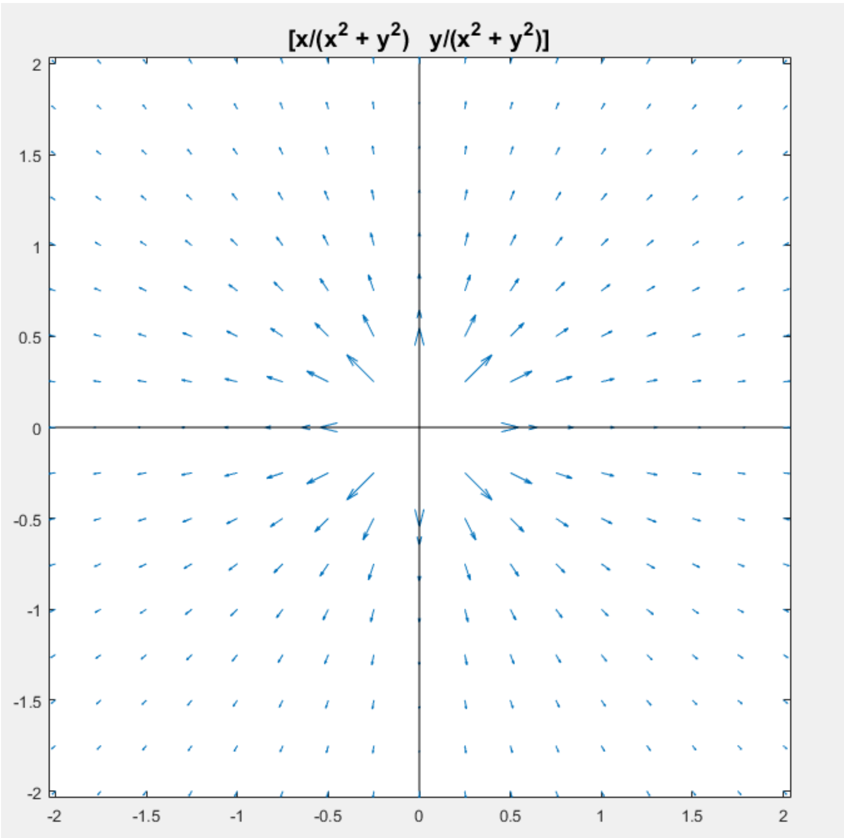
Electric field:

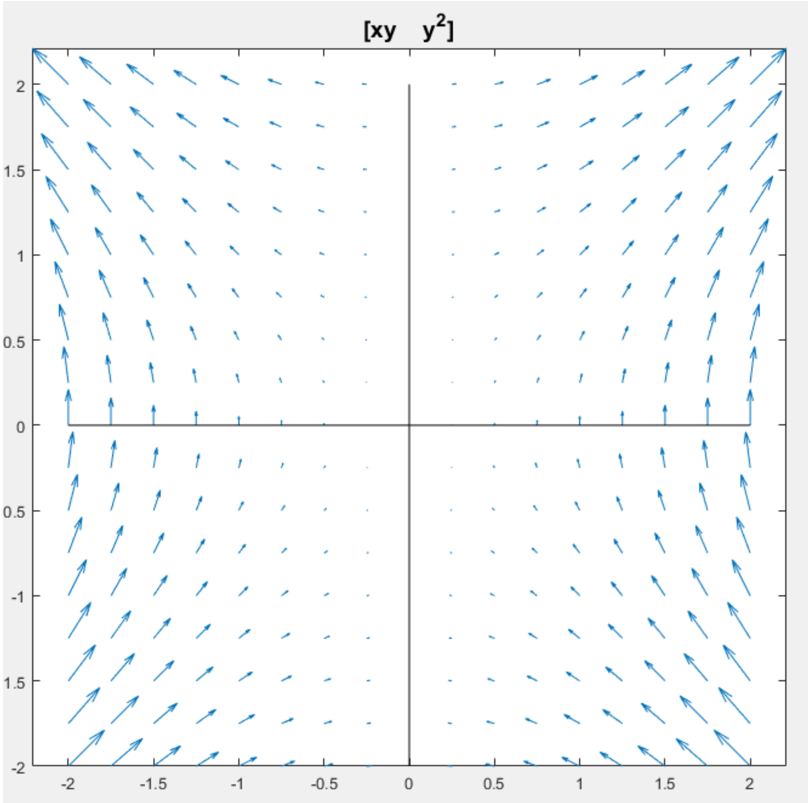
$\boxed{8}$

$\underline{\mathcal{E}}$ in V/m

flux of $\underline{\mathcal{E}}$ in Vm


[-y x]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


[x/(x^2 + y^2) y/(x^2 + y^2)]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



[xy y^2]
2
1.5
1
0.5
0
0.5
-1
1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2