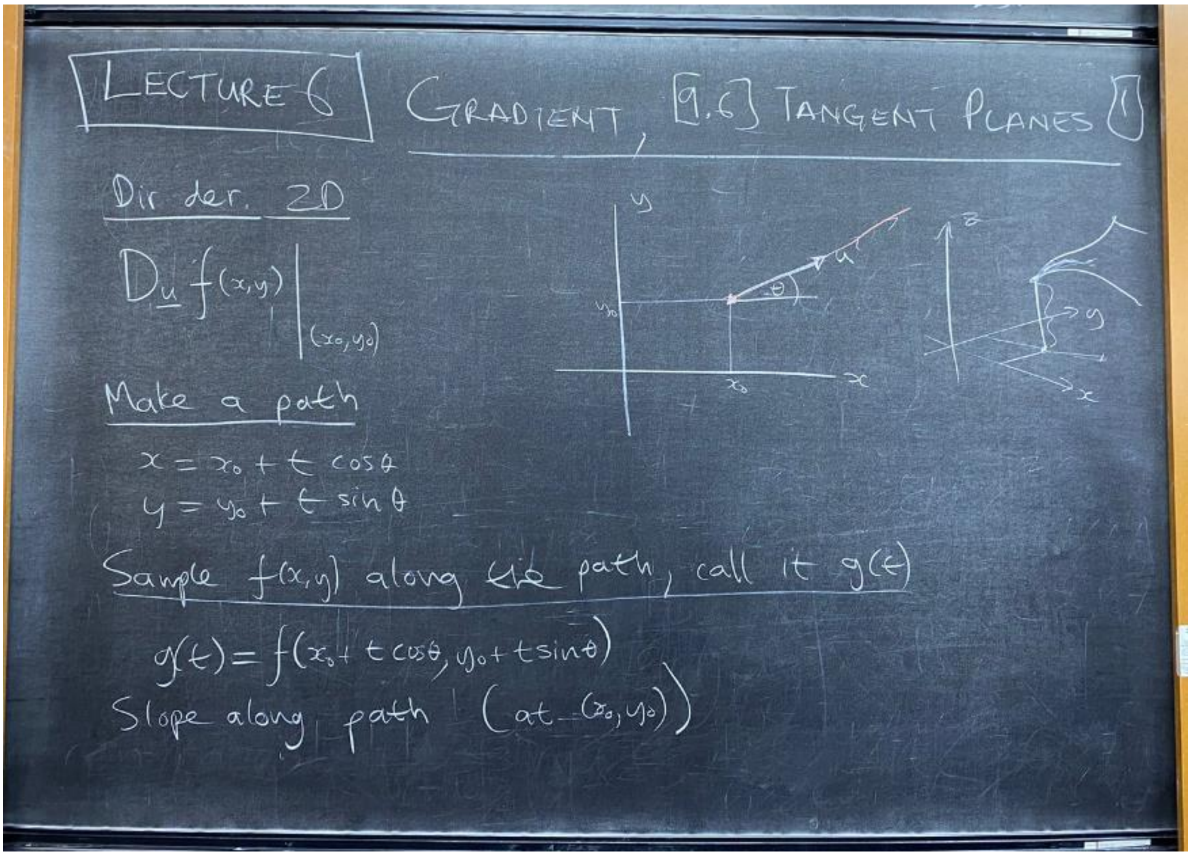
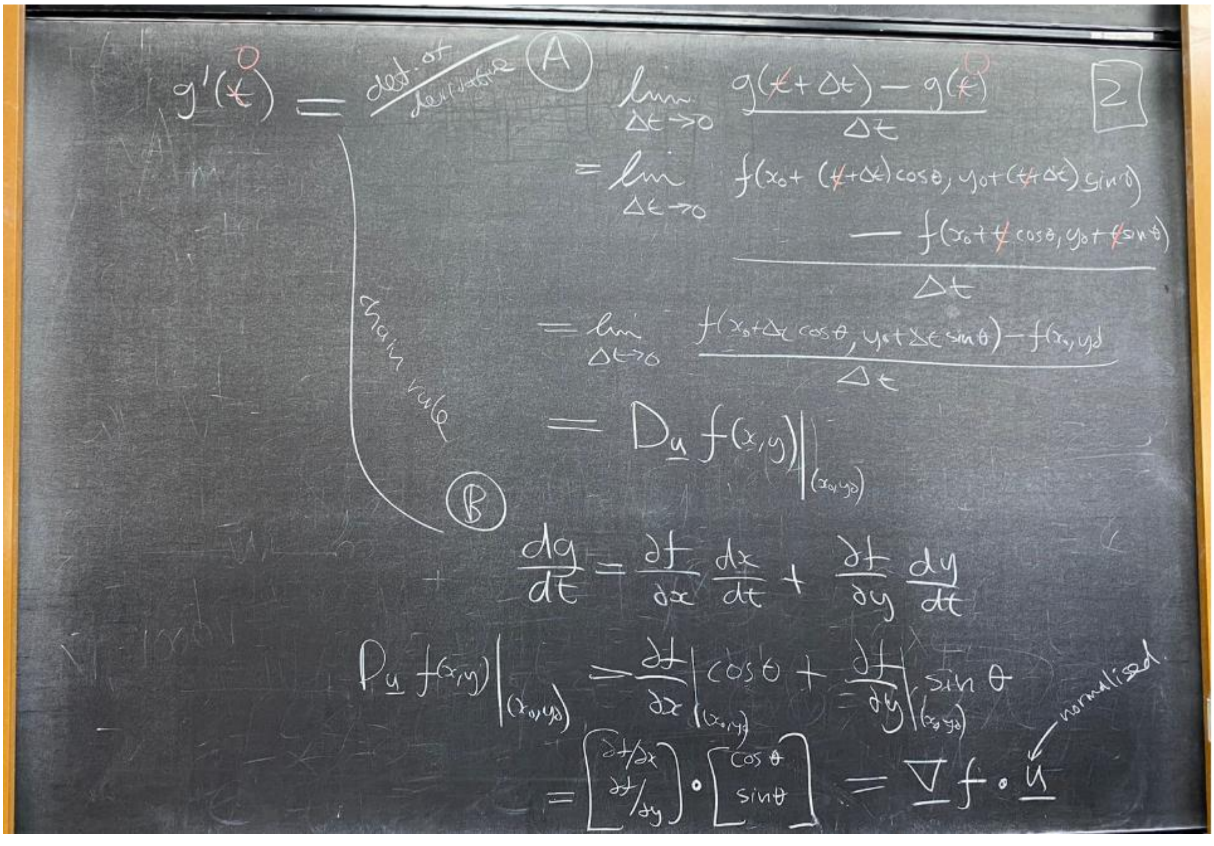
# Appl.Maths. B242-2023: LECTURE 6



## GRADIENT IN 2D

3

$z = f(x,y)$

$\underline{\nabla} f =$ is a vector

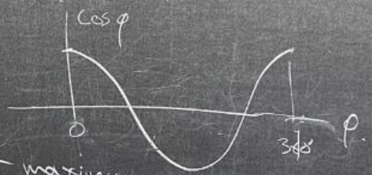
$D_{\underline{u}} f = \underline{\nabla} f \cdot \underline{u}$

$= \|\underline{\nabla} f\| \, \|\underline{u}\| \cos\varphi$


$\underline{a} \cdot \underline{b} = ab \cos\theta$
$= \|\underline{a}\| \, \|\underline{b}\| \cos\theta$

In which direction is $D_{\underline{u}} f$ maximal?

$\longrightarrow$ at $\varphi = 0$



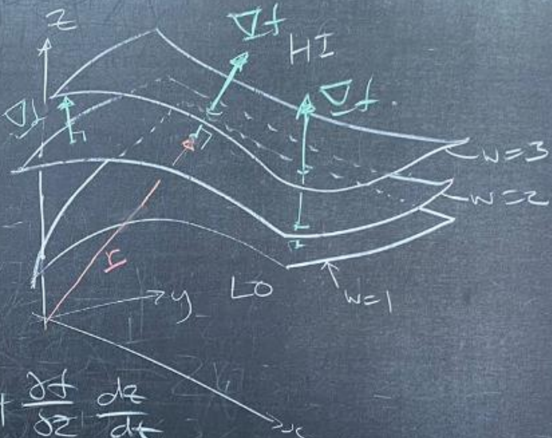

$\underline{\nabla} f$ is a vector
- points in direction of maximum ~~slope~~ increase
- $\|\underline{\nabla} f\|$ is the rate at which $f(x,y)$ increases.

## GRADIENT IN 3D

4

$w = f(x,y,z)$

$$\underline{\nabla} f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{bmatrix}$$




const

$c = f(x,y,z)$

$$\frac{dc}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$0 = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \underline{\nabla} f \cdot \underline{u}$$

$\boxed{5}$

$$0 = \frac{x^2+y^2}{3} - z$$

$$w = \frac{x^2+y^2}{3} - z = f(x,y,z)$$

$$\underline{\nabla} w = \begin{bmatrix} 2x/3 \\ 2y/3 \\ -1 \end{bmatrix} \qquad \underline{\nabla} w \Big|_{\substack{x=1 \\ y=0.6}} = \begin{bmatrix} 0.6667 \\ 0.4 \\ -1 \end{bmatrix}$$

## [9.6] TANGENT PLANES

**Ex:** Find tangent plane to $\frac{x^2+y^2}{z} = \text{const}$ at the point $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

$$\text{const} = \frac{1^2+1^2}{2} = 1$$

$\boxed{6}$

Surface: $1 = \frac{x^2+y^2}{z}$, rewrite $\boxed{z = x^2+y^2}$

Plane: $\underbrace{\underline{n}}_{\text{normal}} \cdot \underline{r} = \underline{n} \cdot \underbrace{\underline{r}_1}_{\text{a point}}$

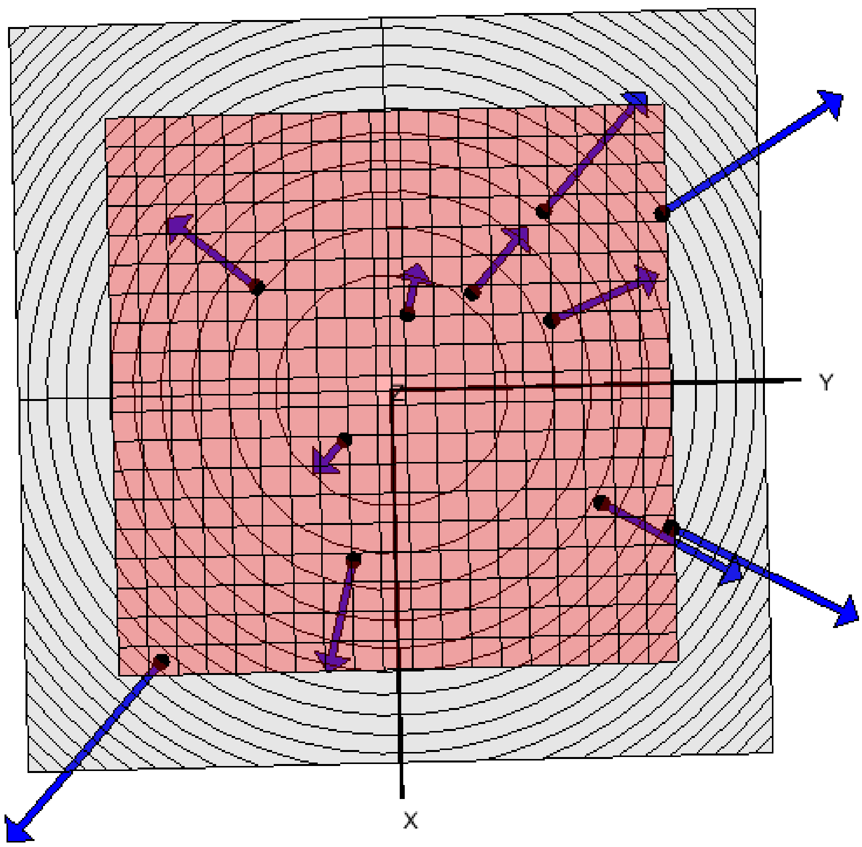
$$\underline{r}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

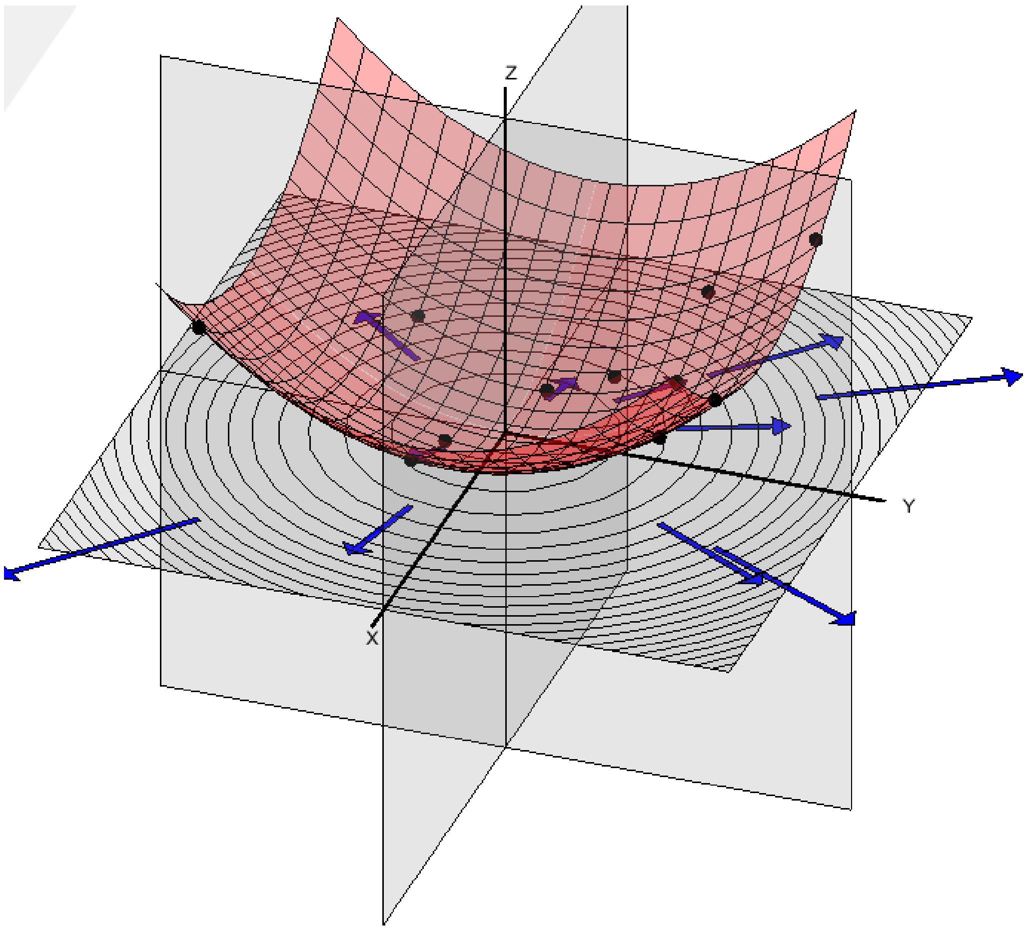
$$w = \frac{x^2+y^2}{z}$$

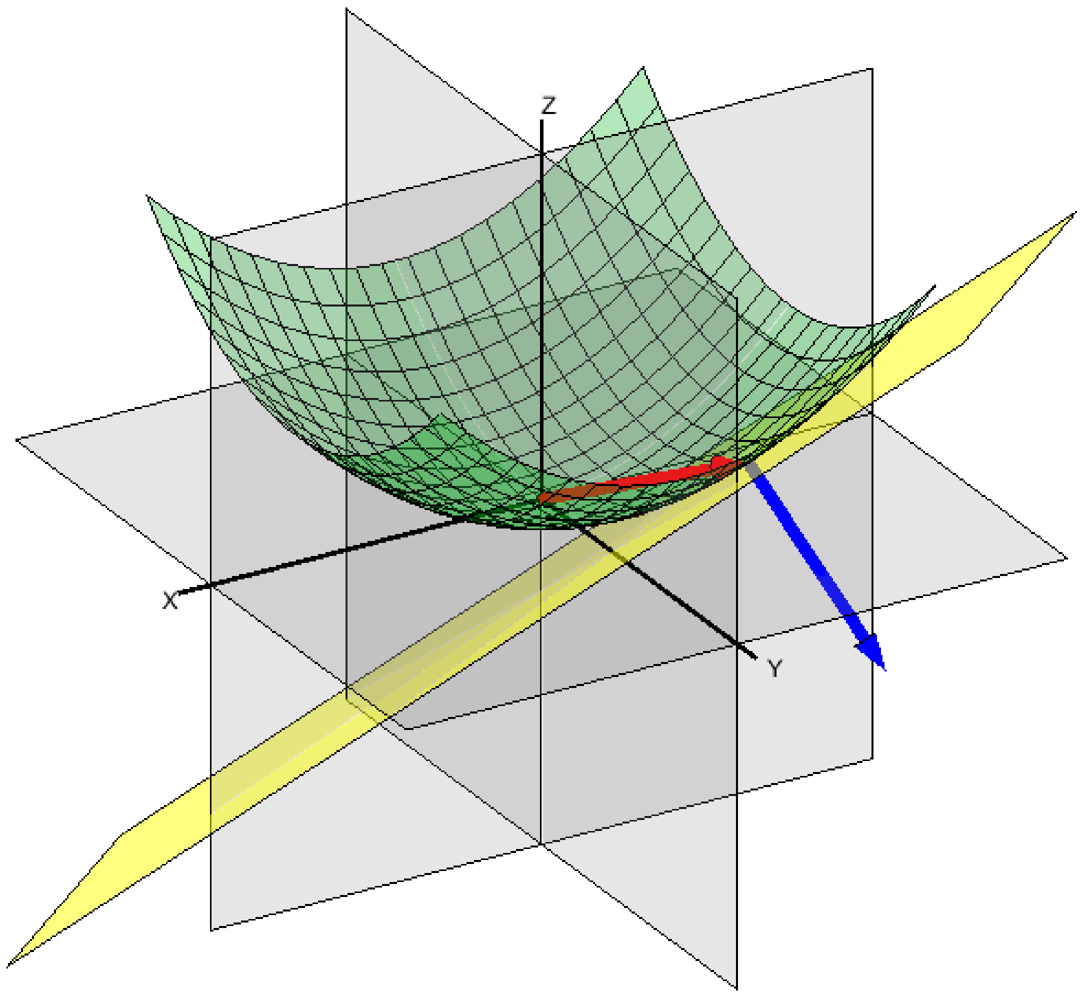
$$\underline{\nabla} w = \begin{bmatrix} 2x/z \\ 2y/z \\ \frac{-(x^2+y^2)}{z^2} \end{bmatrix}, \qquad \underline{\nabla} w \Big|_{\substack{x=1 \\ y=1 \\ z=2}} = \begin{bmatrix} 2\cdot 1/2 \\ 2\cdot 1/2 \\ \frac{-(1+1)}{2^2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\underline{x + y - \tfrac{1}{2} z = 1}$$


Y
X


Z
Y
X


Z
X
Y