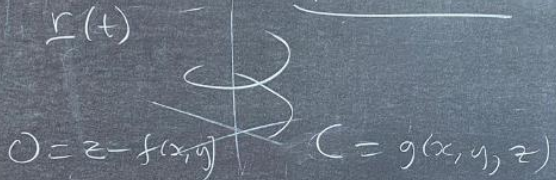


Appl.Maths. B242-2023: LECTURE 5

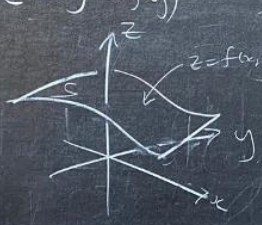
LECTURE 5 | THE DIRECTIONAL DERIVATIVE (1)

SUMMARY:

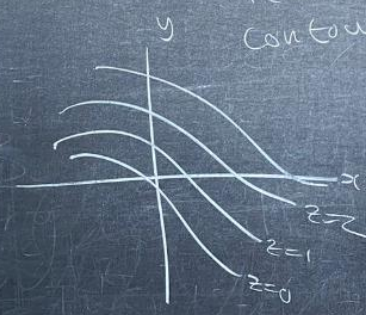
$\underline{r(t)}$ Space curve



$0 = z - f(x, y)$ Surface
 $z = f(x, y)$ 3D diagram



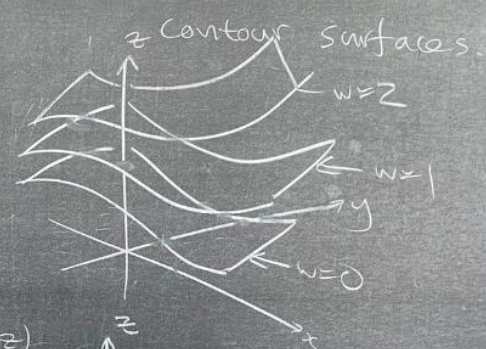
Level curves
Contour curves



Volume function

$w = f(x, y, z)$

~~4D diagram~~



Ex 1:

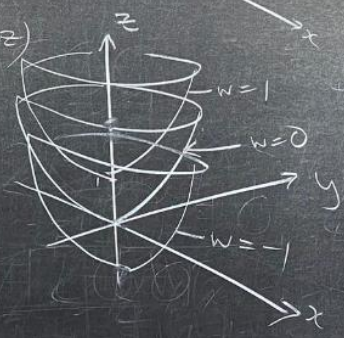
$w = z - x^2 - y^2 = f(x, y, z)$

$z = x^2 + y^2 + w$

Let $w=0$: $z = x^2 + y^2$

Let $w=1$: $z = x^2 + y^2 + 1$

Let $w=-1$: $z = x^2 + y^2 - 1$



"Stacked paraboloids"

Ex 2: $w = \frac{z}{x^2+y^2}$

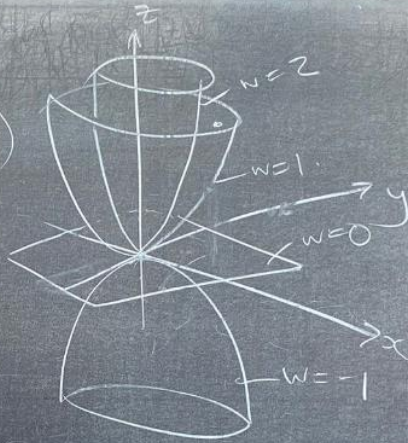
$z = w(x^2+y^2)$

Let $w=1$, $z = (x^2+y^2)$

Let $w=2$, $z = 2(x^2+y^2)$

Let $w=-1$, $z = -(x^2+y^2)$

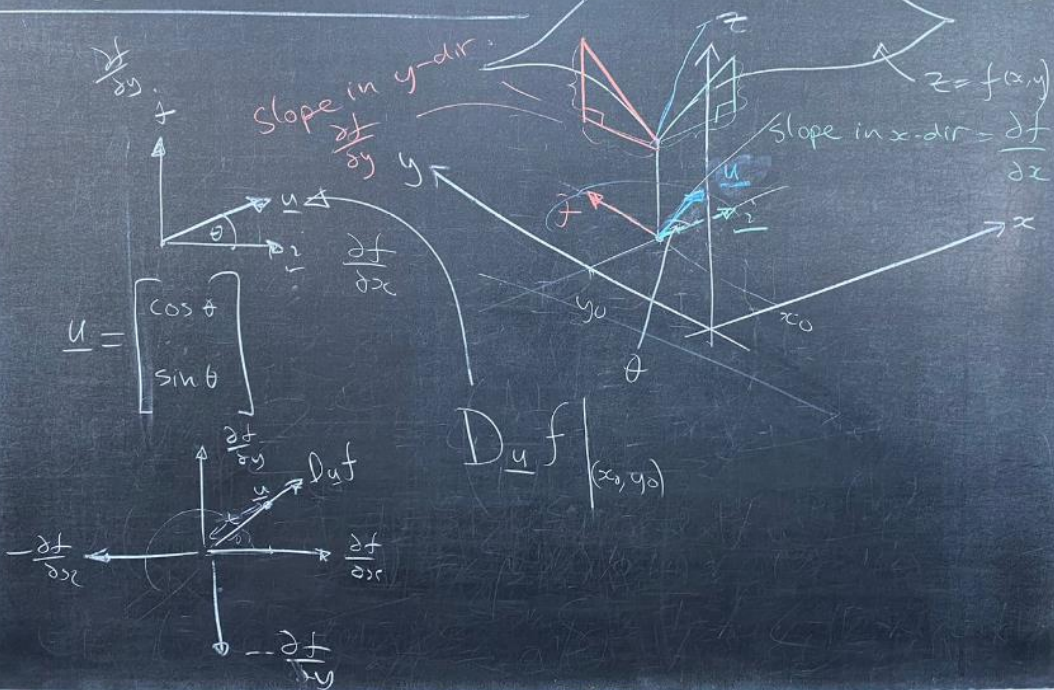
Let $w=0$, $z=0$



Ex 3: $w = x^2 + y^2 + z^2$

Describe the contour surfaces.

[9.5] DIR. DERIVATIVES



$$D_{\underline{u}} f \Big|_{(x_0, y_0)} = \lim_{t \rightarrow 0} \frac{f(x_0 + t \cos \theta, y_0 + t \sin \theta) - f(x_0, y_0)}{t} \quad [5]$$

Formula:

$$D_{\underline{u}} f = \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$$

Check: $\underline{u} = \underline{i}$, $\theta = 0$, $D_{\underline{i}} f = \frac{\partial f}{\partial x} \cdot 1 + \frac{\partial f}{\partial y} \cdot 0 = \frac{\partial f}{\partial x}$.

Rewrite:

$$D_{\underline{u}} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} = \underline{\nabla} f \cdot \underline{u} \Big|_{(x_0, y_0)}$$

↑ vector ↑ \underline{u} (must be normalized)

DEL OPERATOR:

$$\underline{\nabla} = \underline{i} \frac{\partial}{\partial x} + \underline{j} \frac{\partial}{\partial y} + \underline{k} \frac{\partial}{\partial z}$$

returns a vector

$$\nabla = \text{"del"}$$

$$\underline{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$

"del f"

Ex 4 $f(x,y) = \frac{x^2 + y^2}{3}$

7

Find $D_u f \Big|_{(1, \frac{1}{2})}$

where u is in dir 45° $\rightarrow x$
 $u = \begin{bmatrix} \cos 45^\circ \\ \sin 45^\circ \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\nabla f = \begin{bmatrix} \frac{2x}{3} \\ \frac{2y}{3} \end{bmatrix}$$



$$\nabla f \Big|_{(1, \frac{1}{2})} = \begin{bmatrix} \frac{2 \cdot 1}{3} \\ \frac{2 \cdot (\frac{1}{2})}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$D_u f \Big|_{(1, \frac{1}{2})} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} (1) = 0.707 \dots$$

