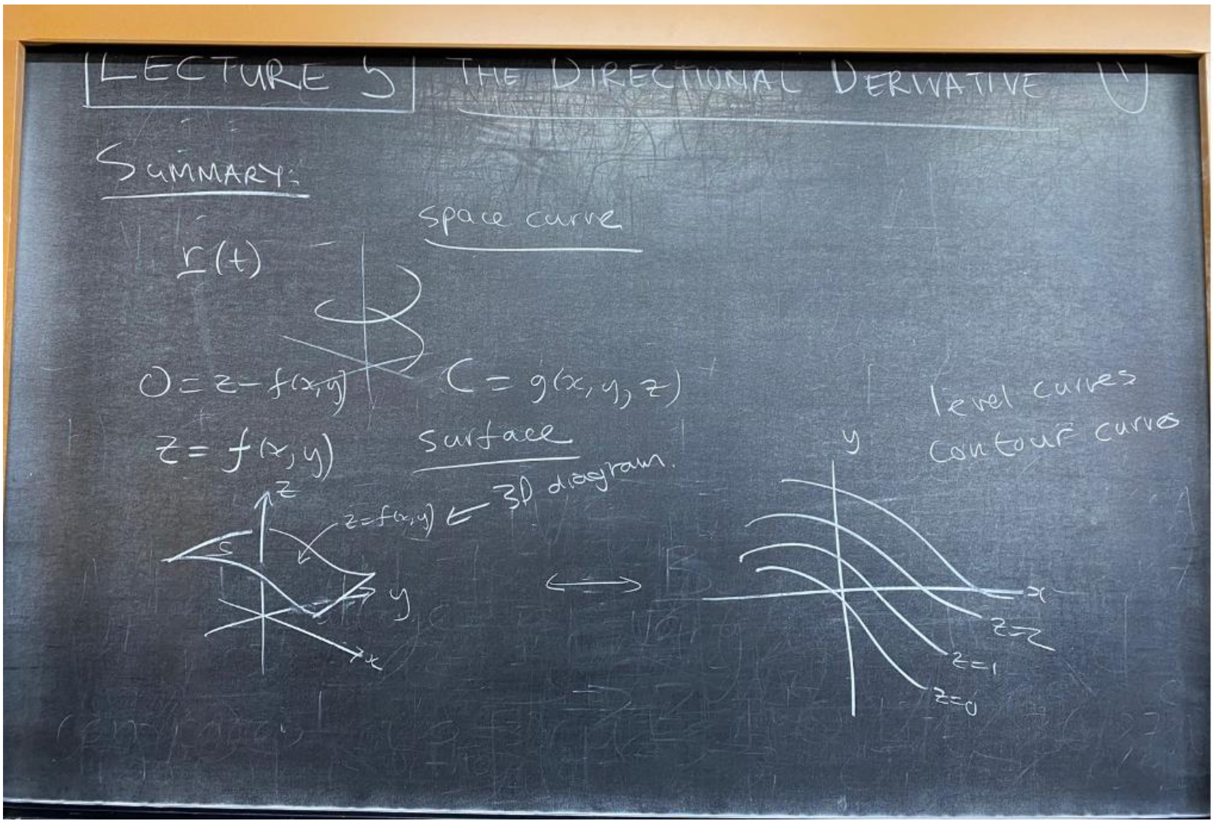
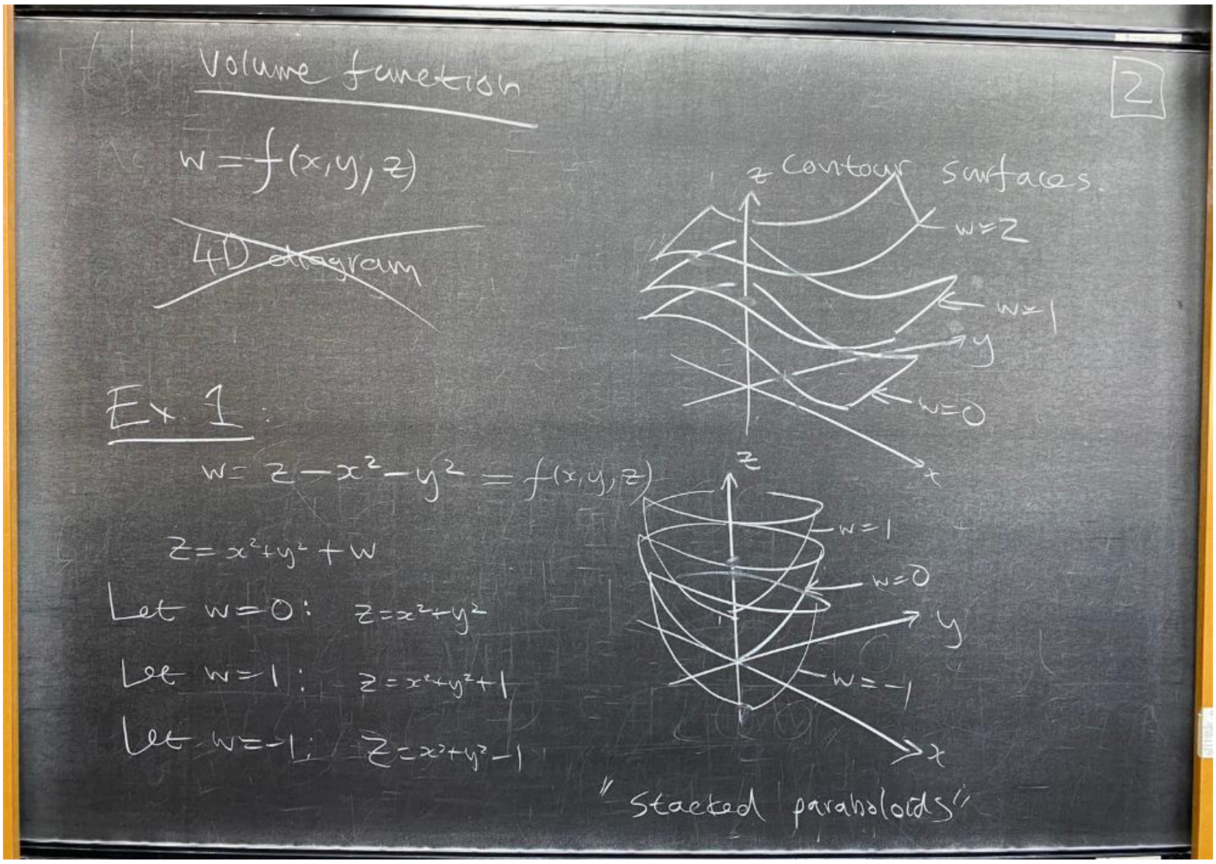
Appl.Maths. B242-2023: LECTURE 5





Ex 2: $\boxed{w = \dfrac{z}{(x^2+y^2)}}$ [3]

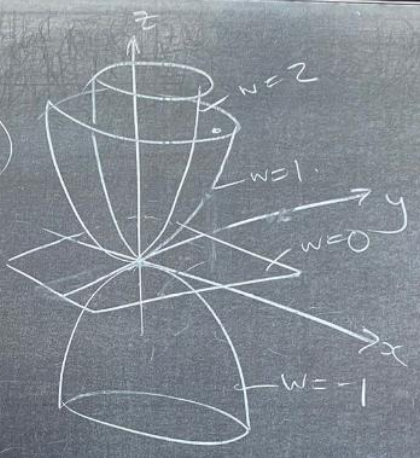
$z = w(x^2+y^2)$

Let $w=1$, $z = (x^2+y^2)$

Let $w=2$, $z = 2(x^2+y^2)$

Let $w=-1$, $z = -1(x^2+y^2)$

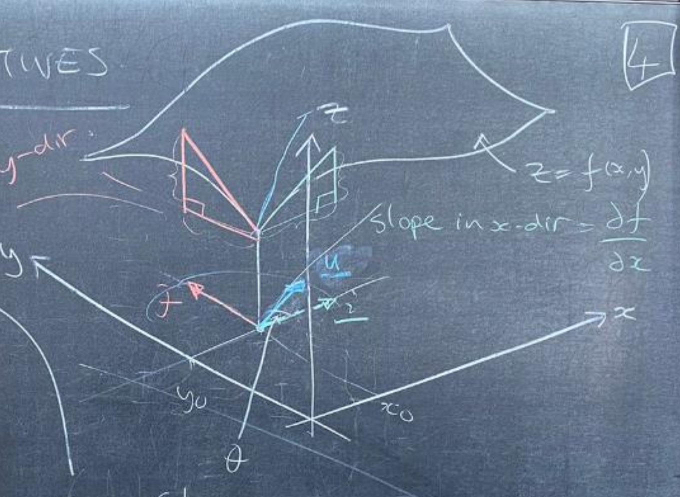
Let $w=0$, $z=0$



Ex 3: $w = x^2+y^2+z^2$

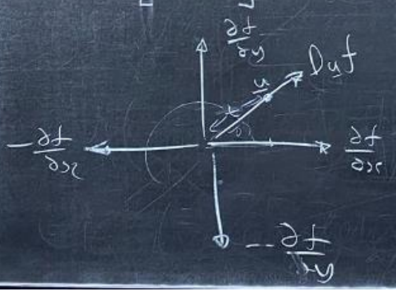
Describe the contour surfaces.

[9.5] DIR. DERIVATIVES [4]



$\underline{u} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

$D_{\underline{u}} f \Big|_{(x_0,y_0)}$



$\boxed{5}$

$$\boxed{\left. D_{\underline{u}} f \right|_{(x_0, y_0)} = \lim_{t \to 0} \frac{f(x_0 + t\cos\theta,\ y_0 + t\sin\theta) - f(x_0, y_0)}{t}}$$

Formula:

$$D_{\underline{u}} f = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

Check: $\underline{u} = \underline{i}$, $\theta = 0$, $D_{\underline{i}} f = \frac{\partial f}{\partial x} \cdot 1 + \frac{\partial f}{\partial y} \cdot 0 = \frac{\partial f}{\partial x}$.

Rewrite:

$$D_{\underline{u}} f = \underbrace{\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}}_{\text{vector}} \cdot \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}_{\underline{u} \text{ (must be normalized)}} = \left. \underline{\nabla} f \cdot \underline{u} \right|_{(x_0, y_0)}$$

$\boxed{6}$

DEL OPERATOR:

$$\underline{\nabla} = \underbrace{\underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}}_{\text{returns a vector}}$$

$\underline{\nabla}$ = "del"

$$\underline{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$

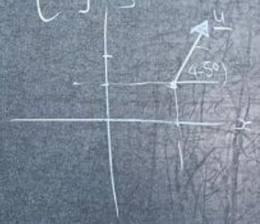
"del f"

$\boxed{7}$

Ex 4: $f(x,y) = \dfrac{x^2+y^2}{3}$

Find $D_{\underline{u}} f \Big|_{(1,\frac{1}{2})}$ where $\underline{u}$ is in dir 45° $\to x$

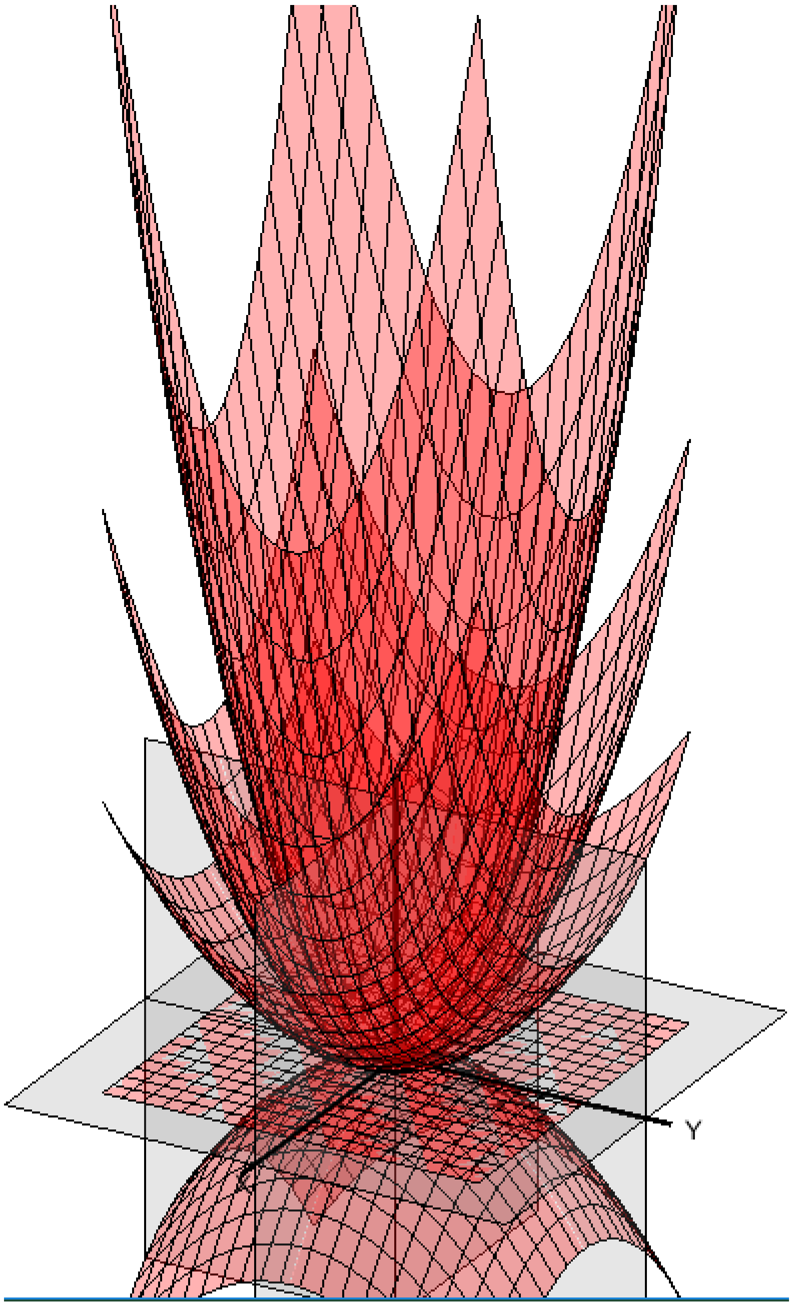
$$\underline{u} = \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

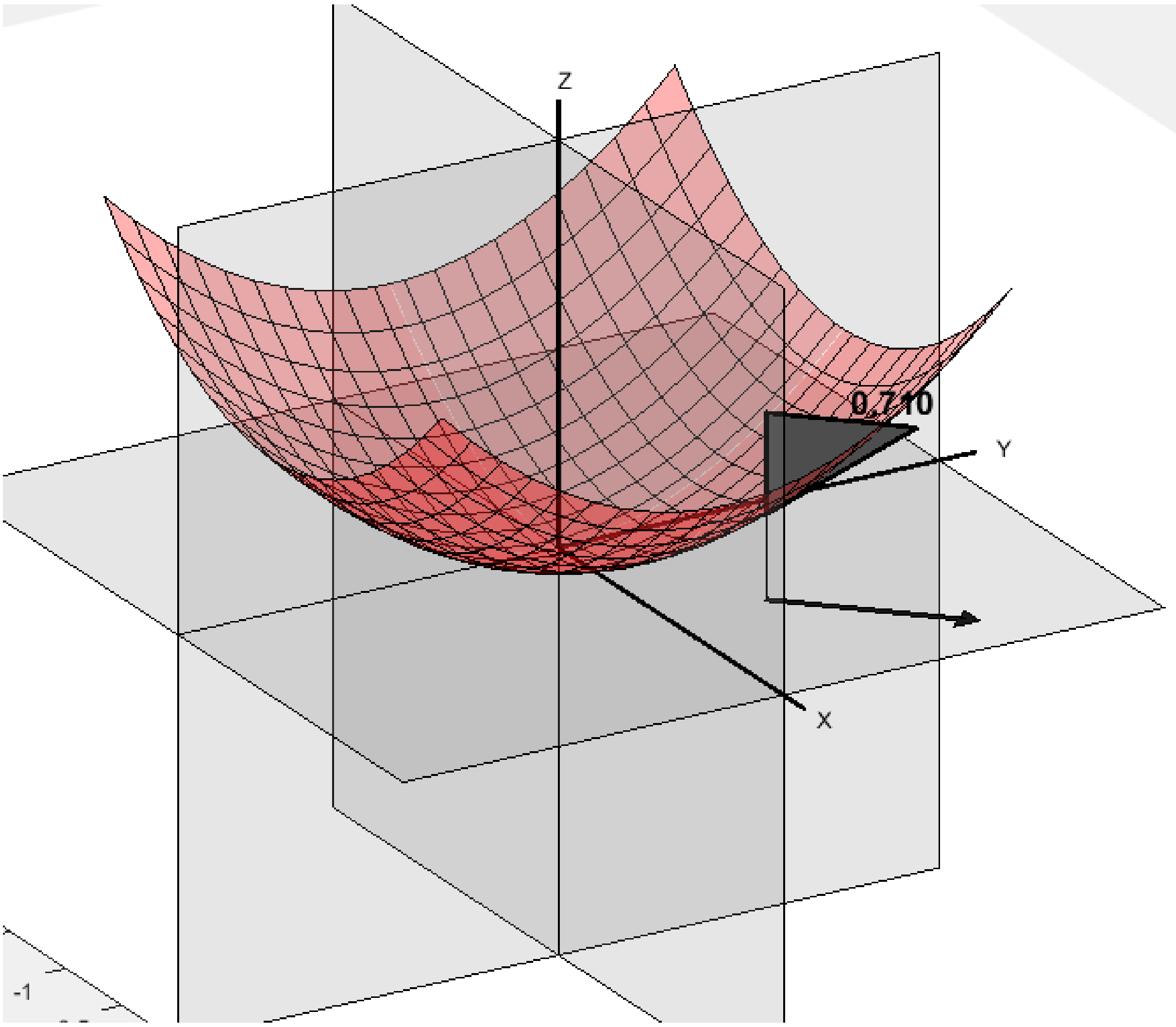


$$\underline{\nabla} f = \begin{bmatrix} \frac{2x}{3} \\ \frac{2y}{3} \end{bmatrix}$$

$$\underline{\nabla} f \Big|_{(1,\frac{1}{2})} = \begin{bmatrix} 2 \cdot 1/3 \\ \frac{2 \cdot (\frac{1}{2})}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$D_{\underline{u}} f \Big|_{(1,\frac{1}{2})} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} (1) = 0.707\ldots\ldots$$


Y


Z
0,710
Y
X
-1