


Appl.Maths. B242-2023: LECTURE 04

LECTURE 4 **9.4 SURFACES (PARTIAL DER'S)** [1]

9.1 Summary



$$\underline{r}(t) = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix}, \quad \underline{r}'(t) = \begin{bmatrix} f'(t) \\ g'(t) \\ h'(t) \end{bmatrix}, \quad t \in [t_0, t_1]$$

$$\text{arclength} = \int_{t=t_0}^{t_1} \|\underline{r}'(t)\| dt = \int_{t=t_0}^{t_1} \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} dt$$

SURFACE

$$z = f(x, y) \quad \longleftrightarrow \quad g(x, y, z) = \text{constant}$$


Ex. 1

$$3x + 2y + 4z = 6 \quad \longleftrightarrow \quad z = \frac{6 - 3x - 2y}{4} = f(x, y)$$

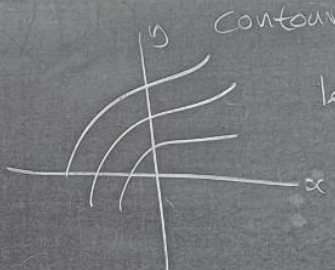
plane with $\underline{n} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$, going through $\begin{bmatrix} 0 \\ 0 \\ 3/4 \end{bmatrix}$

VIZUALISATION [2]

3D



Contours



level curves

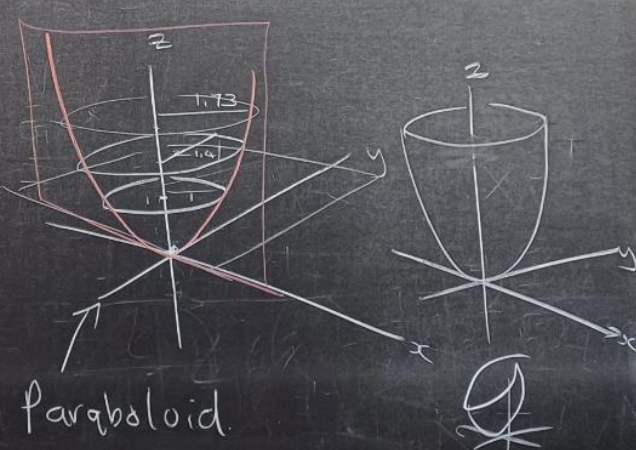
Ex. 2

3D-DIAG

$$x^2 + y^2 - z = 0$$

$$z = x^2 + y^2$$

let $z=0$, $0 = x^2 + y^2$
 let $z=1$, $1 = x^2 + y^2$
 let $z=2$, $2 = x^2 + y^2$
 let $z=3$, $3 = x^2 + y^2$
 let $y=0$, $z = x^2$



Paraboloid

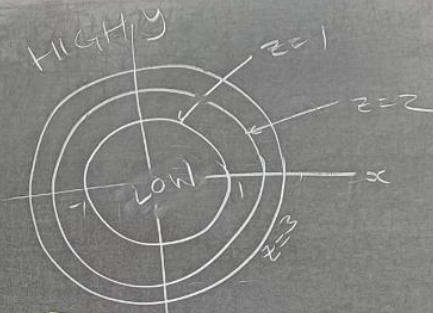
Contours

$$z = x^2 + y^2$$

$$\text{Let } z=1, \quad x^2 + y^2 = 1$$

$$z=3, \quad x^2 + y^2 = (1.73)^2$$

HIGH y



3

Ex 3: $z = x^2 + \frac{1}{2}y$

$$y = -2x + 2z$$

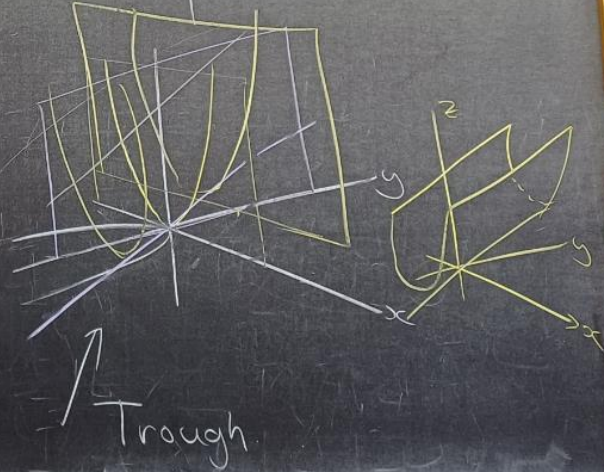
3D-DIAG

$$z=0, \quad y = -2x^2$$

$$y=0, \quad z = x^2$$

$$x=0, \quad z = \frac{1}{2}y$$

$$y=1, \quad z = x^2 + \frac{1}{2}$$



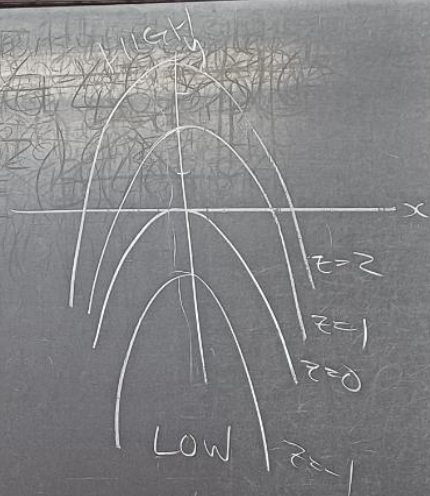
Contours

$$\text{Let } z=0, \quad y = -2x^2$$

$$\text{Let } z=1, \quad y = -2x^2 + 2$$

$$\text{Let } z=2, \quad y = -2x^2 + 4$$

HIGH y

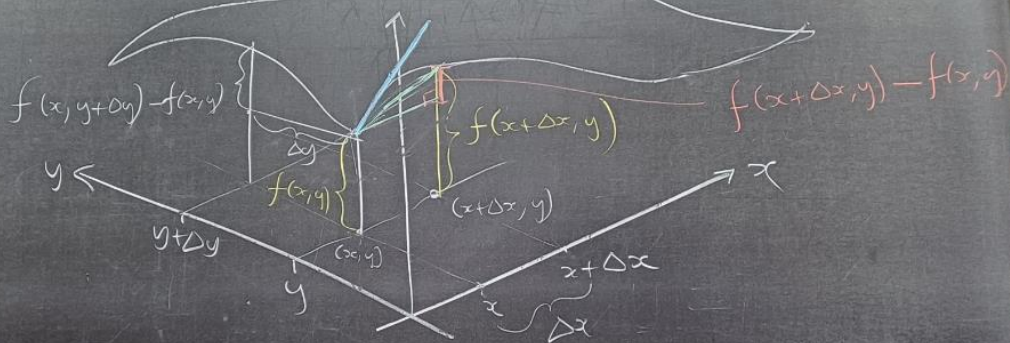


4

PARTIAL DER'S

$$z = f(x, y)$$

5



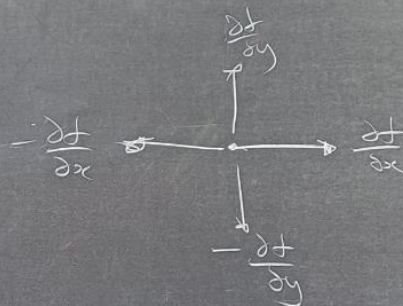
$$\frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} = \text{slope of green line}$$

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} = \text{slope of tangent line}$$

slope in eastward direction

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

= slope northward



6

7

CHAIN RULE

surf S
 $z = f(x, y)$

Path Q on xy -plane
 $\left. \begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned} \right\} t \in [t_0, t_1]$

Path P (on S)
 $\left. \begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned} \right\}$

$z(t) = z = f(x(t), y(t))$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$
