Exercise 9.8

9.8 Exercises

Answers to selected odd-numbered problems begin on page ANS-21.

In Problems 1–4, evaluate $\int_C G(x, y)\,dx$, $\int_C G(x, y)\,dy$, and $\int_C G(x, y)\,ds$ on the indicated curve C.

1. $G(x, y) = 2xy$; $x = 5\cos t$, $y = 5\sin t$, $0 \le t \le \pi/4$
2. $G(x, y) = x^3 + 2xy^2 + 2x$; $x = 2t$, $y = t^2$, $0 \le t \le 1$
3. $G(x, y) = 3x^2 + 6y^2$; $y = 2x + 1$, $-1 \le x \le 0$
4. $G(x, y) = x^2/y^3$; $2y = 3x^{2/3}$, $1 \le x \le 8$

In Problems 5 and 6, evaluate $\int_C G(x, y, z)\,dx$, $\int_C G(x, y, z)\,dy$, $\int_C G(x, y, z)\,dz$, and $\int_C G(x, y, z)\,ds$ on the indicated curve C.

5. $G(x, y, z) = z$; $x = \cos t$, $y = \sin t$, $z = t$, $0 \le t \le \pi/2$
6. $G(x, y, z) = 4xyz$; $x = \frac{1}{3}t^3$, $y = t^2$, $z = 2t$, $0 \le t \le 1$

In Problems 7–10, evaluate $\int_C (2x + y)\,dx + xy\,dy$ on the given curve C between $(-1, 2)$ and $(2, 5)$.

7. $y = x + 3$
8. $y = x^2 + 1$
9.

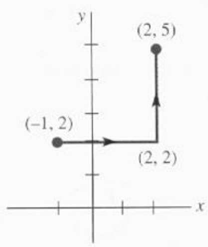

FIGURE 9.8.13 Curve C for Problem 9

10.

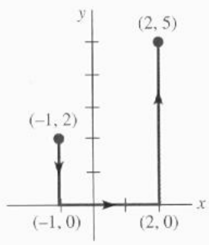

FIGURE 9.8.14 Curve C for Problem 10

In Problems 11–14, evaluate $\int_C y\,dx + x\,dy$ on the given curve C between $(0, 0)$ and $(1, 1)$.

11. $y = x^2$
12. $y = x$
13. C consists of the line segments from $(0, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(1, 1)$.
14. C consists of the line segments from $(0, 0)$ to $(1, 0)$ and from $(1, 0)$ to $(1, 1)$.
15. Evaluate $\int_C (6x^2 + 2y^2)\,dx + 4xy\,dy$, where C is given by $x = \sqrt{t}$, $y = t$, $4 \le t \le 9$.
16. Evaluate $\int_C -y^2\,dx + xy\,dy$, where C is given by $x = 2t$, $y = t^3$, $0 \le t \le 2$.
17. Evaluate $\int_C 2x^3y\,dx + (3x + y)\,dy$, where C is given by $x = y^2$ from $(1, -1)$ to $(1, 1)$.
18. Evaluate $\int_C 4x\,dx + 2y\,dy$, where C is given by $x = y^3 + 1$ from $(0, -1)$ to $(9, 2)$.

In Problems 19 and 20, evaluate $\oint_C (x^2 + y^2)\,dx - 2xy\,dy$ on the given closed curve C.

19.

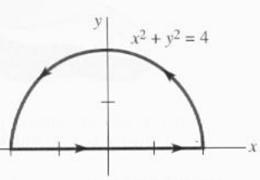

FIGURE 9.8.15 Closed curve C for Problem 19

20.

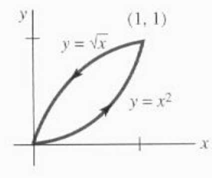

FIGURE 9.8.16 Closed curve C for Problem 20

In Problems 21 and 22, evaluate $\oint_C x^2y^3\,dx - xy^2\,dy$ on the given closed curve C.

21.

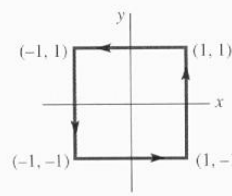

FIGURE 9.8.17 Closed curve C for Problem 21

22.

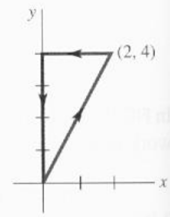

FIGURE 9.8.18 Closed curve C for Problem 22

23. Evaluate $\oint_C (x^2 - y^2)\,ds$, where C is given by
$$x = 5\cos t, \quad y = 5\sin t, \quad 0 \le t \le 2\pi.$$
24. Evaluate $\int_{-C} y\,dx - x\,dy$, where C is given by
$$x = 2\cos t, \quad y = 3\sin t, \quad 0 \le t \le \pi.$$

In Problems 25–28, evaluate $\int_C y\,dx + z\,dy + x\,dz$ on the given curve C between $(0, 0, 0)$ and $(6, 8, 5)$.

25. C consists of the line segments from $(0, 0, 0)$ to $(2, 3, 4)$ and from $(2, 3, 4)$ to $(6, 8, 5)$.
26. $x = 3t$, $y = t^3$, $z = \frac{5}{4}t^2$, $\quad 0 \le t \le 2$
27.

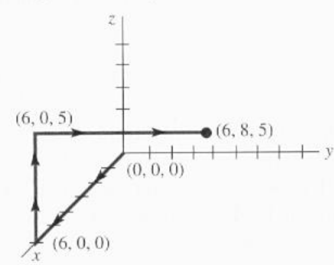

FIGURE 9.8.19 Closed curve C for Problem 27

28.

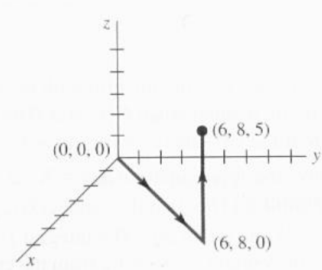

FIGURE 9.8.20 Closed curve C for Problem 28

n Problems 29 and 30, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

29. $\mathbf{F}(x, y) = y^3\mathbf{i} - x^2y\mathbf{j}$; $\mathbf{r}(t) = e^{-2t}\mathbf{i} + e^t\mathbf{j}$, $0 \le t \le \ln 2$

30. $\mathbf{F}(x, y, z) = e^x\mathbf{i} + xe^{xy}\mathbf{j} + xye^{xyz}\mathbf{k}$; $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$

31. Find the work done by the force $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$ acting along $y = \ln x$ from $(1, 0)$ to $(e, 1)$.

32. Find the work done by the force $\mathbf{F}(x, y) = 2xy\mathbf{i} + 4y^2\mathbf{j}$ acting along the piecewise-smooth curve consisting of the line segments from $(-2, 2)$ to $(0, 0)$ and from $(0, 0)$ to $(2, 3)$.

33. Find the work done by the force $\mathbf{F}(x, y) = (x + 2y)\mathbf{i} + (6y - 2x)\mathbf{j}$ acting counterclockwise once around the triangle with vertices $(1, 1)$, $(3, 1)$, and $(3, 2)$.

34. Find the work done by the force $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ acting along the curve given by $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$ from $t = 1$ to $t = 3$.

35. Find the work done by a constant force $\mathbf{F}(x, y) = a\mathbf{i} + b\mathbf{j}$ acting counterclockwise once around the circle $x^2 + y^2 = 9$.

36. In an inverse square force field $\mathbf{F} = c\mathbf{r}/\|\mathbf{r}\|^3$, where c is a constant and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,* find the work done in moving a particle along the line from $(1, 1, 1)$ to $(3, 3, 3)$.

37. Verify that the line integral $\int_C y^2\,dx + xy\,dy$ has the same value on C for each of the following parameterizations:

$$C: x = 2t + 1, \quad y = 4t + 2, \quad 0 \le t \le 1$$
$$C: x = t^2, \quad y = 2t^2, \quad 1 \le t \le \sqrt{3}$$
$$C: x = \ln t, \quad y = 2\ln t, \quad e \le t \le e^3.$$

38. Consider the three curves between $(0, 0)$ and $(2, 4)$:

$$C_1: x = t, \quad y = 2t, \quad 0 \le t \le 2$$
$$C_2: x = t, \quad y = t^2, \quad 0 \le t \le 2$$
$$C_3: x = 2t - 4, \quad y = 4t - 8, \quad 2 \le t \le 3.$$

Show that $\int_{C_1} xy\,ds = \int_{C_3} xy\,ds$, but $\int_{C_1} xy\,ds \ne \int_{C_2} xy\,ds$. Explain.

39. Assume a smooth curve C is described by the vector function $\mathbf{r}(t)$ for $a \le t \le b$. Let acceleration, velocity, and speed be given by $\mathbf{a} = d\mathbf{v}/dt$, $\mathbf{v} = d\mathbf{r}/dt$, and $v = \|\mathbf{v}\|$, respectively. Using Newton's second law $\mathbf{F} = m\mathbf{a}$, show that, in the absence of friction, the work done by $\mathbf{F}$ in moving a particle of constant mass m from point A at $t = a$ to point B at $t = b$ is the same as the change in kinetic energy:

$$K(B) - K(A) = \frac{1}{2}m[v(b)]^2 - \frac{1}{2}m[v(a)]^2.$$

[*Hint*: Consider $\frac{d}{dt}v^2 = \frac{d}{dt}\mathbf{v} \cdot \mathbf{v}$.]

40. If $\rho(x, y)$ is the density of a wire (mass per unit length), then $m = \int_C \rho(x, y)\,ds$ is the mass of the wire. Find the mass of a wire having the shape of the semicircle $x = 1 + \cos t$, $y = \sin t$, $0 \le t \le \pi$, if the density at a point P is directly proportional to distance from the y-axis.

41. The coordinates of the center of mass of a wire with variable density are given by $\bar{x} = M_y/m$, $\bar{y} = M_x/m$, where

$$m = \int_C \rho(x, y)\,ds, \quad M_x = \int_C y\rho(x, y)\,ds$$

and
$$M_y = \int_C x\rho(x, y)\,ds.$$

Find the center of mass of the wire in Problem 40.

42. A force field $\mathbf{F}(x, y)$ acts at each point on the curve C, which is the union of C_1, C_2, and C_3 shown in **FIGURE 9.8.21**. $\|\mathbf{F}\|$ is measured in pounds and distance is measured in feet using the scale given in the figure. Use the representative vectors shown to approximate the work done by $\mathbf{F}$ along C. [*Hint*: Use $W = \int_C \mathbf{F} \cdot \mathbf{T}\,ds$.]

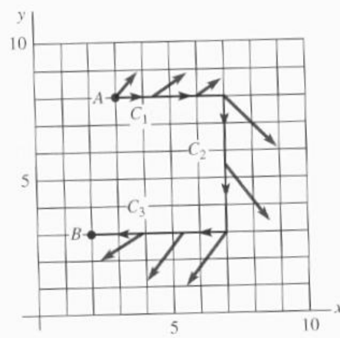

FIGURE 9.8.21 Force field in Problem 42