9.6 Exercises Answers to selected odd-numbered problems begin on page ANS-21.

In Problems 1–12, sketch the level curve or surface passing through the indicated point. Sketch the gradient at the point.

1. $f(x, y) = x - 2y$; $(6, 1)$
2. $f(x, y) = \dfrac{y + 2x}{x}$; $(1, 3)$
3. $f(x, y) = y - x^2$; $(2, 5)$
4. $f(x, y) = x^2 + y^2$; $(-1, 3)$
5. $f(x, y) = \dfrac{x^2}{4} + \dfrac{y^2}{9}$; $(-2, -3)$
6. $f(x, y) = \dfrac{y^2}{x}$; $(2, 2)$
7. $f(x, y) = (x - 1)^2 - y^2$; $(1, 1)$
8. $f(x, y) = \dfrac{y - 1}{\sin x}$; $(\pi/6, \frac{3}{2})$
9. $F(x, y, z) = y + z$; $(3, 1, 1)$
10. $F(x, y, z) = x^2 + y^2 - z$; $(1, 1, 3)$
11. $F(x, y, z) = \sqrt{x^2 + y^2 + z^2}$; $(3, 4, 0)$
12. $F(x, y, z) = x^2 - y^2 + z$; $(0, -1, 1)$

In Problems 13 and 14, find the points on the given surface at which the gradient is parallel to the indicated vector.

13. $z = x^2 + y^2$; $4\mathbf{i} + \mathbf{j} + \frac{1}{2}\mathbf{k}$
14. $x^3 + y^2 + z = 15$; $27\mathbf{i} + 8\mathbf{j} + \mathbf{k}$

In Problems 15–24, find an equation of the tangent plane to the graph of the given equation at the indicated point.

15. $x^2 + y^2 + z^2 = 9$; $(-2, 2, 1)$
16. $5x^2 - y^2 + 4z^2 = 8$; $(2, 4, 1)$
17. $x^2 - y^2 - 3z^2 = 5$; $(6, 2, 3)$
18. $xy + yz + zx = 7$; $(1, -3, -5)$
19. $z = 25 - x^2 - y^2$; $(3, -4, 0)$
20. $xz = 6$; $(2, 0, 3)$
21. $z = \cos(2x + y)$; $(\pi/2, \pi/4, -1/\sqrt{2})$
22. $x^2y^3 + 6z = 10$; $(2, 1, 1)$
23. $z = \ln(x^2 + y^2)$; $(1/\sqrt{2}, 1/\sqrt{2}, 0)$
24. $z = 8e^{-2y} \sin 4x$; $(\pi/24, 0, 4)$

In Problems 25 and 26, find the points on the given surface at which the tangent plane is parallel to the indicated plane.

25. $x^2 + y^2 + z^2 = 7$; $2x + 4y + 6z = 1$
26. $x^2 - 2y^2 - 3z^2 = 33$; $8x + 4y + 6z = 5$
27. Find points on the surface $x^2 + 4x + y^2 + z^2 - 2z = 11$ at which the tangent plane is horizontal.
28. Find points on the surface $x^2 + 3y^2 + 4z^2 - 2xy = 16$ at which the tangent plane is parallel to (a) the xz-plane, (b) the yz-plane, and (c) the xy-plane.

In Problems 29 and 30, show that the second equation is an equation of the tangent plane to the graph of the first equation at (x_0, y_0, z_0).

29. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$; $\dfrac{xx_0}{a^2} + \dfrac{yy_0}{b^2} + \dfrac{zz_0}{c^2} = 1$
30. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$; $\dfrac{xx_0}{a^2} - \dfrac{yy_0}{b^2} + \dfrac{zz_0}{c^2} = 1$
31. Show that every tangent plane to the graph of $z^2 = x^2 + y^2$ passes through the origin.
32. Show that the sum of the x-, y-, and z-intercepts of every tangent plane to the graph of $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{a}$, $a > 0$, is the number a.

In Problems 33 and 34, find parametric equations for the normal line at the indicated point. In Problems 35 and 36, find symmetric equations for the normal line.

33. $x^2 + 2y^2 + z^2 = 4$; $(1, -1, 1)$
34. $z = 2x^2 - 4y^2$; $(3, -2, 2)$
35. $z = 4x^2 + 9y^2 + 1$; $(\frac{1}{2}, \frac{1}{3}, 3)$
36. $x^2 + y^2 - z^2 = 0$; $(3, 4, 5)$
37. Show that every normal line to the graph $x^2 + y^2 + z^2 = a^2$ passes through the origin.
38. Two surfaces are said to be **orthogonal** at a point P of intersection if their normal lines at P are orthogonal. Prove that the surfaces given by $F(x, y, z) = 0$ and $G(x, y, z) = 0$ are orthogonal at P if and only if $F_xG_x + F_yG_y + F_zG_z = 0$.

In Problems 39 and 40, use the result of Problem 38 to show that the given surfaces are orthogonal at a point of intersection.

39. $x^2 + y^2 + z^2 = 25$; $-x^2 + y^2 + z^2 = 0$
40. $x^2 - y^2 + z^2 = 4$; $z = 1/xy^2$

9.7 Curl and Divergence

Introduction In Section 9.1 we introduced the concept of vector function of one variable. In this section we examine vector functions of two and three variables.

Vector Fields Vector functions of two and three variables,

$$\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$$

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$$