Exercise 9.15

9.15 Exercises

Answers to selected odd-numbered problems begin on page ANS-22.

In Problems 1–8, evaluate the given iterated integral.

1. $\int_2^4 \int_{-2}^2 \int_{-1}^1 (x + y + z)\, dx\, dy\, dz$

2. $\int_1^3 \int_1^x \int_2^{xy} 24xy\, dz\, dy\, dx$

3. $\int_0^6 \int_0^{6-x} \int_0^{6-x-z} dy\, dz\, dx$

4. $\int_0^1 \int_0^{1-x} \int_0^{\sqrt{y}} 4x^2z^3\, dz\, dy\, dx$

5. $\int_0^{\pi/2} \int_0^{y^2} \int_0^y \cos\left(\frac{x}{y}\right) dz\, dx\, dy$

6. $\int_0^{\sqrt{2}} \int_{\sqrt{y}}^2 \int_0^{e^{x^2}} x\, dz\, dx\, dy$

7. $\int_0^1 \int_0^1 \int_0^{2-x^2-y^2} xye^z\, dz\, dx\, dy$

8. $\int_0^4 \int_0^{1/2} \int_0^{x^2} \frac{1}{\sqrt{x^2 - y^2}}\, dy\, dx\, dz$

9. Evaluate $\iiint_D z\, dV$, where D is the region in the first octant bounded by the graphs of $y = x$, $y = x - 2$, $y = 1$, $y = 3$, $z = 0$, and $z = 5$.

10. Evaluate $\iiint_D (x^2 + y^2)\, dV$, where D is the region bounded by the graphs of $y = x^2$, $z = 4 - y$, and $z = 0$.

In Problems 11 and 12, change the indicated order of integration to each of the other five orders.

11. $\int_0^2 \int_0^{4-2y} \int_{x+2y}^4 F(x, y, z)\, dz\, dx\, dy$

12. $\int_0^2 \int_0^{\sqrt{36-9x^2}/2} \int_1^3 F(x, y, z)\, dz\, dy\, dx$

In Problems 13 and 14, consider the solid given in the figure. Set up, but do not evaluate, the integrals giving the volume V of the solid using the indicated orders of integration.

13.

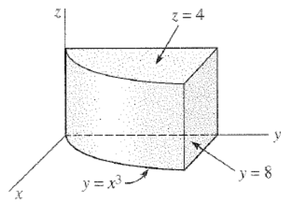
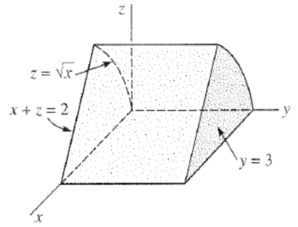


FIGURE 9.15.14 Solid for Problem 13

(a) $dz\, dy\, dx$ **(b)** $dx\, dz\, dy$ **(c)** $dy\, dx\, dz$

14.



24. $x = 2,\ y = x,\ y = 0,\ z = x^2 + y^2,\ z = 0$

25. Find the center of mass of the solid given in **FIGURE 9.15.14** if the density at a point P is directly proportional to the distance from the xy-plane.

26. Find the centroid of the solid in **FIGURE 9.15.15** if the density is constant.

27. Find the center of mass of the solid bounded by the graphs of $x^2 + z^2 = 4$, $y = 0$, and $y = 3$ if the density at a point P is directly proportional to the distance from the xz-plane.

28. Find the center of mass of the solid bounded by the graphs of $y = x^2$, $y = x$, $z = y + 2$, and $z = 0$ if the density at a point P is directly proportional to the distance from the xy-plane.

In Problems 29 and 30, set up, but do not evaluate, the iterated integrals giving the mass of the solid that has the given shape and density.

29. $x^2 + y^2 = 1,\ z + y = 8,\ z - 2y = 2;\ \rho(x, y, z) = x + y + 4$

30. $x^2 + y^2 - z^2 = 1,\ z = -1,\ z = 2;\ \rho(x, y, z) = z^2$ [*Hint*: Do not use $dz\, dy\, dx$.]

31. Find the moment of inertia of the solid in Figure 9.15.14 about the y-axis if the density is as given in Problem 25. Find the radius of gyration.

32. Find the moment of inertia of the solid in Figure 9.15.15 about the x-axis if the density is constant. Find the radius of gyration.

33. Find the moment of inertia about the z-axis of the solid in the first octant that is bounded by the coordinate planes and the graph of $x + y + z = 1$ if the density is constant.

FIGURE 9.15.15 Solid for Problem 14

(a) $dx\,dz\,dy$ (b) $dy\,dx\,dz$ (c) $dz\,dx\,dy$

[*Hint*: Part (c) will require two integrals.]

In Problems 15–20, sketch the region D whose volume V is given by the iterated integral.

15. $\int_0^4 \int_0^3 \int_0^{2-2z/3} dx\,dz\,dy$

16. $4\int_0^3 \int_0^{\sqrt{9-y^2}} \int_4^{\sqrt{25-x^2-y^2}} dz\,dx\,dy$

17. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^5 dz\,dy\,dx$

18. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 dz\,dy\,dx$

19. $\int_0^2 \int_0^{2-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$

20. $\int_1^3 \int_0^{1/x} \int_0^3 dy\,dz\,dx$

In Problems 21–24, find the volume of the solid bounded by the graphs of the given equations.

21. $x = y^2,\ 4 - x = y^2,\ z = 0,\ z = 3$

22. $x^2 + y^2 = 4,\ z = x + y$, the coordinate planes, first octant

23. $y = x^2 + z^2,\ y = 8 - x^2 - z^2$

34. Find the moment of inertia about the y-axis of the solid bounded by the graphs of $z = y, z = 4 - y, z = 1, z = 0, x = 2$, and $x = 0$ if the density at a point P is directly proportional to the distance from the yz-plane.

In Problems 35–38, convert the point given in cylindrical coordinates to rectangular coordinates.

35. $\left(10, \frac{3\pi}{4}, 5\right)$ **36.** $\left(2, \frac{5\pi}{6}, -3\right)$

37. $\left(\sqrt{3}, \frac{\pi}{3}, -4\right)$ **38.** $\left(4, \frac{7\pi}{4}, 0\right)$

In Problems 39–42, convert the point given in rectangular coordinates to cylindrical coordinates.

39. $(1, -1, -9)$ **40.** $(2\sqrt{3}, 2, 17)$

41. $(-\sqrt{2}, \sqrt{6}, 2)$ **42.** $(1, 2, 7)$

In Problems 43–46, convert the given equation to cylindrical coordinates.

43. $x^2 + y^2 + z^2 = 25$ **44.** $x + y - z = 1$

45. $x^2 + y^2 - z^2 = 1$ **46.** $x^2 + z^2 = 16$

In Problems 47–50, convert the given equation to rectangular coordinates.

47. $z = r^2$ **48.** $z = 2r\sin\theta$

49. $r = 5\sec\theta$ **50.** $\theta = \pi/6$

In Problems 51–58, use triple integrals and cylindrical coordinates. In Problems 51–54, find the volume of the solid that is bounded by the graphs of the given equations.

51. $x^2 + y^2 = 4,\ x^2 + y^2 + z^2 = 16,\ z = 0$

52. $z = 10 - x^2 - y^2,\ z = 1$

53. $z = x^2 + y^2,\ x^2 + y^2 = 25,\ z = 0$

54. $y = x^2 + z^2,\ 2y = x^2 + z^2 + 4$

55. Find the centroid of the homogeneous solid that is bounded by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.

56. Find the center of mass of the solid that is bounded by the graphs of $y^2 + z^2 = 16, x = 0$, and $x = 5$ if the density at a point P is directly proportional to distance from the yz-plane.

57. Find the moment of inertia about the z-axis of the solid that is bounded above by the hemisphere $z = \sqrt{9 - x^2 - y^2}$ and below by the plane $z = 2$ if the density at a point P is inversely proportional to the square of the distance from the z-axis.

58. Find the moment of inertia about the x-axis of the solid that is bounded by the cone $z = \sqrt{x^2 - y^2}$ and the plane $z = 1$ if the density at a point P is directly proportional to the distance from the z-axis.

In Problems 59–62, convert the point given in spherical coordinates to (a) rectangular coordinates and (b) cylindrical coordinates.

59. $\left(\frac{2}{3}, \frac{\pi}{2}, \frac{\pi}{6}\right)$ **60.** $\left(5, \frac{5\pi}{4}, \frac{2\pi}{3}\right)$

61. $\left(8, \frac{\pi}{4}, \frac{3\pi}{4}\right)$ **62.** $\left(\frac{1}{3}, \frac{5\pi}{3}, \frac{\pi}{6}\right)$

In Problems 63–66, convert the points given in rectangular coordinates to spherical coordinates.

63. $(-5, -5, 0)$ **64.** $(1, -\sqrt{3}, 1)$

65. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ **66.** $\left(-\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$

In Problems 67–70, convert the given equation to spherical coordinates.

67. $x^2 + y^2 + z^2 = 64$ **68.** $x^2 + y^2 + z^2 = 4z$

69. $z^2 = 3x^2 + 3y^2$ **70.** $-x^2 - y^2 + z^2 = 1$

In Problems 71–74, convert the given equation to rectangular coordinates.

71. $\rho = 10$ **72.** $\phi = \pi/3$

73. $\rho = 2\sec\phi$ **74.** $\rho\sin^2\phi = \cos\phi$

In Problems 75–82, use triple integrals and spherical coordinates. In Problems 75–78, find the volume of the solid that is bounded by the graphs of the given equations.

75. $z = \sqrt{x^2 + y^2},\ x^2 + y^2 + z^2 = 9$

76. $x^2 + y^2 + z^2 = 4,\ y = x,\ y = \sqrt{3}x,\ z = 0$, first octant

77. $z^2 = 3x^2 + 3y^2,\ x = 0,\ y = 0,\ z = 2$, first octant

78. Inside $x^2 + y^2 + z^2 = 1$ and outside $z^2 = x^2 + y^2$

79. Find the centroid of the homogeneous solid that is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2z$.

80. Find the center of mass of the solid that is bounded by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the plane $z = 0$ if the density at a point P is directly proportional to the distance from the xy-plane.

81. Find the mass of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$ and below by the plane $z = 4$ if the density at a point P is inversely proportional to the distance from the origin. [*Hint*: Express the upper ϕ limit of integration as an inverse cosine.]

82. Find the moment of inertia about the z-axis of the solid that is bounded by the sphere $x^2 + y^2 + z^2 = a^2$ if the density at a point P is directly proportional to the distance from the origin.