Exercise 9.14

9.14 Exercises

Answers to selected odd-numbered problems begin on page ANS-22.

In Problems 1–4, verify Stokes' theorem. Assume that the surface S is oriented upward.

1. $\mathbf{F} = 5y\mathbf{i} - 5x\mathbf{j} + 3\mathbf{k}$; S that portion of the plane $z = 1$ within the cylinder $x^2 + y^2 = 4$
2. $\mathbf{F} = 2z\mathbf{i} - 3x\mathbf{j} + 4y\mathbf{k}$; S that portion of the paraboloid $z = 16 - x^2 - y^2$ for $z \geq 0$
3. $\mathbf{F} = z\,\mathbf{i} + x\mathbf{j} + y\mathbf{k}$; S that portion of the plane $2x + y + 2z = 6$ in the first octant
4. $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; S that portion of the sphere $x^2 + y^2 + z^2 = 1$ for $z \geq 0$

In Problems 5–12, use Stokes' theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$. Assume C is oriented counterclockwise as viewed from above.

5. $\mathbf{F} = (2z + x)\mathbf{i} + (y - z)\mathbf{j} + (x + y)\mathbf{k}$; C the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$
6. $\mathbf{F} = z^2 y \cos xy\,\mathbf{i} + z^2 x(1 + \cos xy)\,\mathbf{j} + 2z \sin xy\,\mathbf{k}$; C the boundary of the plane $z = 1 - y$ shown in **FIGURE 9.14.7**.

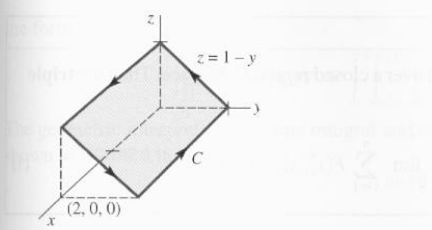

FIGURE 9.14.7 Curve C for Problem 6

7. $\mathbf{F} = xy\mathbf{i} + 2yz\,\mathbf{j} + xz\,\mathbf{k}$; C the boundary given in Problem 6
8. $\mathbf{F} = (x + 2z)\,\mathbf{i} + (3x + y)\mathbf{j} + (2y - z)\mathbf{k}$; C the curve of intersection of the plane $x + 2y + z = 4$ with the coordinate planes
9. $\mathbf{F} = y^3\mathbf{i} - x^3\mathbf{j} + z^3\mathbf{k}$; C the trace of the cylinder $x^2 + y^2 = 1$ in the plane $x + y + z = 1$
10. $\mathbf{F} = x^2y\mathbf{i} + (x + y^2)\mathbf{j} + xy^2z\,\mathbf{k}$; C the boundary of the surface shown in **FIGURE 9.14.8**

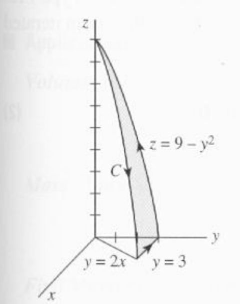

FIGURE 9.14.8 Curve C for Problem 10

11. $\mathbf{F} = x\mathbf{i} + x^3y^2\mathbf{j} + z\mathbf{k}$; C the boundary of the semi-ellipsoid $z = \sqrt{4 - 4x^2 - y^2}$ in the plane $z = 0$
12. $\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$; C the curve of intersection of the plane $x + y + z = 0$ and the sphere $x^2 + y^2 + z^2 = 1$ [*Hint*: Recall that the area of an ellipse $x^2/a^2 + y^2/b^2 = 1$ is πab.]

In Problems 13–16, use Stokes' theorem to evaluate $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS$. Assume that the surface S is oriented upward.

13. $\mathbf{F} = 6yz\mathbf{i} + 5x\mathbf{j} + yze^{x^2}\mathbf{k}$; S that portion of the paraboloid $z = \frac{1}{4}x^2 + y^2$ for $0 \leq z \leq 4$
14. $\mathbf{F} = y\mathbf{i} + (y - x)\mathbf{j} + z^2\mathbf{k}$; S that portion of the sphere $x^2 + y^2 + (z - 4)^2 = 25$ for $z \geq 0$
15. $\mathbf{F} = 3x^2\mathbf{i} + 8x^3y\mathbf{j} + 3x^2y\mathbf{k}$; S that portion of the plane $z = x$ that lies inside the rectangular cylinder defined by the planes $x = 0$, $y = 0$, $x = 2$, $y = 2$
16. $\mathbf{F} = 2xy^2z\mathbf{i} + 2x^2yz\mathbf{j} + (x^2y^2 - 6x)\mathbf{k}$; S that portion of the plane $z = y$ that lies inside the cylinder $x^2 + y^2 = 1$
17. Use Stokes' theorem to evaluate

$$\oint_C z^2 e^{x^2} dx + xy^2\, dy + \tan^{-1} y\, dz$$

where C is the circle $x^2 + y^2 = 9$, by finding a surface S with C as its boundary and such that the orientation of C is counterclockwise as viewed from above.

18. Consider the surface integral $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS$, where $\mathbf{F} = xyz\,\mathbf{k}$ and S is that portion of the paraboloid $z = 1 - x^2 - y^2$ for $z \geq 0$ oriented upward.
    - **(a)** Evaluate the surface integral by the method of Section 9.13; that is, do not use Stokes' theorem.
    - **(b)** Evaluate the surface integral by finding a simpler surface that is oriented upward and has the same boundary as the paraboloid.
    - **(c)** Use Stokes' theorem to verify the result in part (b).