Exercise 9.11

9.11 Exercises

Answers to selected odd-numbered problems begin on page ANS-22.

In Problems 1–4, use a double integral in polar coordinates to find the area of the region bounded by the graphs of the given polar equations.

1. $r = 3 + 3\sin\theta$

2. $r = 2 + \cos\theta$

3. $r = 2\sin\theta,\ r = 1$, common area

4. $r = 8\sin 4\theta$, one petal

In Problems 5–10, find the volume of the solid bounded by the graphs of the given equations.

5. One petal of $r = 5\cos 3\theta,\ z = 0,\ z = 4$

6. $x^2 + y^2 = 4,\ z = \sqrt{9 - x^2 - y^2},\ z = 0$

7. Between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$, $z = \sqrt{16 - x^2 - y^2},\ z = 0$

8. $z = \sqrt{x^2 + y^2},\ x^2 + y^2 = 25,\ z = 0$

9. $r = 1 + \cos\theta,\ z = y,\ z = 0$, first octant

10. $r = \cos\theta,\ z = 2 + x^2 + y^2,\ z = 0$

In Problems 11–16, find the center of mass of the lamina that has the given shape and density.

11. $r = 1,\ r = 3,\ x = 0,\ y = 0$, first quadrant; $\rho(r, \theta) = k$ (constant)

12. $r = \cos\theta$; density at point P directly proportional to the distance from the pole

13. $y = \sqrt{3}x,\ y = 0,\ x = 3;\ \rho(r, \theta) = r^2$

14. $r = 4\cos 2\theta$, petal on the polar axis; $\rho(r, \theta) = k$ (constant)

15. Outside $r = 2$ and inside $r = 2 + 2\cos\theta,\ y = 0$, first quadrant; density at a point P inversely proportional to the distance from the pole

16. $r = 2 + 2\cos\theta,\ y = 0$, first and second quadrants; $\rho(r, \theta) = k$ (constant)

In Problems 17–20, find the indicated moment of inertia of the lamina that has the given shape and density.

17. $r = a;\ \rho(r, \theta) = k$ (constant); I_x

18. $r = a$; $\rho(r, \theta) = \dfrac{1}{1 + r^4}$; I_x

19. Outside $r = a$ and inside $r = 2a\cos\theta$; density at a point P inversely proportional to the cube of the distance from the pole; I_y

20. Outside $r = 1$ and inside $r = 2\sin 2\theta$, first quadrant; $\rho(r, \theta) = \sec^2\theta$; I_y

In Problems 21–24, find the **polar moment of inertia**

$$I_0 = \iint_R r^2 \rho(r, \theta)\, dA = I_x + I_y$$

of the lamina that has the given shape and density.

21. $r = a$; $\rho(r, \theta) = k$ (constant) [*Hint*: Use Problem 17 and the fact that $I_x = I_y$.]

22. $r = \theta$, $0 \le \theta \le \pi$, $y = 0$; density at a point P proportional to the distance from the pole

23. $r\theta = 1$, $\frac{1}{3} \le \theta \le 1$, $r = 1$, $r = 3$, $y = 0$; density at a point P inversely proportional to the distance from the pole [*Hint*: Integrate first with respect to θ.]

24. $r = 2a\cos\theta$; $\rho(r, \theta) = k$ (constant)

In Problems 25–32, evaluate the given iterated integral by changing to polar coordinates.

25. $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$

26. $\displaystyle\int_{0}^{\sqrt{2}/2}\int_{0}^{\sqrt{1-y^2}} \frac{y^2}{\sqrt{x^2 + y^2}}\, dx\, dy$

27. $\displaystyle\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy$

28. $\displaystyle\int_{-\sqrt{\pi}}^{\sqrt{\pi}}\int_{0}^{\sqrt{\pi-x^2}} \sin(x^2 + y^2)\, dy\, dx$

29. $\displaystyle\int_{0}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \frac{x^2}{x^2 + y^2}\, dy\, dx + \int_{1}^{2}\int_{0}^{\sqrt{4-x^2}} \frac{x^2}{x^2 + y^2}\, dy\, dx$

30. $\displaystyle\int_{0}^{1}\int_{0}^{\sqrt{2y-y^2}} (1 - x^2 - y^2)\, dx\, dy$

31. $\displaystyle\int_{-5}^{5}\int_{0}^{\sqrt{25-x^2}} (4x + 3y)\, dy\, dx$

32. $\displaystyle\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{1}{1 + \sqrt{x^2 + y^2}}\, dx\, dy$

33. The liquid hydrogen tank in the space shuttle has the form of a right circular cylinder with a semi-ellipsoidal cap at each end. The radius of the cylindrical part of the tank is 4.2 m. Find the volume of the tank shown in **FIGURE 9.11.6**.

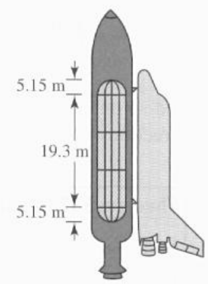

FIGURE 9.11.6 Fuel tank in Problem 33

34. Evaluate $\iint_R (x + y)\, dA$ over the region shown in **FIGURE 9.11.7**.

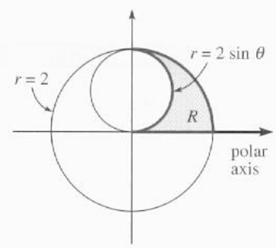

FIGURE 9.11.7 Region R for Problem 34

35. The improper integral $\int_0^\infty e^{-x^2}\, dx$ is important in the theory of probability, statistics, and other areas of applied mathematics. If I denotes the integral, then

$$I = \int_0^\infty e^{-x^2}\, dx \quad \text{and} \quad I = \int_0^\infty e^{-y^2}\, dy$$

and consequently

$$I^2 = \left(\int_0^\infty e^{-x^2} dx\right)\left(\int_0^\infty e^{-y^2} dy\right) = \int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, dx\, dy.$$

Use polar coordinates to evaluate the last integral. Find the value of I.