Exercise 9.10

9.10 Exercises

Answers to selected odd-numbered problems begin on page ANS-21.

In Problems 1–8, evaluate the given partial integral.

1. $\displaystyle\int_{-1}^{3} (6xy - 5e^{y})\,dx$
2. $\displaystyle\int_{1}^{2} \tan xy\,dy$
3. $\displaystyle\int_{1}^{3x} x^3 e^{xy}\,dy$
4. $\displaystyle\int_{\sqrt{y}}^{y^3} (8x^3y - 4xy^2)\,dx$
5. $\displaystyle\int_{0}^{2x} \frac{xy}{x^2 + y^2}\,dy$
6. $\displaystyle\int_{x^3}^{x} e^{2y/x}\,dy$
7. $\displaystyle\int_{\tan y}^{\sec y} (2x + \cos y)\,dx$
8. $\displaystyle\int_{\sqrt{y}}^{1} y \ln x\,dx$

In Problems 9–12, sketch the region of integration for the given iterated integral.

9. $\displaystyle\int_{0}^{2}\int_{1}^{2x+1} f(x, y)\,dy\,dx$
10. $\displaystyle\int_{1}^{4}\int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\,dx\,dy$
11. $\displaystyle\int_{-1}^{3}\int_{0}^{\sqrt{16-y^2}} f(x, y)\,dx\,dy$
12. $\displaystyle\int_{-1}^{2}\int_{-x^2}^{x^2+1} f(x, y)\,dy\,dx$

In Problems 13–22, evaluate the double integral over the region R that is bounded by the graphs of the given equations. Choose the most convenient order of integration.

13. $\displaystyle\iint_R x^3y^2\,dA;\ y = x,\ y = 0,\ x = 1$
14. $\displaystyle\iint_R (x + 1)\,dA;\ y = x,\ x + y = 4,\ x = 0$
15. $\displaystyle\iint_R (2x + 4y + 1)\,dA;\ y = x^2,\ y = x^3$
16. $\displaystyle\iint_R xe^{y}\,dA;$ R the same as in Problem 13
17. $\displaystyle\iint_R 2xy\,dA;\ y = x^3,\ y = 8,\ x = 0$
18. $\displaystyle\iint_R \frac{x}{\sqrt{y}}\,dA;\ y = x^2 + 1,\ y = 3 - x^2$
19. $\displaystyle\iint_R \frac{y}{1 + xy}\,dA;\ y = 0,\ y = 1,\ x = 0,\ x = 1$
20. $\displaystyle\iint_R \sin\frac{\pi x}{y}\,dA;\ x = y^2,\ x = 0,\ y = 1,\ y = 2$
21. $\displaystyle\iint_R \sqrt{x^2 + 1}\,dA;\ x = y,\ x = -y,\ x = \sqrt{3}$
22. $\displaystyle\iint_R x\,dA;\ y = \tan^{-1}x,\ y = 0,\ x = 1$

23. Consider the solid bounded by the graphs of $x^2 + y^2 = 4$, $z = 4 - y$, and $z = 0$ shown in **FIGURE 9.10.11**. Choose and evaluate the correct integral representing the volume V of the solid.

    (a) $\displaystyle 4\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}} (4 - y)\,dy\,dx$

    (b) $\displaystyle 2\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} (4 - y)\,dy\,dx$

    (c) $\displaystyle 2\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} (4 - y)\,dx\,dy$

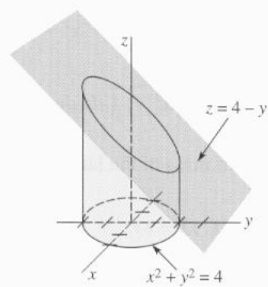

FIGURE 9.10.11 Solid for Problem 23

24. Consider the solid bounded by the graphs of $x^2 + y^2 = 4$ and $y^2 + z^2 = 4$. An eighth of the solid is shown in **FIGURE 9.10.12**. Choose and evaluate the correct integral representing the volume V of the solid.

    (a) $\displaystyle 4\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4 - y^2)^{1/2}\,dy\,dx$

    (b) $\displaystyle 8\int_{0}^{2}\int_{0}^{\sqrt{4-y^2}} (4 - y^2)^{1/2}\,dx\,dy$

    (c) $\displaystyle 8\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}} (4 - x^2)^{1/2}\,dy\,dx$

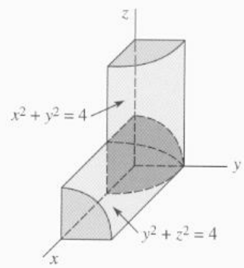

FIGURE 9.10.12 Solid for Problem 24

In Problems 25–34, find the volume of the solid bounded by the graphs of the given equations.

25. $2x + y + z = 6, x = 0, y = 0, z = 0$, first octant

26. $z = 4 - y^2, x = 3, x = 0, y = 0, z = 0$, first octant

27. $x^2 + y^2 = 4, x - y + 2z = 4, x = 0, y = 0, z = 0$, first octant

28. $y = x^2, y + z = 3, z = 0$

29. $z = 1 + x^2 + y^2, 3x + y = 3, x = 0, y = 0, z = 0$, first octant

30. $z = x + y, x^2 + y^2 = 9, x = 0, y = 0, z = 0$, first octant

31. $yz = 6, x = 0, x = 5, y = 1, y = 6, z = 0$

32. $z = 4 - x^2 - \frac{1}{4}y^2, z = 0$

33. $z = 4 - y^2, x^2 + y^2 = 2x, z = 0$

34. $z = 1 - x^2, z = 1 - y^2, x = 0, y = 0, z = 0$, first octant

In Problems 35–40, evaluate the given iterated integral by reversing the order of integration.

35. $\int_0^1 \int_x^1 x^2\sqrt{1 + y^4}\, dy\, dx$ **36.** $\int_0^1 \int_{2y}^2 e^{-y/x}\, dx\, dy$

37. $\int_0^2 \int_{y^2}^4 \cos\sqrt{x^3}\, dx\, dy$ **38.** $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x\sqrt{1 - x^2 - y^2}\, dy\, dx$

39. $\int_0^1 \int_x^1 \frac{1}{1 + y^4}\, dy\, dx$ **40.** $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1}\, dx\, dy$

In Problems 41–50, find the center of mass of the lamina that has the given shape and density.

41. $x = 0,\ x = 4,\ y = 0,\ y = 3;\ \rho(x, y) = xy$

42. $x = 0,\ y = 0,\ 2x + y = 4;\ \rho(x, y) = x^2$

43. $y = x,\ x + y = 6,\ y = 0;\ \rho(x, y) = 2y$

44. $y = |x|,\ y = 3;\ \rho(x, y) = x^2 + y^2$

45. $y = x^2,\ x = 1,\ y = 0;\ \rho(x, y) = x + y$

46. $x = y^2,\ x = 4;\ \rho(x, y) = y + 5$

47. $y = 1 - x^2,\ y = 0$; density at a point P directly proportional to the distance from the x-axis

48. $y = \sin x,\ 0 \le x \le \pi,\ y = 0$; density at a point P directly proportional to the distance from the y-axis

49. $y = e^x,\ x = 0,\ x = 1,\ y = 0;\ \rho(x, y) = y^3$

50. $y = \sqrt{9 - x^2}, y = 0;\ \rho(x, y) = x^2$

In Problems 51–54, find the moment of inertia about the x-axis of the lamina that has the given shape and density.

51. $x = y - y^2,\ x = 0;\ \rho(x, y) = 2x$

52. $y = x^2,\ y = \sqrt{x};\ \rho(x, y) = x^2$

53. $y = \cos x,\ -\pi/2 \le x \le \pi/2,\ y = 0;\ \rho(x, y) = k$ (constant)

54. $y = \sqrt{4 - x^2}, x = 0, y = 0$, first quadrant; $\rho(x, y) = y$

In Problems 55–58, find the moment of inertia about the y-axis of the lamina that has the given shape and density.

55. $y = x^2,\ x = 0,\ y = 4$, first quadrant; $\rho(x, y) = y$

56. $y = x^2,\ y = \sqrt{x};\ \rho(x, y) = x^2$

57. $y = x,\ y = 0,\ y = 1,\ x = 3;\ \rho(x, y) = 4x + 3y$

58. Same R and density as in Problem 47

In Problems 59 and 60, find the radius of gyration about the indicated axis of the lamina that has the given shape and density.

59. $x = \sqrt{a^2 - y^2},\ x = 0;\ \rho(x, y) = x$; y-axis

60. $x + y = a,\ a > 0,\ x = 0,\ y = 0;\ \rho(x, y) = k$ (constant); x-axis

61. A lamina has the shape of the region bounded by the graph of the ellipse $x^2/a^2 + y^2/b^2 = 1$. If its density is $\rho(x, y) = 1$, find:

(a) the moment of inertia about the x-axis of the lamina,

(b) the moment of inertia about the y-axis of the lamina,

(c) the radius of gyration about the x-axis [*Hint*: The area of the ellipse is πab], and

(d) the radius of gyration about the y-axis.

62. A cross section of an experimental airfoil is the lamina shown in **FIGURE 9.10.13**. The arc ABC is elliptical, whereas the two arcs AD and CD are parabolic. Find the moment of inertia about the x-axis of the lamina under the assumption that the density is $\rho(x, y) = 1$.

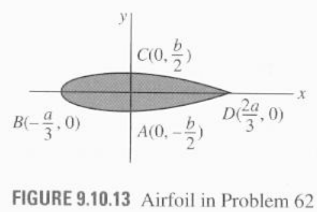

FIGURE 9.10.13 Airfoil in Problem 62

The **polar moment of inertia** of a lamina with respect to the origin is defined to be

$$I_0 = \iint_R (x^2 + y^2)\rho(x, y)\, dA = I_x + I_y.$$

In Problems 63–66, find the polar moment of inertia of the lamina that has the given shape and density.

63. $x + y = a,\ a > 0,\ x = 0,\ y = 0;\ \rho(x, y) = k$ (constant)

64. $y = x^2,\ y = \sqrt{x};\ \rho(x, y) = x^2$ [*Hint*: See Problems 52 and 56.]

65. $x = y^2 + 2,\ x = 6 - y^2$; density at a point P inversely proportional to the square of the distance from the origin

66. $y = x,\ y = 0,\ y = 3,\ x = 4;\ \rho(x, y) = k$ (constant)

67. Find the radius of gyration in Problem 63.

68. Show that the polar moment of inertia about the center of a thin homogeneous rectangular plate of mass m, width w, and length l is $I_0 = m(l^2 + w^2)/12$.