9.1 Exercises Answers to selected odd-numbered problems begin on page ANS-19.

In Problems 1–10, graph the curve traced by the given vector function.

1. $\mathbf{r}(t) = 2\sin t\mathbf{i} + 4\cos t\mathbf{j} + t\mathbf{k};\ t \geq 0$
2. $\mathbf{r}(t) = \cos t\mathbf{i} + t\mathbf{j} + \sin t\mathbf{k};\ t \geq 0$
3. $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + \cos t\mathbf{k};\ t \geq 0$
4. $\mathbf{r}(t) = 4\mathbf{i} + 2\cos t\mathbf{j} + 3\sin t\mathbf{k}$
5. $\mathbf{r}(t) = \langle e^t, e^{2t}\rangle$
6. $\mathbf{r}(t) = \cosh t\mathbf{i} + 3\sinh t\mathbf{j}$
7. $\mathbf{r}(t) = \langle \sqrt{2}\sin t, \sqrt{2}\sin t, 2\cos t\rangle;\ 0 \leq t \leq \pi/2$
8. $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + t\mathbf{k}$
9. $\mathbf{r}(t) = e^t\cos t\mathbf{i} + e^t\sin t\mathbf{j} + e^t\mathbf{k}$
10. $\mathbf{r}(t) = \langle t\cos t, t\sin t, t^2\rangle$

In Problems 11–14, find the vector function that describes the curve C of intersection between the given surfaces. Sketch the curve C. Use the indicated parameter.

11. $z = x^2 + y^2,\ y = x;\ x = t$
12. $x^2 + y^2 - z^2 = 1,\ y = 2x;\ x = t$
13. $x^2 + y^2 = 9,\ z = 9 - x^2;\ x = 3\cos t$
14. $z = x^2 + y^2,\ z = 1;\ x = \sin t$
15. Given that $\mathbf{r}(t) = \dfrac{\sin 2t}{t}\mathbf{i} + (t-2)^5\mathbf{j} + t\ln t\mathbf{k}$, find $\lim_{t\to 0^-}\mathbf{r}(t)$.
16. Given that $\lim_{t\to a} r_1(t) = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\lim_{t\to a}\mathbf{r}_2(t) = 2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$, find:
    (a) $\lim_{t\to a}[-4\mathbf{r}_1(t) + 3\mathbf{r}_2(t)]$
    (b) $\lim_{t\to a}\mathbf{r}_1(t)\cdot\mathbf{r}_2(t)$.

In Problems 17–20, find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ for the given vector function.

17. $\mathbf{r}(t) = \ln t\mathbf{i} + \mathbf{j},\ t > 0$
18. $\mathbf{r}(t) = \langle t\cos t - \sin t, t + \cos t\rangle$
19. $\mathbf{r}(t) = \langle te^{2t}, t^3, 4t^2 - t\rangle$
20. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + \tan^{-1}t\mathbf{k}$

In Problems 21–24, graph the curve C that is described by $\mathbf{r}$ and graph $\mathbf{r}'$ at the indicated value of t.

21. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 6\sin t\mathbf{j};\ t = \pi/6$
22. $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j};\ t = -1$
23. $\mathbf{r}(t) = 2\mathbf{i} + t\mathbf{j} + \dfrac{4}{1+t^2}\mathbf{k};\ t = 1$
24. $\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 2t\mathbf{k};\ t = \pi/4$

In Problems 25 and 26, find parametric equations of the tangent line to the given curve at the indicated value of t.

25. $x = t,\ y = \frac{1}{2}t^2,\ z = \frac{1}{3}t^3;\ t = 2$
26. $x = t^3 - t,\ y = \dfrac{6t}{t+1},\ z = (2t+1)^2;\ t = 1$

In Problems 27–32, find the indicated derivative. Assume that all vector functions are differentiable.

27. $\dfrac{d}{dt}[\mathbf{r}(t)\times\mathbf{r}'(t)]$
28. $\dfrac{d}{dt}[\mathbf{r}(t)\cdot(t\mathbf{r}(t))]$
29. $\dfrac{d}{dt}[\mathbf{r}(t)\cdot(\mathbf{r}'(t)\times\mathbf{r}''(t))]$
30. $\dfrac{d}{dt}[\mathbf{r}_1(t)\times(\mathbf{r}_2(t)\times\mathbf{r}_3(t))]$
31. $\dfrac{d}{dt}\left[\mathbf{r}_1(2t) + \mathbf{r}_2\left(\dfrac{1}{t}\right)\right]$
32. $\dfrac{d}{dt}[t^3\mathbf{r}(t^2)]$

In Problems 33–36, evaluate the given integral.

33. $\displaystyle\int_{-1}^{2}(t\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k})\,dt$
34. $\displaystyle\int_{0}^{4}(\sqrt{2t+1}\mathbf{i} - \sqrt{t}\mathbf{j} + \sin\pi t\mathbf{k})\,dt$
35. $\displaystyle\int(te^t\mathbf{i} - e^{-2t}\mathbf{j} + te^{t^2}\mathbf{k})\,dt$
36. $\displaystyle\int\frac{1}{1+t^2}(\mathbf{i} + t\mathbf{j} + t^2\mathbf{k})\,dt$

In Problems 37–40, find a vector function $\mathbf{r}$ that satisfies the indicated conditions.

37. $\mathbf{r}'(t) = 6\mathbf{i} + 6t\mathbf{j} + 3t^2\mathbf{k};\ r(0) = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$
38. $\mathbf{r}'(t) = t\sin t^2\mathbf{i} - \cos 2t\mathbf{j};\ \mathbf{r}(0) = \frac{3}{2}\mathbf{i}$
39. $\mathbf{r}''(t) = 12t\mathbf{i} - 3t^{-1/2}\mathbf{j} + 2\mathbf{k};\ \mathbf{r}'(1) = \mathbf{j},\ \mathbf{r}(1) = 2\mathbf{i} - \mathbf{k}$
40. $\mathbf{r}''(t) = \sec^2 t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k};$
    $\mathbf{r}'(0) = \mathbf{i} + \mathbf{j} + \mathbf{k},\ \mathbf{r}(0) = -\mathbf{j} + 5\mathbf{k}$

In Problems 41–44, find the length of the curve traced by the given vector function on the indicated interval.

41. $\mathbf{r}(t) = a\cos t\mathbf{i} + a\sin t\mathbf{j} + ct\mathbf{k};\ 0 \leq t \leq 2\pi$
42. $\mathbf{r}(t) = t\mathbf{i} + t\cos t\mathbf{j} + t\sin t\mathbf{k};\ 0 \leq t \leq \pi$
43. $\mathbf{r}(t) = e^t\cos 2t\mathbf{i} + e^t\sin 2t\mathbf{j} + e^t\mathbf{k};\ 0 \leq t \leq 3\pi$
44. $\mathbf{r}(t) = 3t\mathbf{i} + \sqrt{3}t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k};\ 0 \leq t \leq 1$
45. Express the vector equation of a circle $\mathbf{r}(t) = a\cos t\mathbf{i} + a\sin t\mathbf{j}$ as a function of arc length s. Verify that $\mathbf{r}'(s)$ is a unit vector.
46. If $\mathbf{r}(s)$ is the vector function given in (4), verify that $\mathbf{r}'(s)$ is a unit vector.
47. Suppose $\mathbf{r}$ is a differentiable vector function for which $\|\mathbf{r}(t)\| = c$ for all t. Show that the tangent vector $\mathbf{r}'(t)$ is perpendicular to the position vector $\mathbf{r}(t)$ for all t.
48. In Problem 47, describe geometrically the kind of curve C for which $\|\mathbf{r}(t)\| = c$.

Miscellaneous Problems

49. Prove Theorem 9.1.4(*ii*).
50. Prove Theorem 9.1.4(*iii*).
51. Prove Theorem 9.1.4(*iv*).
52. If $\mathbf{v}$ is a constant vector and $\mathbf{r}$ is integrable on $[a, b]$, prove that $\int_a^b \mathbf{v}\cdot\mathbf{r}(t)\,dt = \mathbf{v}\cdot\int_a^b \mathbf{r}(t)\,dt$.