Stellenbosch University Faculty of Engineering

Module Framework

This document should be read with the following documents:

- ***Stellenbosch University Calendar Parts 1 and 11.***
- ***Faculty of Engineering Assessment Rules***
- ***Faculty of Engineering General Stipulations for Undergraduate Modules***[1]

Module name & code **Numerical Methods** **36323 876** Year: **PG**	Lecturer(s): Title. **Prof Nick Hale** Room: **A410** e-mail: **nickhale@sun.ac.za**	Approved by Programme Coordinator: Date: 29 January 2020

1 Assessment Details

Calculation of final marks (according to formulas in the Faculty of Engineering's Assessment Rules):

* 5 x tutorial mark (5% each of final mark, drop lowest. Total = 20%)
* Homework Assignment 1: 40%
* Homework assignment 2: 40%

2 Module Objectives

A student who has successfully completed this module can:

* Have an understanding of matrix and vector norms and their properties.
* Define concepts of forward and backward error, and their relationship to the matrix condition number when solving linear systems.
* Understand and implement basic algorithms for solving and factorising linear systems (i.e., Gaussian elimination, LU factorisations).
* Define what is meant by an 'over-determined' linear system and describe what is meant by a 'least squares' solution and how this might be computed.
* Understand other factorisations of dense matrices, such as the QR factorisation, singular value decomposition (SVD) and eigenvalue decompositions, and various algorithms to compute these.
* Describe what is meant by a 'sparse' matrix, and the implications of sparsity on selecting a numerical method for solving a linear system.
* Discretise some simple partial differential equations using finite difference approximations to arrive at structured, sparse linear systems.
* Derive, implement, and compare various algorithms for solving such linear systems, e.g., Jacobi, Gauss-Seidel, and conjugate gradients.

[1] Available on SUNLearn for modules offered by Faculty of Engineering, in the block titled "General Programme Information" on the right-hand side

3 Module Content and Schedule

The module focuses on matrix computations. We study the effective solution of linear systems, involving both square and rectangular matrices (least-squares). Direct as well as iterative methods are considered, with the emphasis on sparse matrices and matrices with structure. Numerical methods for the eigenvalue problem are also considered. Pitfalls such as numerical instability and ill-conditioning are pointed out. Model problems are taken from partial differential equations, data analysis and image processing. Theory, algorithmic aspects, and applications are emphasized in equal parts. Our primary textbook will be M.T. Heath, Scientific Computing: An Introductory Survey, McGraw-Hill, 1997.

Schedule

Day 1:

- Chapter 1: Fundamentals of Scientific Computing
- Chapter 2: Dense linear algebra (existence and uniqueness of solutions, vector and matrix norms, condition numbers, error bounds)

Day2:

- Chapter 2 continued: Dense linear algebra (solving linear systems, Guassian elimination, LU factorisation, pivotting, complexity, solving modified problems, SPD matrices and Cholesky factorisations, sparse direct methods)

Day 3:

- Chapter 3: Least-squares problems (normal equations, QR factorisation, Householder transformations, Givens rotations, Gram-Schmidt orthogonalization)

Day 4:

- Chapter 4: Eigenvalue problems (power iteration and related methods, deflations, QR algorithm, Arnoldi and Lanczos)

Day 5:

- Chapter 11: Solving sparse linear systems (finite difference model problem, fixed point methods, splitting methods, symmetric preconditioners, steepest descent, conjugate gradients, preconditioning, GMRES)

4 Other Module-Specific Information

- The contact time will be split in to approximately four hours of lectures in the morning, and four hours of labs/tutorials in the afternoons where we will work on examples related to the content covered that morning and a short tutorial-style test. Homework assignments will be completed in the student's own time and be submitted before the given deadline.

5 Deadlines

Tasks, assignments, tutorials, presentations and projects must be submitted by specified due dates and times. If it appears that you are unable to meet a deadline, a written motivation for an extension must be submitted at least 24 hours before the original submission deadline. Extensions are subject to penalty of up to 10 marks per day (or part thereof) with respect to the original due date. An extension to an assignment or project deadline of longer than one week (7 calendar days) will not be granted. Failure to meet a submission deadline is likely to result in no (zero) marks being allocated to the task set.

6 Plagiarism Avoidance

The purpose of the Stellenbosch University Policy on Plagiarism is to "*Set standards of academic conduct for the use of the work of others and the reuse of one's own published work; promote academic integrity*." The policy document is available at

https://www.sun.ac.za/english/learning-teaching/ctl/Documents/SU%20Plagiarism%20Policy_2016.pdf

Please refer to the Writing Guide, Appendix F: Referencing and plagiarism on how to avoid plagiarism (the handle is "Hints for thesis writing").

http://www.sun.ac.za/english/faculty/eng/mechanical-mechatronic/postgrad/postgrad-current

Please refer to the current edition of the Study Guide for M and PhD students, section 3.1 regarding departmental assessment criteria for the adjudication of plagiarism.

http://www.sun.ac.za/english/faculty/eng/mechanical-mechatronic/postgrad/postgrad-current

Alleged cases of plagiarism are handled according to the Stellenbosch University ("SU") Procedure for the investigation and management of allegations of plagiarism. This document is available at

https://www.sun.ac.za/english/research-innovation/Research-Development/Documents/Policies%20and%20Guidelines/ENGLISH/SU%20Procedure%20for%20the%20investigation%20and%20management%20of%20allegations%20of%20plagiarism_2016.pdf

Less serious cases of plagiarism are dealt with on faculty/departmental level following internal faculty processes. The Plagiarism Procedure for the Faculty of Engineering of Stellenbosch University is available at

https://learn.sun.ac.za/pluginfile.php/1225885/block_html/content/Plagiarism%20Procedures%20Faculty%20of%20Engineering%20V2a_E.pdf

7 References and Suggested Reading

- M. T. Heath, Scientific Computing: An Introductory Survey, McGraw-Hill, 1997
- L. N. Trefethen and D. B. Bau, Numerical Linear Algebra, SIAM, 1997
- G. Strang, Introduction to Linear Algebra, Cambridge University Press, 2016
- H. Wendland, Numerical Linear Algebra, Cambridge University Press, 2017