CHAPTER 8: Image compression   (page 547)

Definition: Reduction of amount of data used to represent an image by reducing redundant data, so that the image can be stored or transferred more efficiently

Compression: $\begin{cases} \textbf{Lossless coding:} & \textbf{Error free compression} \\ \textbf{Lossy coding:} & \textbf{Error containing compression} \end{cases}$

Motivation: Amount of data required to represent a 2-hour SD TV movie...

$$30 \, \frac{\textbf{frames}}{\textbf{sec}} \times (720 \times 480) \, \frac{\textbf{pixels}}{\textbf{frame}} \times 3 \, \frac{\textbf{bytes}}{\textbf{pixel}} = 31,104,000 \, \textbf{bytes/sec}$$

$$31,104,000 \, \frac{\textbf{bytes}}{\textbf{sec}} \times (60)^2 \, \frac{\textbf{sec}}{\textbf{hour}} \times 2 \, \textbf{hours} \approx 2.24 \times 10^{11} \, \textbf{bytes} \approx 224 \, \textbf{GB}$$

Twenty-seven 8.5 **GB dual-layer DVDs are required to store this! To store this on a single DVD, a compression factor of** 26.3 **is required. Even more compression is required for HD TV (**$1920 \times 1080 \times 3$ **bytes/image)**

Other applications: Web page images, televideo conferencing, remote sensing, document and medical imaging, **FAX**

8.1: Fundamentals

- **Objective is to get rid of redundant data (data $\neq$ information)**

  - **Compression ratio:** $C = \dfrac{b}{b'}$ • **Relative data redundancy:** $R = 1 - \dfrac{1}{C}$

$\left. \begin{array}{l} b = \text{ amount of data in uncompressed data set} \\ b' = \text{ amount of data in compressed data set} \end{array} \right\} \approx$ **the same information**

If $b' = b$, then $C = 1$ and $R = 0$
If $b : b' = 10 : 1$, then $R = 0.9 \Rightarrow 90$% of data redundant

Three types of redundancy: $\left\{ \begin{array}{ll} \textbf{(1)} & \textbf{Coding redundancy} \\ \textbf{(2)} & \textbf{Spatial and temporal redundancy} \\ \textbf{(3)} & \textbf{Irrelevant information} \end{array} \right.$

Coding redundancy

Code $\equiv$ system of symbols (letters, numbers) that represent information (set of events)

Piece of information (event) assigned sequence of code symbols $\Rightarrow$ code word

Number of symbols in a code word is its length

Usually the 8-bit codes that represent image intensities contain more bits than required

Spatial and temporal redundancy

Pixels in images are often correlated spatially and information is unnecessarily replicated

In video sequences, temporally correlated pixels also duplicate information

Irrelevant information

Images often contain redundant information that is ignored by the human visual system (psycho-visual redundancy)

a b c

FIGURE 8.1 Computer generated $256 \times 256 \times 8$ bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)

8.1.1 Coding redundancy

Consider the histogram

$$p_r(r_k) = \frac{n_k}{MN}, \quad k = 0, 1, \ldots, L-1$$

and let

$$\ell(r_k) \equiv \begin{array}{l} \textbf{number of bits used to represent} \\ \textbf{the gray scale value } r_k, \end{array}$$

then

$$L_{\text{ave}} = \sum_{k=0}^{L-1} \ell(r_k)\, p(r_k) \equiv \begin{array}{l} \textbf{average number of bits} \\ \textbf{required to represent} \\ \textbf{a pixel in the image} \end{array}$$

Therefore, for an $M \times N$ image, we have that

$$MNL_{\text{ave}} \equiv \begin{array}{l} \textbf{total number a bits required} \\ \textbf{to code the entire image} \end{array}$$

When each grey scale value is represented by an m-bit (natural) binary code, then $L_{\text{ave}} = m$

Example 8.1: Variable length coding (reversible)

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

TABLE 8.1
Example of
variable-length
coding.

Natural 8-bit code: $L_{\text{ave}} = 8$ **bits**

Variable length code: $L_{\text{ave}} = 0.25(2) + 0.47(1) + 0.25(3) + 0.03(3) = 1.81$ **bits**

$$C = \frac{8}{1.81} \approx 4.42$$

$$R = 1 - \frac{1}{4.42} = 0.774 \quad \Rightarrow \quad 77.4\% \text{ of data is redundant}$$

Strategy: When $p(r_k)$ is large, $\ell_2(r_k)$ should be small, and vice versa

Note: Best fixed-length code is $\{00, 01, 10, 11\}$ — the resulting compression is $4:1$, which is still 10% less than $4.42:1$ for variable-length code

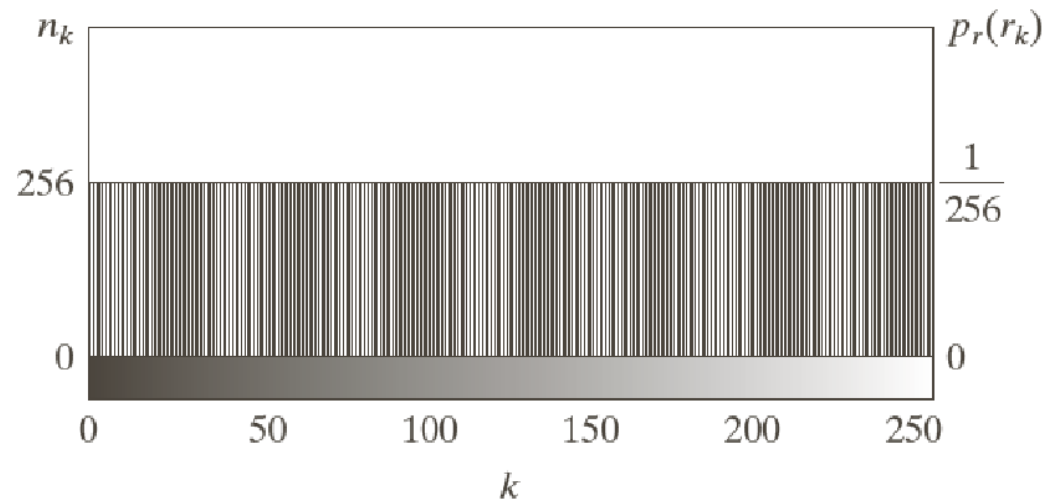## 8.1.2 Spatial and temporal redundancy

FIGURE 8.2 The intensity histogram of the image in Fig. 8.1(b).

(1) All intensities equally probable

(2) Line intensities random $\Rightarrow$ pixels independent in vertical direction

(3) Pixels in a specific line are maximally correlated (horizontally)

• **Fig 8.1 (b) can not be compressed by variable-length coding alone**

• **Spatial redundancy can be eliminated by utilising run-length pairs $(128:1)$ or the differences between adjacent pixels (reversible mappings)**

• **In most images, the pixels are correlated spatially (x and y directions), and in time (videos)**

8.1.3 Irrelevant information

Fig 8.1 (c): "Appears" homogeneous $\Rightarrow$ compression of $65\,536 : 1$ may be possible!
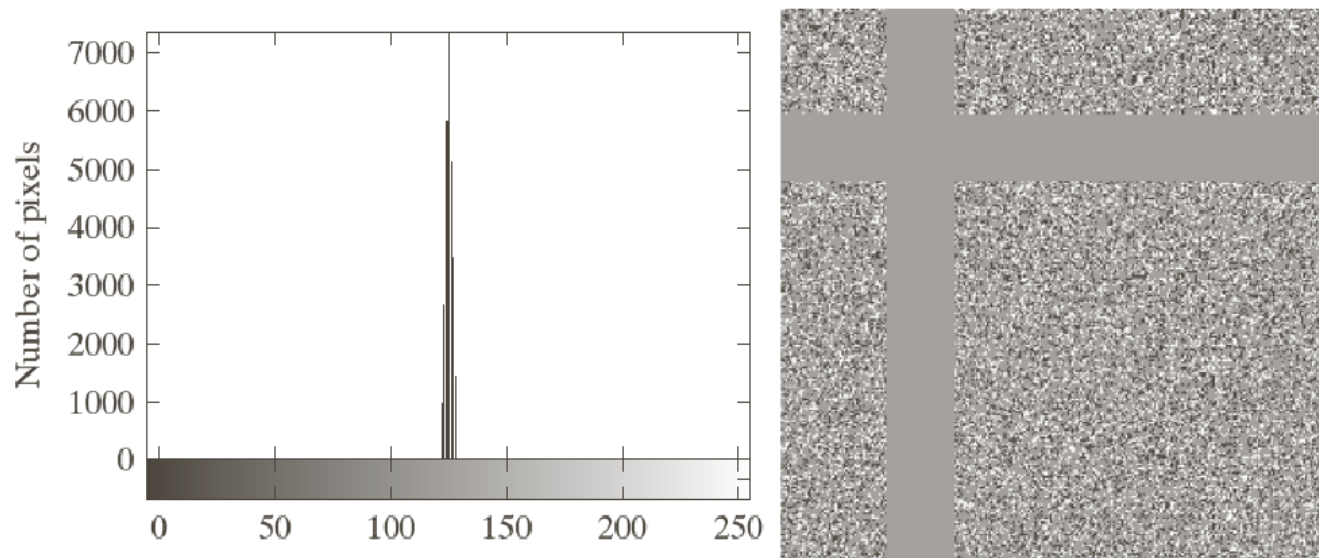

FIGURE 8.3
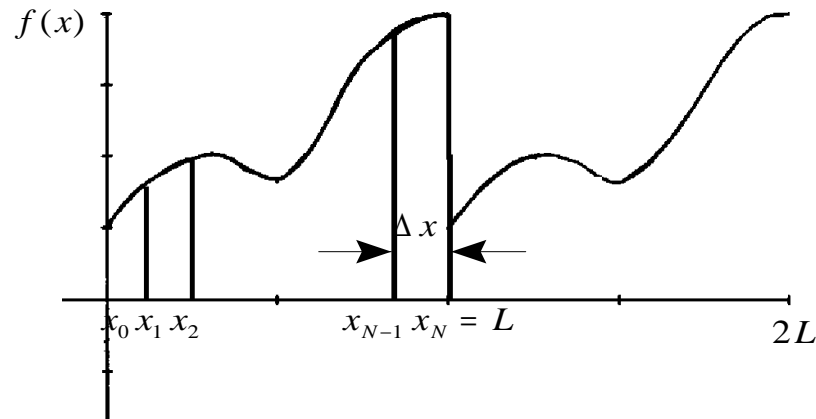(a) Histogram of the image in Fig. 8.1(c) and (b) a histogram equalized version of the image.

Visually redundant data may be removed depending on the application

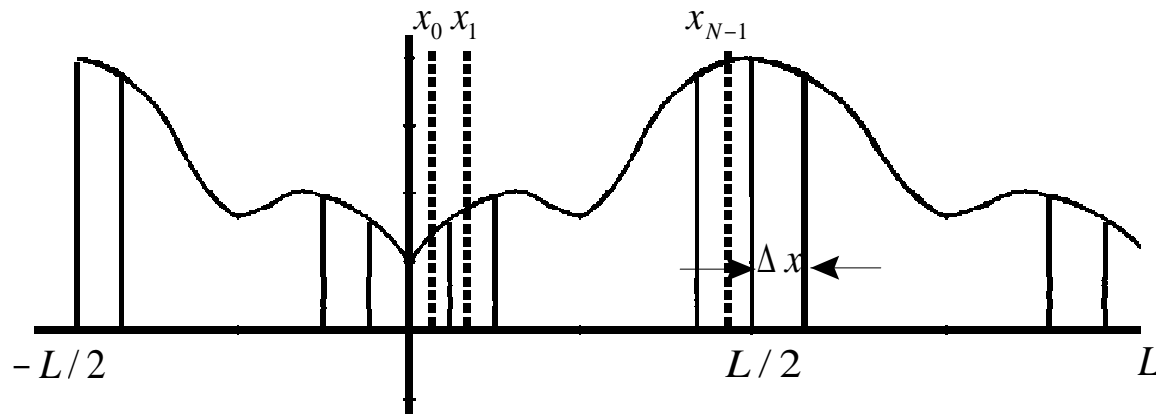**This removal of data is <u>irreversible</u> and referred to as <u>quantization</u>**

The Discrete Cosine Transform (DCT)

We derived the discrete Fourier transform using the example

In general, the periodic continuation of $f(x)$ is discontinuous, which implies that the Fourier coefficients c_n decrease like $1/n$

We therefore rather consider the following

The periodic continuation of $f(x)$ is now guaranteed to be continuous, which implies that the Fourier coefficients c_n will decrease at least like $1/n^2$

$$
\begin{aligned}
a_0 &= \frac{1}{L} \int_{-L/2}^{L/2} f(x)\,dx \\
&= \frac{2}{L} \int_{0}^{L/2} f(x)\,dx \\
&\approx \frac{2\Delta x}{L} \left[f(x_0) + f(x_1) + \ldots + f(x_{N-1}) \right] \quad \textbf{(Rectangle rule)} \\
&= \frac{2\Delta x}{L} \sum_{j=0}^{N-1} f(x_j)
\end{aligned}
$$

But

$$
\Delta x = (L/2)/N = L/(2N) \;\Rightarrow\; (2\Delta x)/L = 1/N,
$$

therefore

$$
\hbar_0 = a_0 = \frac{1}{N} \sum_{j=0}^{N-1} f_j
$$

For $n = 1, 2, \ldots, N - 1$, **we have that**

$$
\begin{aligned}
a_n &= \frac{2}{L} \int_{-L/2}^{L/2} f(x) \cos\left\{\frac{2n\pi x}{L}\right\} dx \\
&= \frac{4}{L} \int_{0}^{L/2} f(x) \cos\left\{\frac{2n\pi x}{L}\right\} dx \\
&\approx \frac{4\,\Delta x}{L} \left\{ f(x_0) \cos\left\{\frac{2n\pi x_0}{L}\right\} + \ldots + f(x_{N-1}) \cos\left\{\frac{2n\pi x_{N-1}}{L}\right\} \right\} \\
&= \frac{4\,\Delta x}{L} \sum_{j=0}^{N-1} f_j \cos\left\{\frac{2n\pi x_j}{L}\right\}
\end{aligned}
$$

But $\quad \dfrac{2\,\Delta x}{L} = \dfrac{1}{N} \implies \dfrac{4\,\Delta x}{L} = \dfrac{2}{N} \quad$ **and** $\quad x_j = \left\{\dfrac{2j+1}{2}\right\} \Delta x, \quad$ **that is**

$$
x_0 = (1/2)\,\Delta x
$$

$$
x_1 = (3/2)\,\Delta x
$$

$$
x_2 = (5/2)\,\Delta x, \quad \textbf{etc.}
$$

Therefore

$$x_j = \left\{ \frac{2j+1}{2} \right\} \frac{L}{2N} \quad \Rightarrow \quad \frac{2x_j}{L} = \frac{(2j+1)}{2N}$$

and

$$\hbar_n = a_n = \frac{2}{N} \sum_{j=0}^{N-1} f_j \cos \left\{ \frac{(2j+1)\, n\pi}{2N} \right\}, \quad n = 1, 2, \ldots, N-1$$

To summarize...

$$\hbar_n = \alpha_n^2 \sum_{j=0}^{N-1} f_j \cos \left\{ \frac{(2j+1)\, n\pi}{2N} \right\}$$

where

$$\alpha_n^2 = \begin{cases} \dfrac{1}{N}, & n = 0 \\[2ex] \dfrac{2}{N}, & n = 1, 2, \ldots, N-1 \end{cases}$$

Multiply on both sides with $\cos\left\{\dfrac{(2k+1)\,n\pi}{2N}\right\}$ **and** $\displaystyle\sum_{n=0}^{N-1}$ **:**

$$\sum_{n=0}^{N-1} \hbar_n \, \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

$$= \sum_{n=0}^{N-1} \alpha_n^2 \sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

$$= \sum_{j=0}^{N-1} f_j \sum_{n=0}^{N-1} \alpha_n^2 \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\}$$

It can be shown that

$$\sum_{n=0}^{N-1} \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right] \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = \begin{cases} \frac{N+1}{2}, & j=k \\ \frac{1}{2}, & j\neq k \end{cases}$$

thus

$$\sum_{n=0}^{N-1} \alpha_n^2 \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\} \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = \begin{cases} \left\{\frac{2}{N}\right\}\left\{\frac{N+1}{2}\right\} - \frac{1}{N} = 1, & j=k \\ \left\{\frac{2}{N}\right\}\left\{\frac{1}{2}\right\} - \frac{1}{N} = 0, & j\neq k \end{cases}$$

This implies that

$$\sum_{n=0}^{N-1} \hbar_n \cos\left\{\frac{(2k+1)\,n\pi}{2N}\right\} = f_k \times 1$$

and

$$f_j = \sum_{n=0}^{N-1} \hbar_n \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\}$$
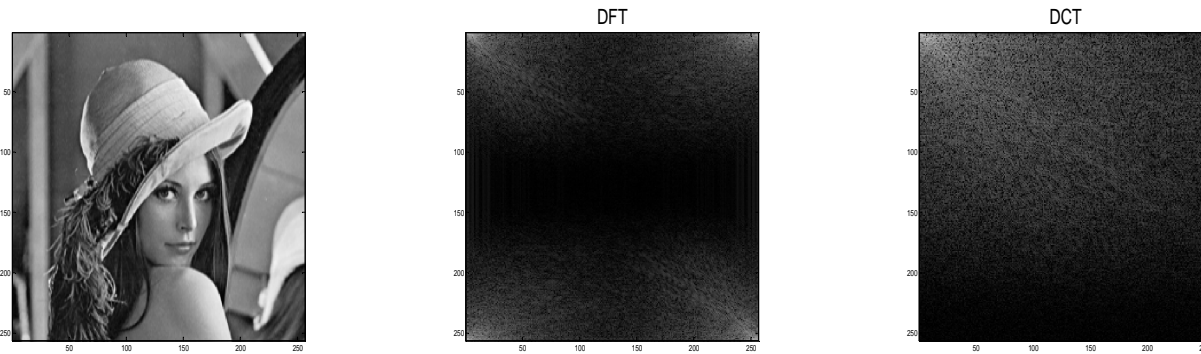
Now let $C_n = \dfrac{1}{\alpha_n}\hbar_n$**, then**

$$C_n = \mathbf{DCT}\{f_j\} = \alpha_n \sum_{j=0}^{N-1} f_j \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\}$$

$$f_j = \mathbf{DICT}\{C_n\} = \sum_{n=0}^{N-1} \alpha_n\, C_n \cos\left\{\frac{(2j+1)\,n\pi}{2N}\right\}$$

where

$$\alpha_n = \begin{cases} \sqrt{\dfrac{1}{N}}, & n=0 \\[2ex] \sqrt{\dfrac{2}{N}}, & n=1,2,\ldots,N-1 \end{cases}$$

8.1.4 Measuring image information (READ)

8.1.5 Fidelity criteria   Objective fidelity criteria

Root-mean-square error

$$e_{\mathrm{rms}} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\,\hat{f}(x,y) - f(x,y)\,]^2 \right]^{1/2}$$

Mean-square signal-to-noise ratio

$$\mathrm{SNR}_{\mathrm{ms}} = \frac{\displaystyle\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\displaystyle\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\,\hat{f}(x,y) - f(x,y)\,]^2}$$

Subjective fidelity criteria

• **Often more appropriate!!!**

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

TABLE 8.2
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)