5.7 Inverse Filtering

(page 373)

- **The simplest approach...**

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- **Problems: (1) We do not know $N(u,v)$**
  (2) $H(u,v)$ often has zero-values or small values

- **Quick fix: Limit filter frequencies to values near the origin**

Example 5.11: Inverse filtering

$$H(u,v) = e^{-k((u-M/2)^2+(v-N/2)^2)^{5/6}}$$

$$k = 0.0025; \quad M = N = 480$$

Discussion of Figure 5.27

(a): Full inverse filtering useless due to small values of $H(u,v)$

Quick fix: Attenuate $\dfrac{G(u,v)}{H(u,v)}$ outside radii of 40, 70 and 85 respectively, using Butterworth lowpass filters of order 10

(b): Radius of $40 \Rightarrow$ "Blurred"; (c): Radius of $70 \Rightarrow$ Optimal;

(d): Radius of $85 \Rightarrow$ Noise emphasized

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

How can we improve on direct inverse filtering?

5.8 Minimun mean square error (Wiener) filtering

- Provides for degradation function AND noise
- Objective is to minimize mean square error $e^2 = E\left\{ (f - \hat{f})^2 \right\}$
- Conditions:

(1) Noise and image are uncorrelated

(2) One or the other has zero mean

(3) Gray levels in $\hat{f}$ are linear function of gray levels in g

- Minimum of error function in frequency domain

$$\begin{aligned}
\hat{F}(u,v) &= \left\{ \frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_\eta(u,v)} \right\} G(u,v) \\
&= \left\{ \frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right\} G(u,v) \\
&= \left\{ \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right\} G(u,v)
\end{aligned}$$

- Term in brackets: Wiener filter / Minimum mean square error filter/ Least square error filter

- **No problem with zeros unless $H(u,v)$ and $S_\eta(u,v)$ are both zero**

  $H(u,v)$ **= degradation function**

  $H^*(u,v)$ **= complex conjugate of** $H(u,v)$

  $|H(u,v)|^2 = H^*(u,v)\ H(u,v)$

  $S_\eta(u,v) = |N(u,v)|^2$ **= power spectrum of the noise**

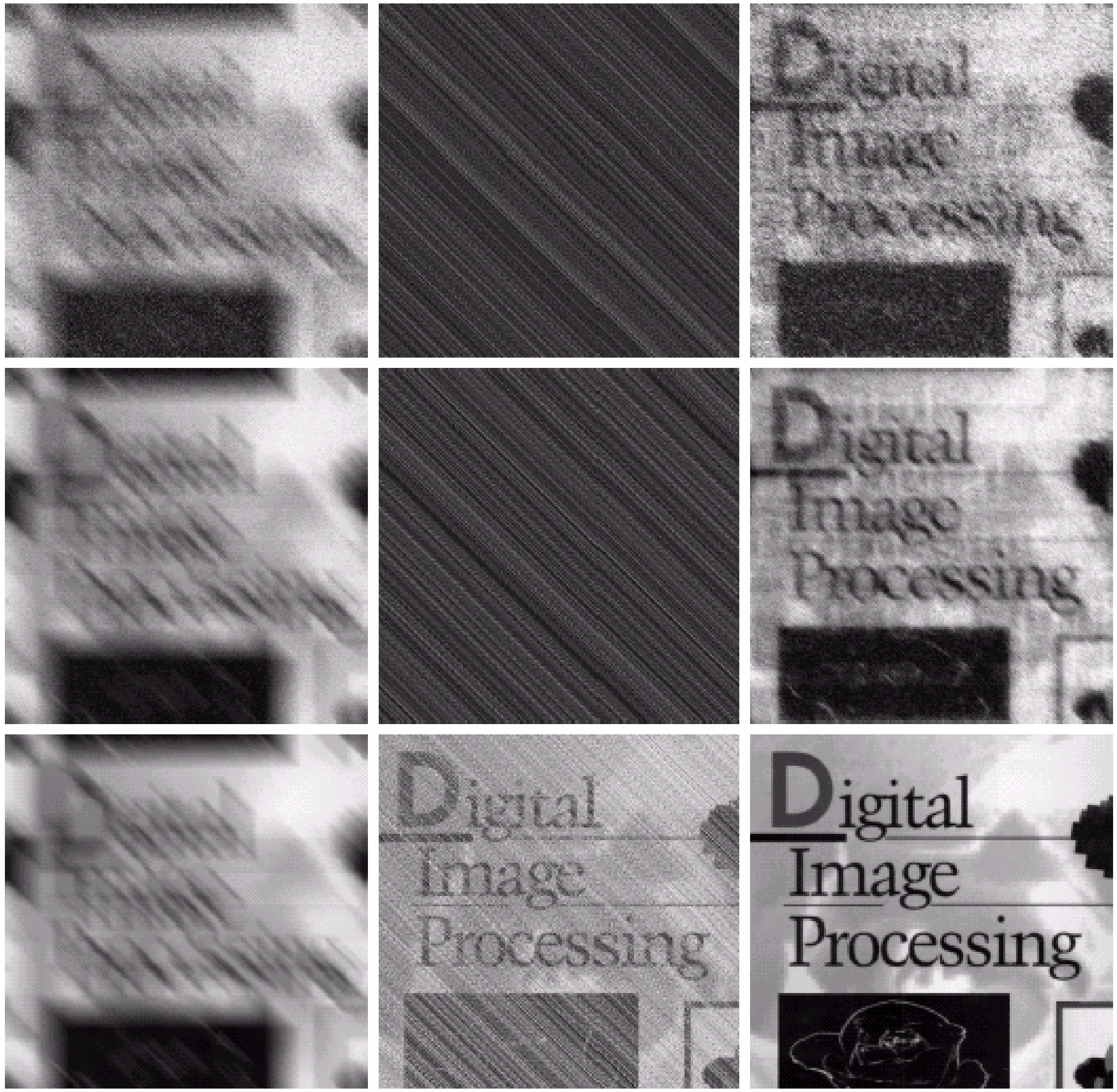
  $S_f(u,v) = |F(u,v)|^2$ **= power spectrum of undegraded image**

- **When noise is zero, Wiener filter $=$ inverse filter**
- **Since $S_\eta(u,v) = |N(u,v)|^2$ and $S_f(u,v) = |F(u,v)|^2$ are seldom known, the Wiener filter is frequently approximated by**

$$\hat{F}(u,v) = \left\{ \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right\} G(u,v), \quad K \textbf{ specified}$$

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

5.9 Constrained Least Square Filtering

- **Wiener filter:** $|N(u,v)|^2$ **and** $|F(u,v)|^2$ **must be known**
  Constant estimate of ratio not always suitable
- **This method requires knowledge of only the mean and variance of the noise**
- **This filter is optimal for each image, while the Wiener filter is optimal in an average sense**
- **Degradation process in matrix notation:**

$$\mathbf{g} = \mathbf{H}\,\mathbf{f} + \eta$$

$$\mathbf{g},\ \mathbf{f},\ \eta : MN \times 1$$

$$\mathbf{H} : MN \times MN$$

- $\mathbf{H}$ **is sensitive to noise** $\Rightarrow$ **Optimality of restoration based on measure of smoothness: Laplacian**
- **Find minimum of criterion function,** C**, defined as** $C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x,y)]^2$ **subject to the constraint**

$$\left|\mathbf{g} - \mathbf{H}\,\hat{\mathbf{f}}\right|^2 = |\eta|^2$$

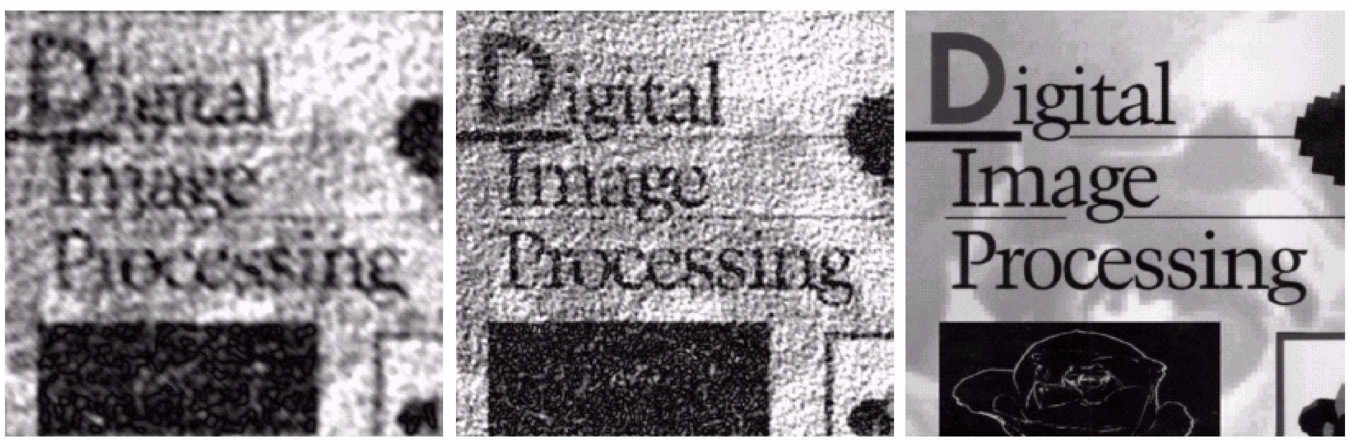
- **Frequency domain solution of optimization problem is**

$$\hat{F}(u,v) = \left\{ \frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right\} G(u,v),$$

where γ **has to be adjusted to satisfy constraint, and** $P(u,v)$ **is Fourier transform of padded version of**

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- **When** $\gamma = 0$**, constrained least squares filter** $=$ **inverse filter**



a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

5.10 Geometric mean filter

$$\hat{F}(u,v) = \left\{ \frac{H^*(u,v)}{|H(u,v)|^2} \right\}^{\alpha} \left\{ \frac{H^*(u,v)}{|H(u,v)|^2 + \beta \left\{ \dfrac{S_\eta(u,v)}{S_f(u,v)} \right\}} \right]^{1-\alpha} G(u,v)$$

$\alpha,\ \beta$ **positive and real**

$\alpha = 1:$	**Inverse filter**
$\alpha = 0:$	**Parametric Wiener filter**
$\alpha = 0$ **and** $\beta = 1$**:**	**Standard Wiener filter**
$\alpha = 1/2$**:**	**Geometric mean: reason for filter's name**
$\alpha < 1/2$ **and** $\beta = 1$**:**	**Performance tend toward Wiener filter**
$\alpha > 1/2$ **and** $\beta = 1$**:**	**Performance tend toward inverse filter**
$\alpha = 1/2$ **and** $\beta = 1$**:**	**Spectrum equalization filter**