11.3 Regional descriptors

- Common practice to combine boundary and regional descriptors

11.3.1 Some simple descriptors

- Area: Number of pixels in region
- Perimeter: Length of boundary
- Compactness: Perimeter^2/Area
- Mean and median gray levels
- Min and max gray level values
- Number of pixels with values above or below mean

11.3.2 Topological Descriptors

- Topology: Study of properties of a figure that are unaffected by any deformation
- Euler number: $E = C - H$
  - Number of connected components: C
  - Number of holes: H

FIGURE 11.22 Infrared images of the Americas at night. (Courtesy of NOAA.)

Region no. (from top)	Ratio of lights per region to total lights
1	0.204
2	0.640
3	0.049
4	0.107

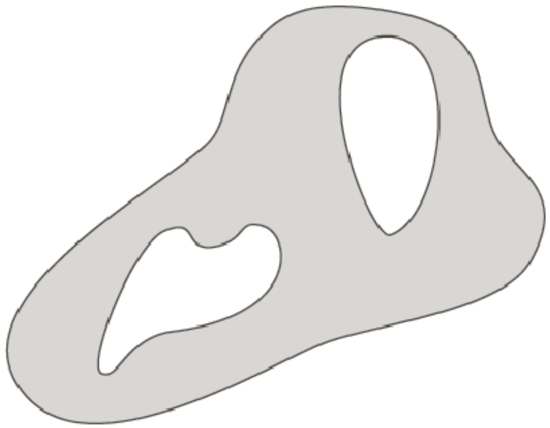


FIGURE 11.23 A region with two holes.

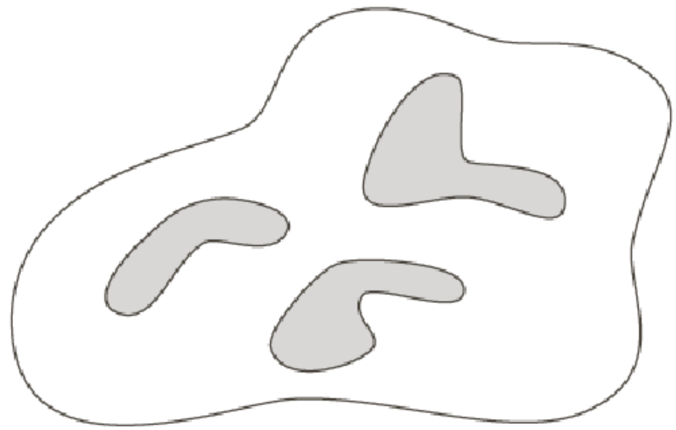
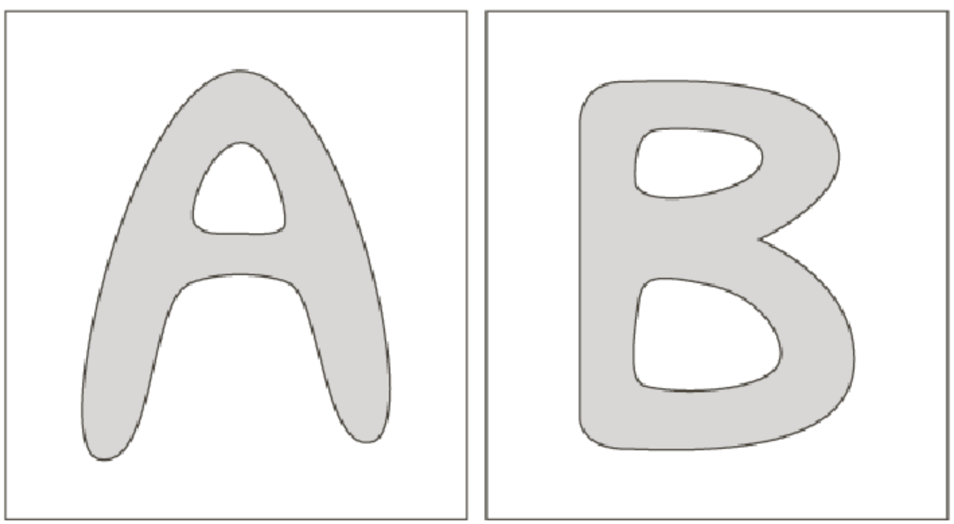


FIGURE 11.24 A region with three connected components.



a b

FIGURE 11.25 Regions with Euler numbers equal to 0 and −1, respectively.

- **Polygonal networks: Euler formula...**

$$\begin{aligned} V - Q + F &= C - H \\ &= E \end{aligned}$$

V: **Number of vertices**
Q: **Number of edges**
F: **Number of faces**

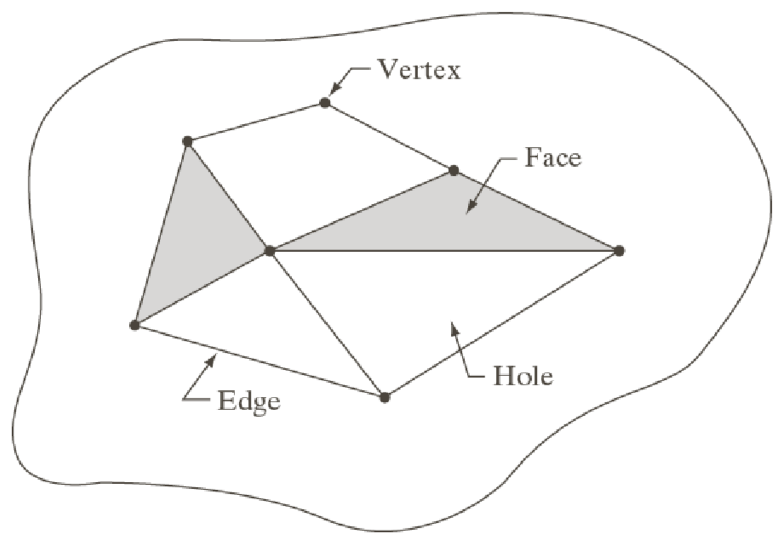
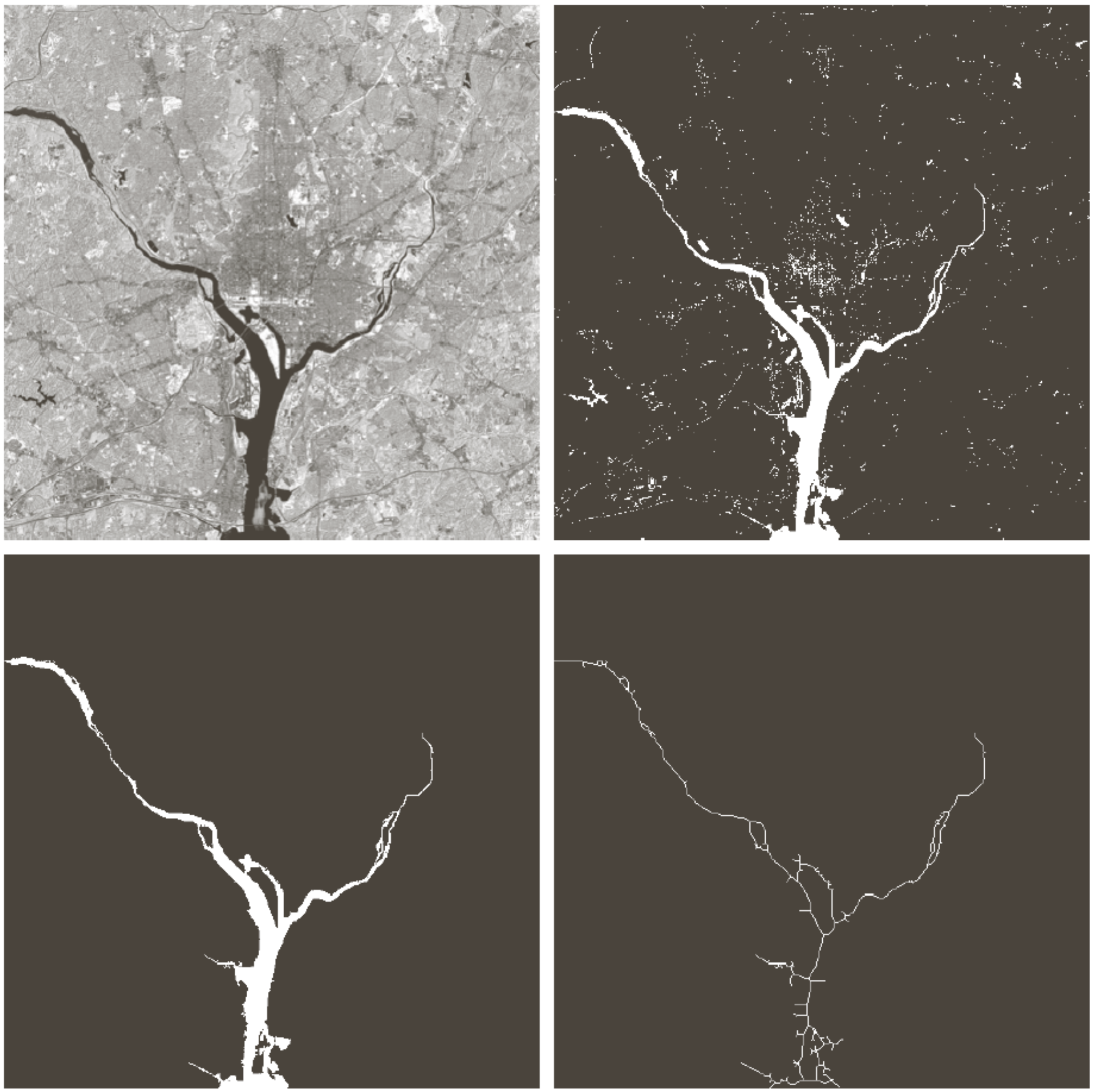


FIGURE 11.26 A region containing a polygonal network.

$$\begin{aligned} 7 - 11 + 2 &= 1 - 3 \\ &= -2 \end{aligned}$$

Example 11.9



a b
c d

FIGURE 11.27 (a) Infrared image of the Washington, D.C. area. (b) Thresholded image. (c) The largest connected component of (b). Skeleton of (c).

$$E = C - H$$

$$1552 = 1591 - 39$$

11.3.3 Texture

(1) Statistical approaches (2) Structural approaches (3) Spectral approaches

Statistical approaches

When $p(z_i),\ i = 0, \ldots, L-1$ represents a histogram of gray-levels, the nth moment of z about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n \, p(z_i)$$

where m is the mean value of z...

$$m = \sum_{i=0}^{L-1} z_i \, p(z_i)$$

- **Relative smoothness...**

$$R(z) = 1 - \frac{1}{1 + \sigma^2(z)}$$

- **The third moment...**

$$\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 \, p(z_i)$$

- **The fourth moment...**

$$\mu_4(z) = \sum_{i=0}^{L-1} (z_i - m)^4 \, p(z_i)$$

... measure of histogram's flatness

- **Measure of uniformity...**

$$U(z) = \sum_{i=0}^{L-1} p^2(z_i)$$

... is maximum for an image in which all grey levels are equal

- **Average entropy measure...**

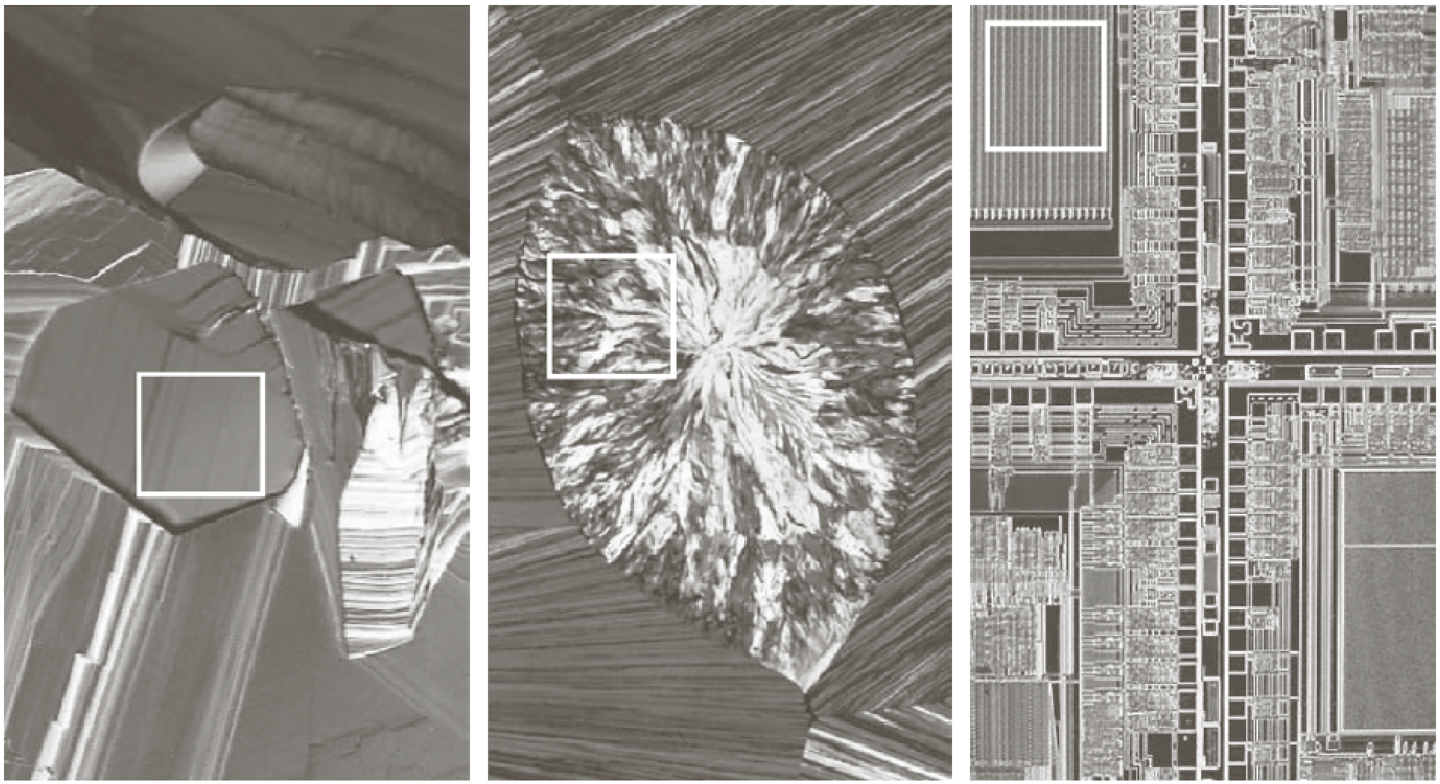
$$e(z) = -\sum_{i=0}^{L-1} p(z_i) \, \log_2 p(z_i)$$

... measure of variability and is 0 for a constant image

Measures of texture computed using only histograms suffer from the limitation that they carry no information regarding the relative position of pixels with respect to each other

- **Gray-level co-occurrence matrices: READ**

Example 11.10: Texture measures based on histograms



a b c

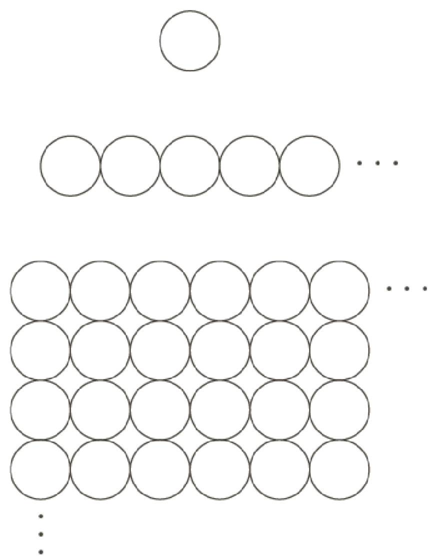
FIGURE 11.28 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

TABLE 11.2 Texture measures for the subimages shown in Fig. 11.28.

Structural approaches

A simple "texture primitive" can be used to form more complex texture patterns by means of some rules that limit the number of possible arrangements of the primitive(s)



a
b
c

FIGURE 11.34 (a) Texture primitive. (b) Pattern generated by the rule $S \rightarrow aS$. (c) 2-D texture pattern generated by this and other rules.

Spectral approaches

- **Three features of Fourier spectrum that is useful for texture description...**

(1) **Prominent peaks $\rightarrow$ principal direction of texture patterns**

(2) **Location of peaks $\rightarrow$ fundamental spatial period**

(3) **Elimination of periodic components $\rightarrow$ non-periodic image elements $\rightarrow$ statistical descriptors**

- Spectrum is symmetric about origin ⇒ only half of frequency plane needs to be considered ⇒ every periodic pattern associated with only one peak
- Consider spectrum in polar coordinates $S(r, \theta)$
  - For each direction θ, consider 1-dimensional $S_\theta(r)$
  - For each frequency r, consider 1-dimensional $S_r(\theta)$
- More global description obtained by summation...

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r)$$

$$S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$

... where R_0 is the radius of a circle centered at the origin

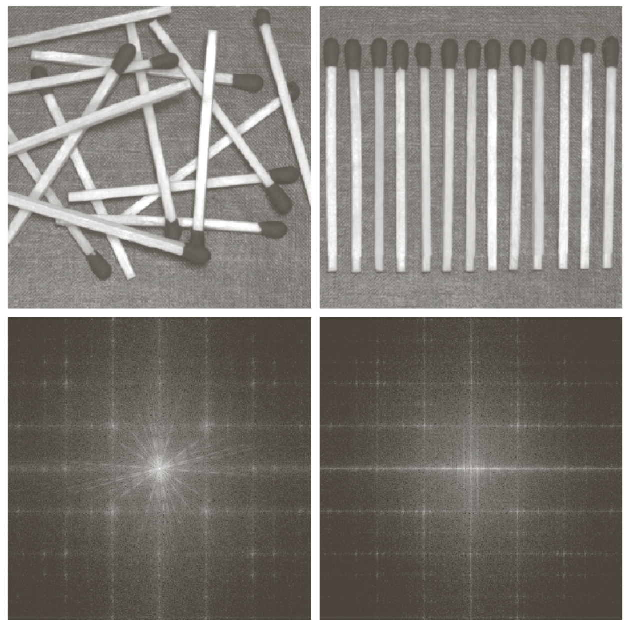
Descriptors of these functions themselves can be computed in order to characterize their behavior quantitatively, for example...

(1) location of the highest value

(2) mean and variance

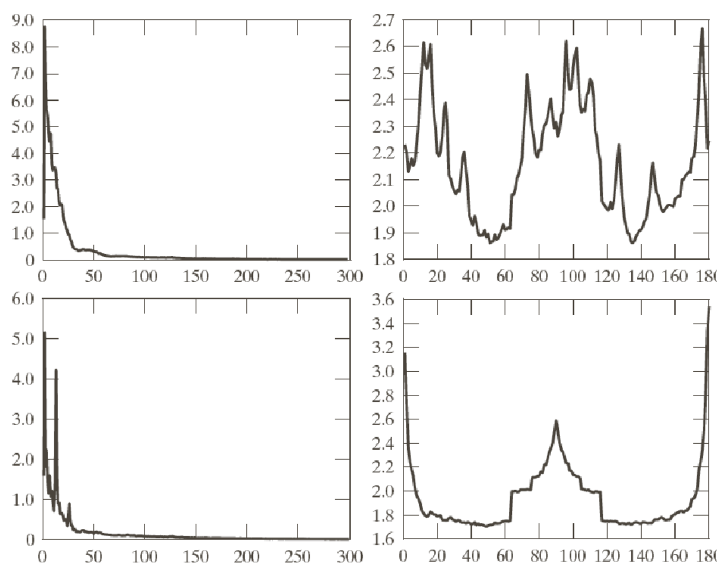
(3) distance between mean and highest value

Example 11.12: Spectral texture



a b
c d

FIGURE 11.35 (a) and (b) Images of random and ordered objects. (c) and (d) Corresponding Fourier spectra. All images are of size 600 × 600 pixels.

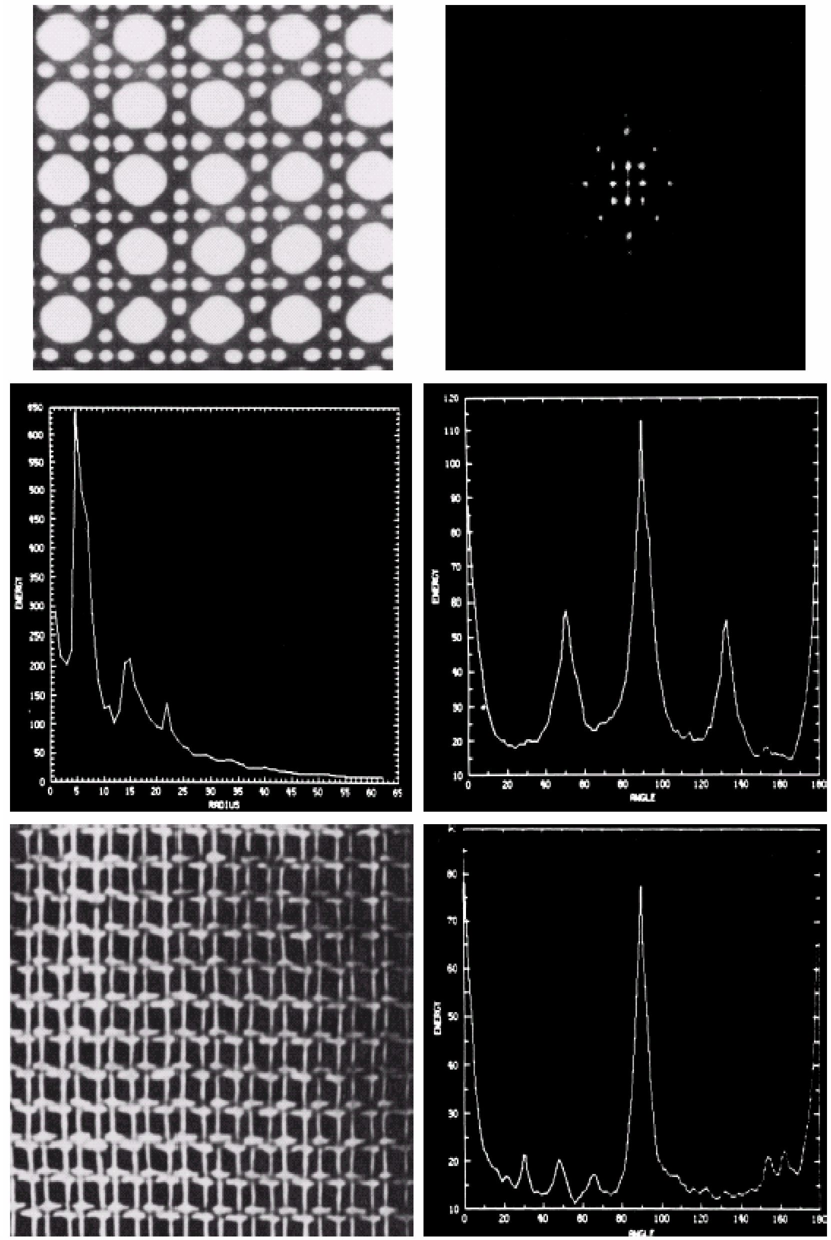


a b
c d

FIGURE 11.36 Plots of (a) $S(r)$ and (b) $S(\theta)$ for Fig. 11.35(a). (c) and (d) are plots of $S(r)$ and $S(\theta)$ for Fig. 11.35(b). All vertical axes are $\times 10^5$.

Example in 2nd edition:

Spectral texture



a b
c d
e f

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)