10.3.6 Multiple thresholds

Otsu's method extended to K classes, $C_1, C_2, \ldots, C_K$

Between-class variance:

$$\sigma_B^2 = \sum_{k=1}^{K} P_k (m_k - m_G)^2, \quad \textbf{where} \quad P_k = \sum_{i \in C_k} p_i \quad \textbf{and} \quad m_k = \sum_{i \in C_k} i p_i$$

The K classes are separated by $K-1$ thresholds whose values $k_1^*, k_2^*, \ldots k_{K-1}^*$ maximize

$$\sigma_B^2(k_1^*, k_2^*, \ldots, k_{K-1}^*) = \max_{0<k_1<k_2<\ldots<k_{K-1}<L-1} \sigma_B^2(k_1, k_2, \ldots, k_{K-1})$$

Otsu's method extended to three classes, C_1, C_2, C_3

Between-class variance:

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 + P_3(m_3 - m_G)^2$$

$$P_1 = \sum_{i=0}^{k_1} p_i, \quad P_2 = \sum_{i=k_1+1}^{k_2} p_i, \quad P_3 = \sum_{i=k_2+1}^{L-1} p_i$$

$$m_1 = \frac{1}{P_1}\sum_{i=0}^{k_1} ip_i, \quad m_2 = \frac{1}{P_2}\sum_{i=k_1+1}^{k_2} ip_i, \quad m_3 = \frac{1}{P_3}\sum_{i=k_2+1}^{L-1} ip_i$$

The following relationships also hold:

$$P_1 m_1 + P_2 m_2 + P_3 m_3 = m_G, \qquad P_1 + P_2 + P_3 = 1$$

The three classes are separated by two thresholds whose values k_1^* and k_2^* maximize

$$\sigma_B^2(k_1^*, k_2^*) = \max_{0<k_1<k_2<L-1} \sigma_B^2(k_1, k_2)$$

Algorithm

(1) Let $k_1 = 1$

(2) Increment k_2 through all its values greater than k_1 and less than $L-1$

(3) Increment k_1 to its next value and increment k_2 through all its values greater than k_1 and less than $L-1$

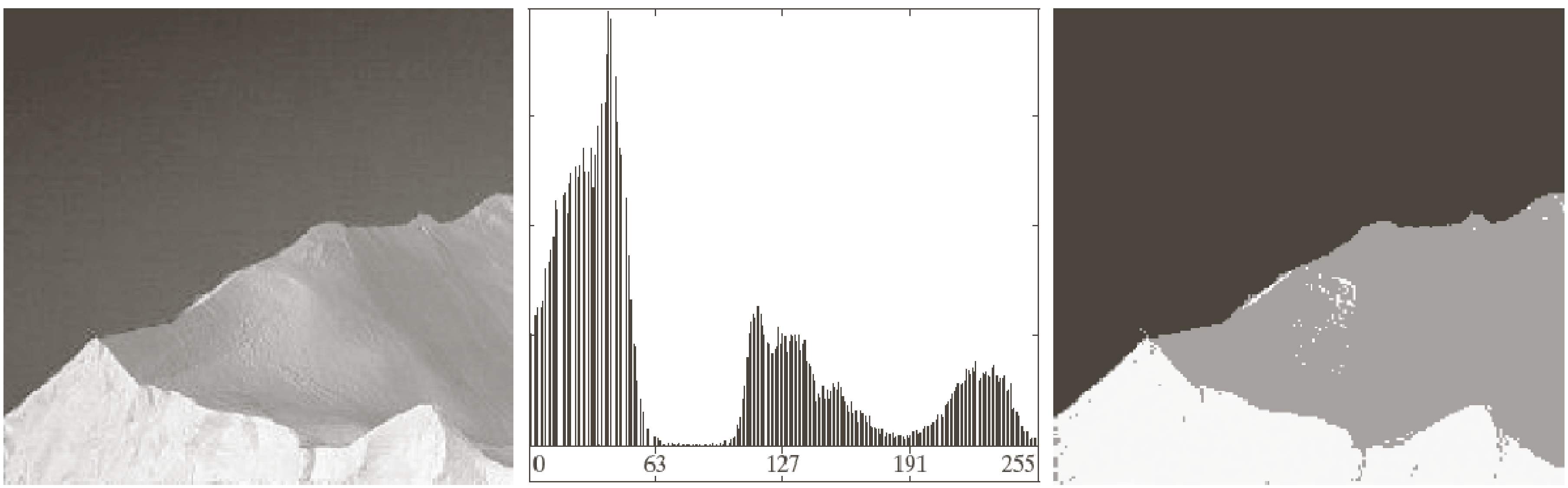
(4) Repeat until $k_1 = L-3$

This results in a 2-D array $\sigma_B^2(k_1, k_2)$, after which k_1^* and k_2^* that correspond to the maximum value in the array, are selected

Segmentation is as follows: $g(x,y) = \begin{cases} a, & \textbf{if } f(x,y) \le k_1^* \\ b, & \textbf{if } k_1^* < f(x,y) \le k_2^* \\ c, & \textbf{if } f(x,y) > k_2^* \end{cases}$

Separability measure: $\eta(k_1^*, k_2^*) = \dfrac{\sigma_B^2(k_1^*, k_2^*)}{\sigma_G^2}$

Example 10.19: Multiple global thresholding

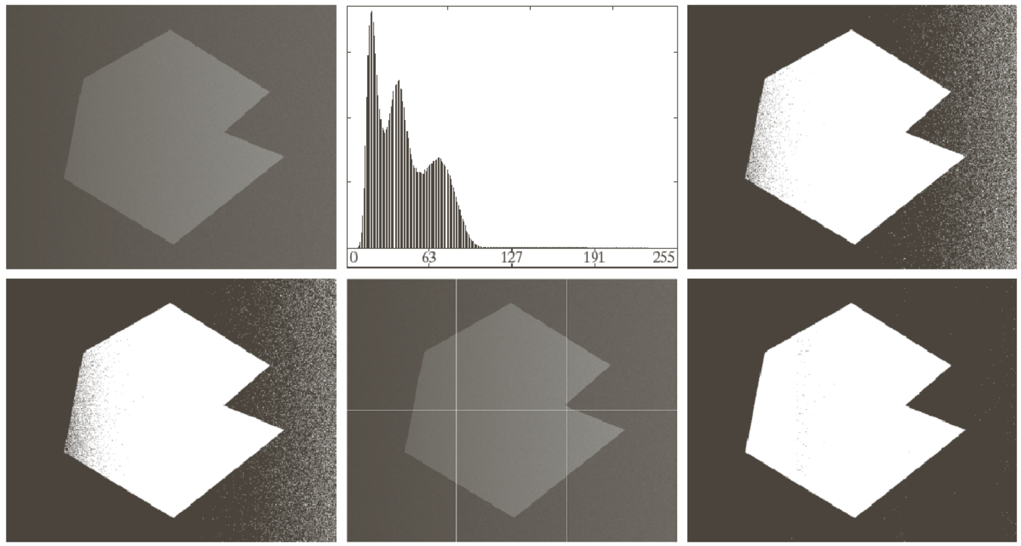


a b c

FIGURE 10.45 (a) Image of iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

10.3.7 Variable thresholding

- **Image partitioning**



a b c
d e f

FIGURE 10.46 (a) Noisy, shaded image and (b) its histogram. (c) Segmentation of (a) using the iterative global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (e) Image subdivided into six subimages. (f) Result of applying Otsu's method to each subimage individually.

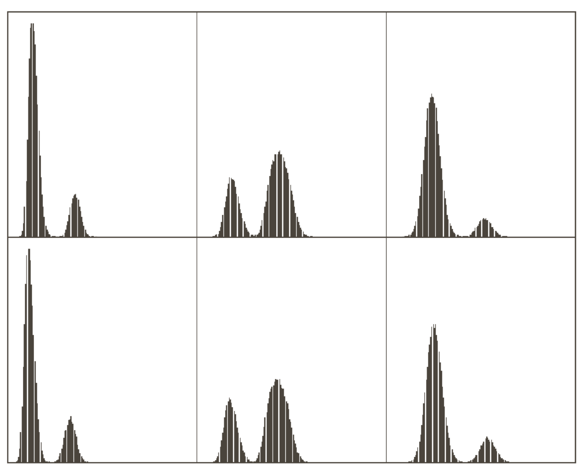


FIGURE 10.47 Histograms of the six subimages in Fig. 10.46(e).

10.3.7 Variable thresholding

• **Local image properties**

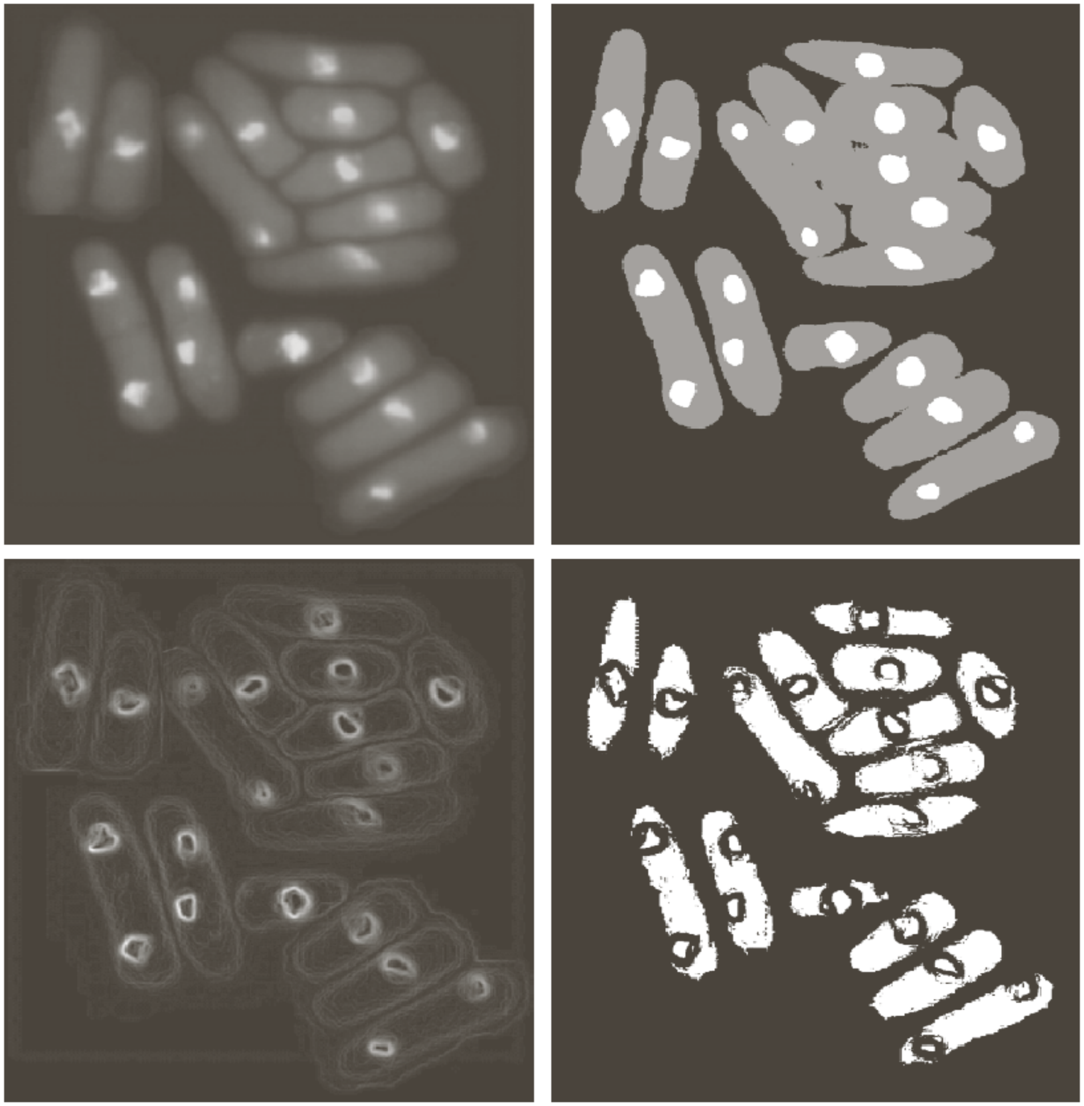
Examples: (1) $T_{xy} = a\,\sigma_{xy} + b\,m_{xy}$ **(2)** $T_{xy} = a\,\sigma_{xy} + b\,m_G$

$$g(x,y) = \begin{cases} 1, & \textbf{if } f(x,y) > T_{xy} \\ 0, & \textbf{if } f(x,y) \leq T_{xy} \end{cases}$$

$$g(x,y) = \begin{cases} 1, & \textbf{if } Q\textbf{(local parameters) is true} \\ 0, & \textbf{if } Q\textbf{(local parameters) is false} \end{cases}$$

$$Q(\sigma_{xy}, m_{xy}) = \begin{cases} \textbf{true,} & \textbf{if } f(x,y) > a\,\sigma_{xy} \textbf{ AND } f(x,y) > b\,m_{xy} \\ \textbf{false,} & \textbf{otherwise} \end{cases}$$

Example 10.21: Variable thresholding based on local image properties



a b
c d

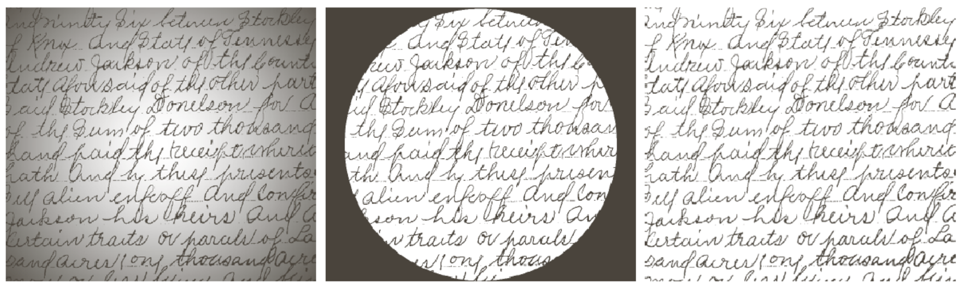
FIGURE 10.48 (a) Image from Fig. 10.43. (b) Image segmented using the dual thresholding approach discussed in Section 10.3.6. (c) Image of local standard deviations. (d) Result obtained using local thresholding.

10.3.7 Variable thresholding

• **Moving averages**

$$m(k+1) = \frac{1}{n}\sum_{i=k+2-n}^{k+1} z_i = m(k) + \frac{1}{n}(z_{k+1} - z_{k-n})$$

$$g(x,y) = \begin{cases} 1, & \textbf{if } f(x,y) > T_{xy} \\ 0, & \textbf{if } f(x,y) \le T_{xy} \end{cases}, \qquad T_{xy} = bm_{xy}$$



a b c

FIGURE 10.49 (a) Text image corrupted by spot shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages.

(10.3.8 Multi-variable thresholding: See 6.7.2)

a b c

FIGURE 10.50 (a) Text image corrupted by sinusoidal shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages.

10.4 Region-based segmentation

10.4.1 Region-growing

- Start from a set of seed points and from these points grow the regions by appending to each seed those neighbouring pixels that have similar properties
- The selection of the seed points depends on the problem. When a priory information is not available, clustering techniques can be used: compute the above mentioned properties at every pixel and use the centroids of clusters
- The selection of similarity criteria depends on the problem under consideration and the type of image data that is available
- Descriptors must be used in conjunction with connectivity (adjacency) information
- Formulation of a "stopping rule". Growing a region should stop when no more pixels satisfy the criteria for inclusion in that region.
- When a model of the expected results is partially available, the consideration of additional criteria like the size of the region, the likeliness between a candidate pixel and the pixels grown so far, and the shape of the region can improve the performance of the algorithm.

Algorithm (based on 8-connectivity):

- $f(x,y) \equiv$ **input image array;**
- $S(x,y) \equiv$ **seed array (1s at locations of seed points and 0s elsewhere);**
- $Q \equiv$ **predicate to be applied at each location (x,y)**

(1) **Find all connected components in $S(x,y)$ and erode each connected component to one pixel; label all such pixels found as 1. All other pixels in S are labelled 0.**

(2) **Form and image f_Q such that, at a pair of coordinates (x,y), let $f_Q(x,y) = 1$ if the input image satisfies the given predicate, Q, at those coordinates; otherwise let $f_Q(x,y) = 0$.**

(3) **Let g be an image formed by appending to each seed point in S all the 1-valued points in f_Q that are 8-connected to that seed point.**

(4) **Label each connected component in g with a different region label (e.g. 1, 2, 3, $\ldots$). This constitutes the segmented image obtained by region growing.**

Example 10.23: Segmentation by region growing

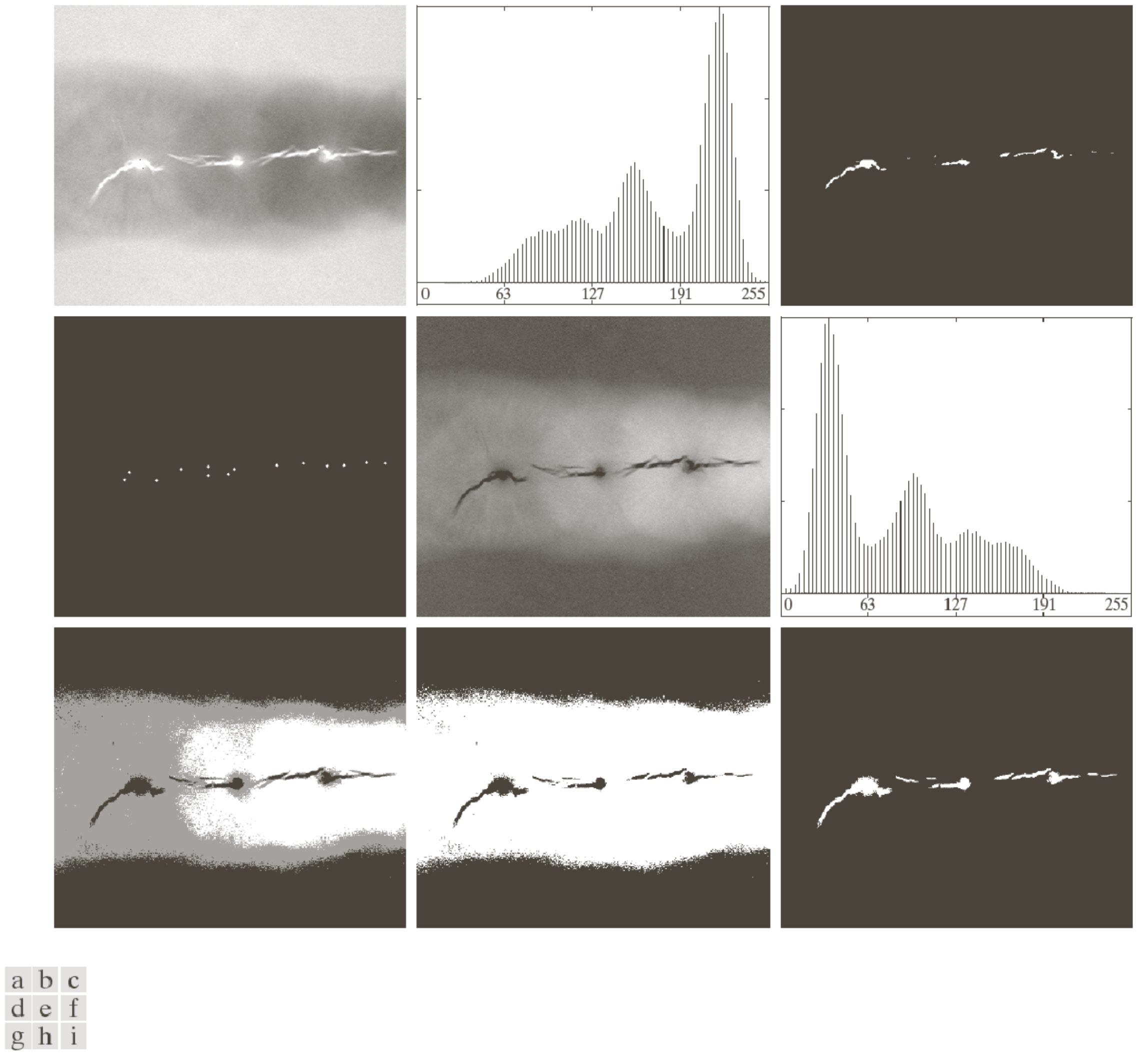


FIGURE 10.51 (a) X-ray image of a defective weld. (b) Histogram. (c) Initial seed image. (d) Final seed image (the points were enlarged for clarity). (e) Absolute value of the difference between (a) and (c). (f) Histogram of (e). (g) Difference image thresholded using dual thresholds. (h) Difference image thresholded with the smallest of the dual thresholds. (i) Segmentation result obtained by region growing. (Original image courtesy of X-TEK Systems, Ltd.)

10.4.2 Region splitting and merging

Subdivide an image initially into a set of arbitrary, disjoint regions and then merge and/or split the regions in an attempt to satisfy the necessary conditions

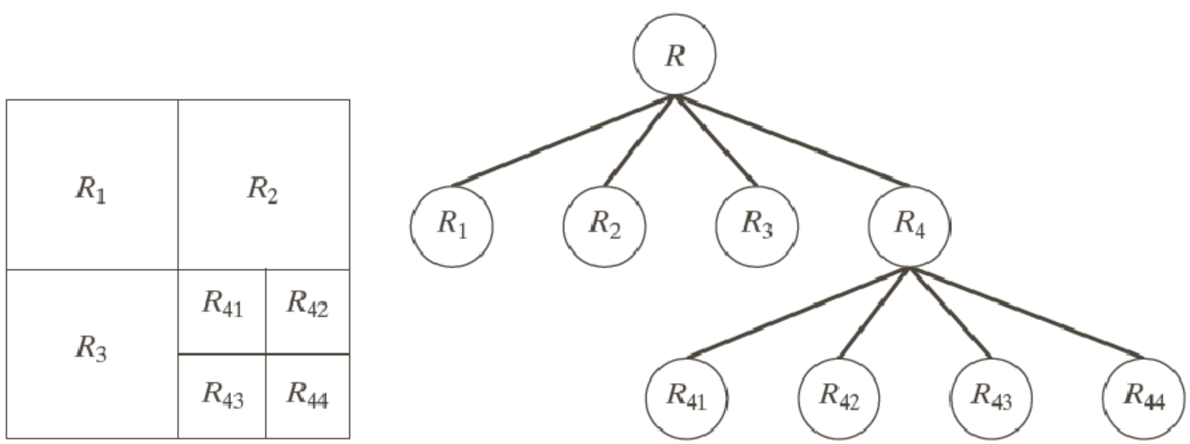
Let R represent entire image region and select a predicate Q

(1) **Split into four disjoint quadrants any region R_i for which $Q(R_i) =$ FALSE**

(2) **Merge any adjacent regions R_j and R_k for which $Q(R_j \cup R_k) =$ TRUE**

(3) **Stop when no further merging or splitting is possible**

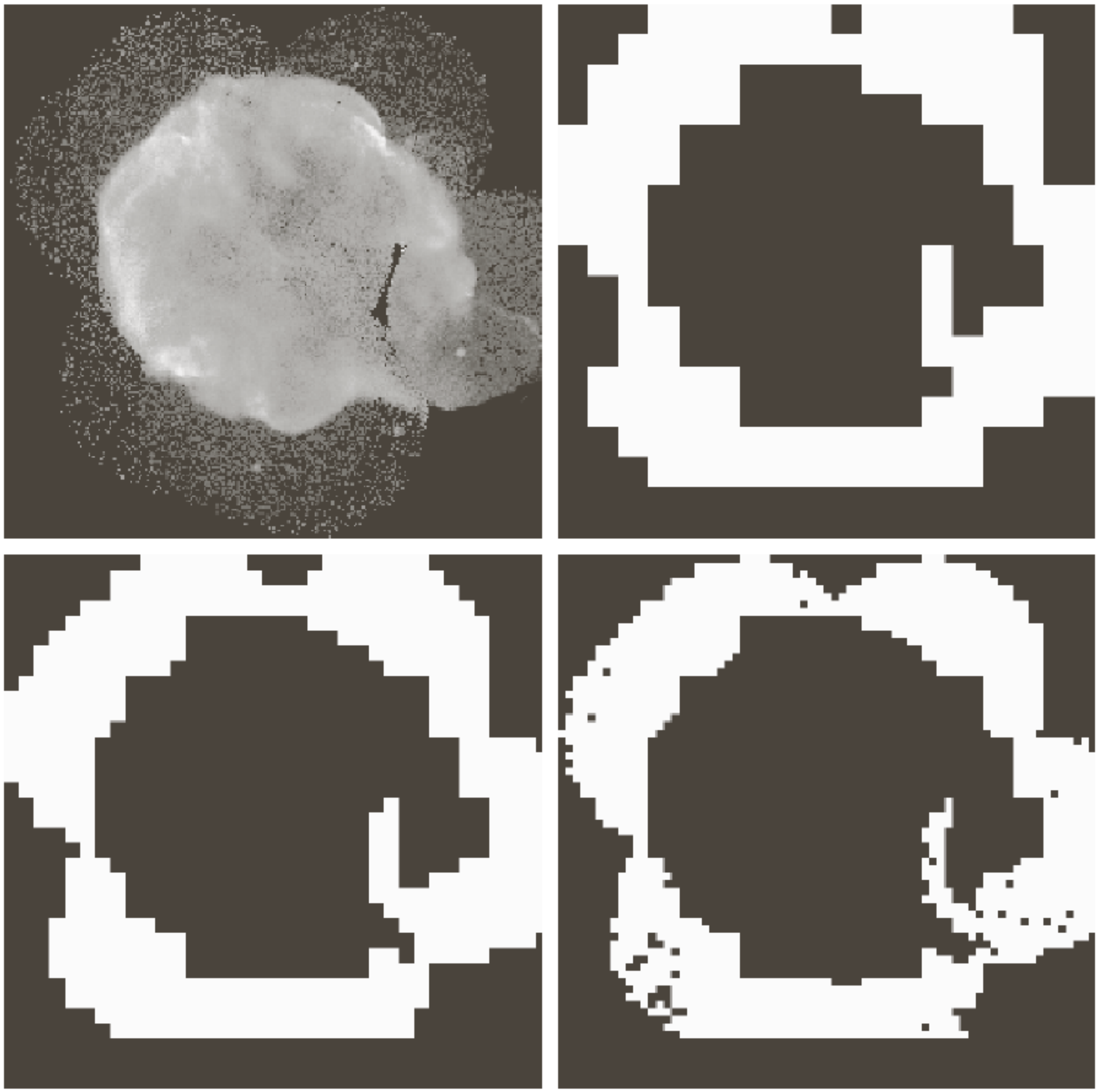
Several variations of this theme are possible



a b

FIGURE 10.52 (a) Partitioned image. (b) Corresponding quadtree. R represents the entire image region.

Example 10.24: Segmentation by region splitting and merging



a b
c d

FIGURE 10.53 (a) Image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of limiting the smallest allowed quadregion to sizes of 32×32, 16×16, and 8×8 pixels, respectively. (Original image courtesy of NASA.)