10.3 Thresholding

10.3.1 Foundation

A thresholded image $g(x, y)$ is defined as

$$g(x,y) = \begin{cases} 1, & \textbf{if } f(x,y) > T \\ 0, & \textbf{if } f(x,y) \leq T \end{cases},$$

where 1 is object and 0 is background

Global thresholding: T is constant applicable over whole image

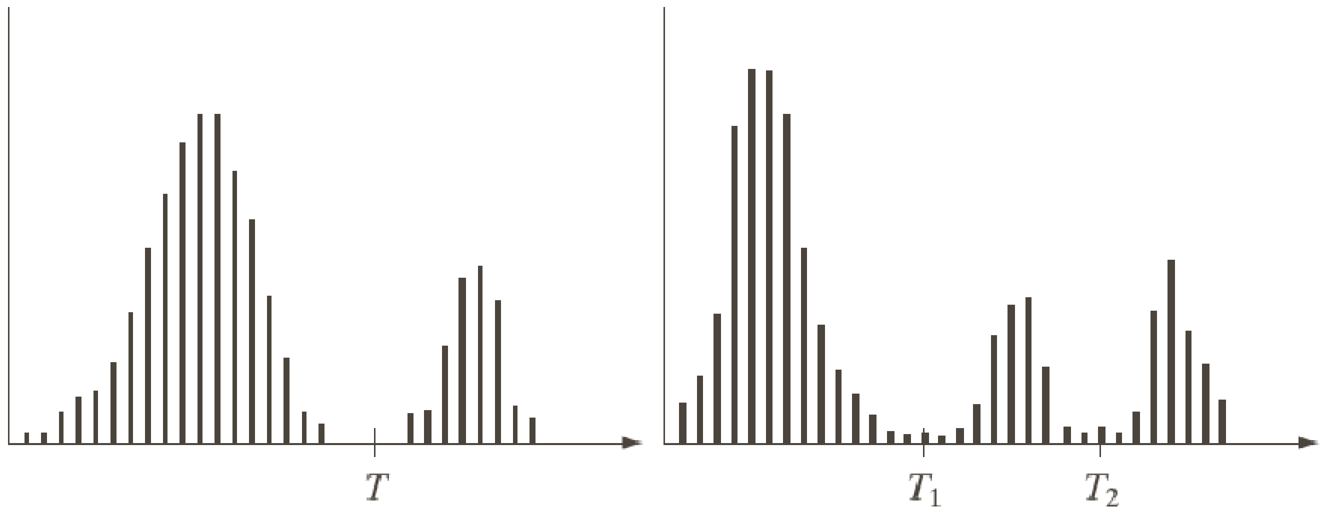
Variable (local/regional) thresholding: T changes over an image

Dynamic (adaptive) thresholding: T depends on spatial coordinates (x, y)

Multiple thresholding:

$$g(x,y) = \begin{cases} a, & \textbf{if } f(x,y) > T_2 \\ b, & \textbf{if } T_1 < f(x,y) \leq T_2 \\ c, & \textbf{if } f(x,y) \leq T_1 \end{cases},$$

Segmentation requiring more than two thresholds is very difficult and variable thresholding (10.3.7) or region growing (10.4) is often preferred



a b

FIGURE 10.35 Intensity histograms that can be partitioned (a) by a single threshold, and (b) by dual thresholds.

Width and depth of valleys (in histogram) affect success of thresholding

Properties of valleys are affected by:

(1) **separation between peaks;**

(2) **noise content;**

(3) **relative sizes of objects and background;**

(4) **uniformity of illumination source;**

(5) **uniformity of reflectance properties of image**

The role of noise in image thresholding

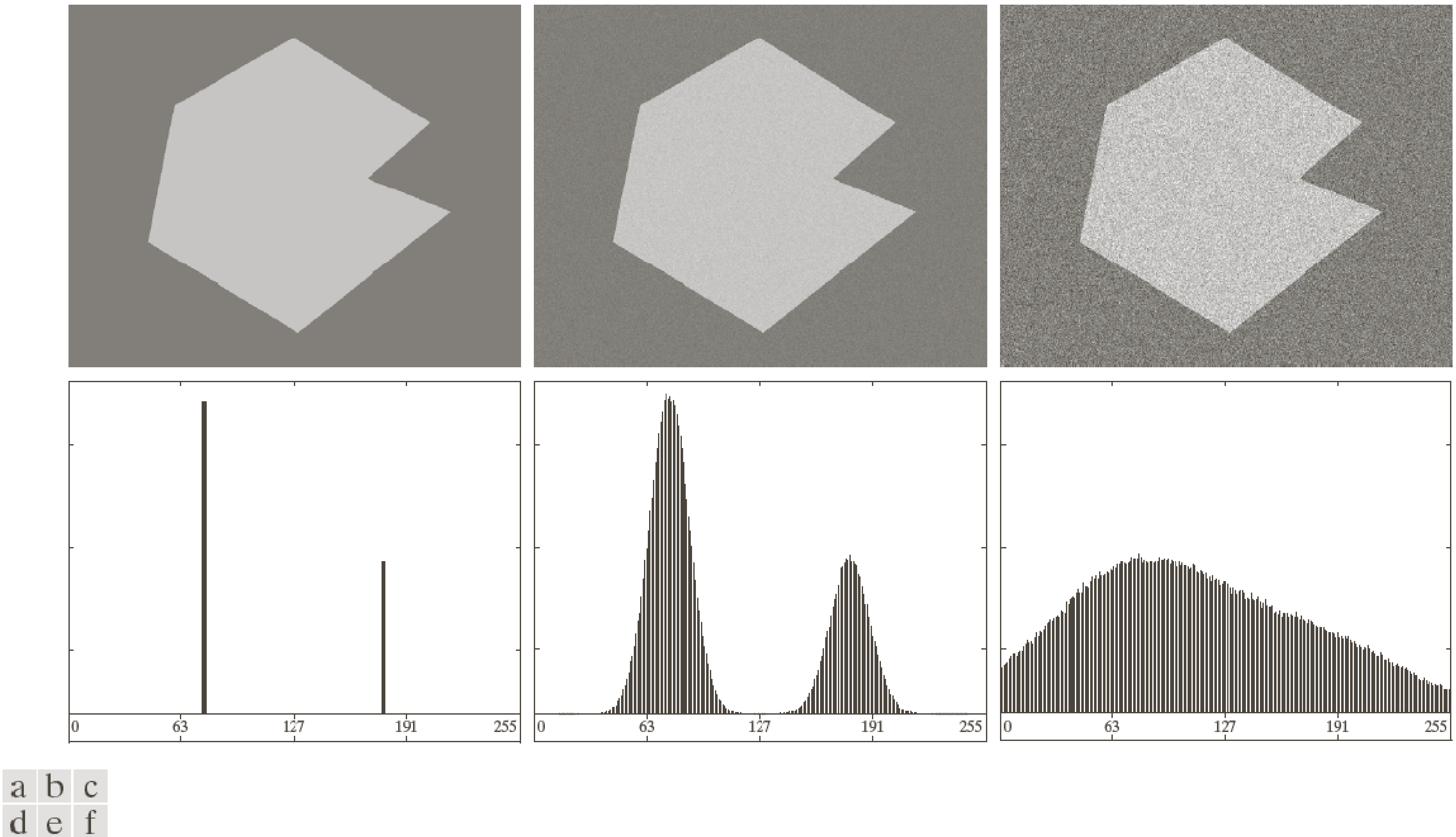


FIGURE 10.36 (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.

The role of illumination and reflectance

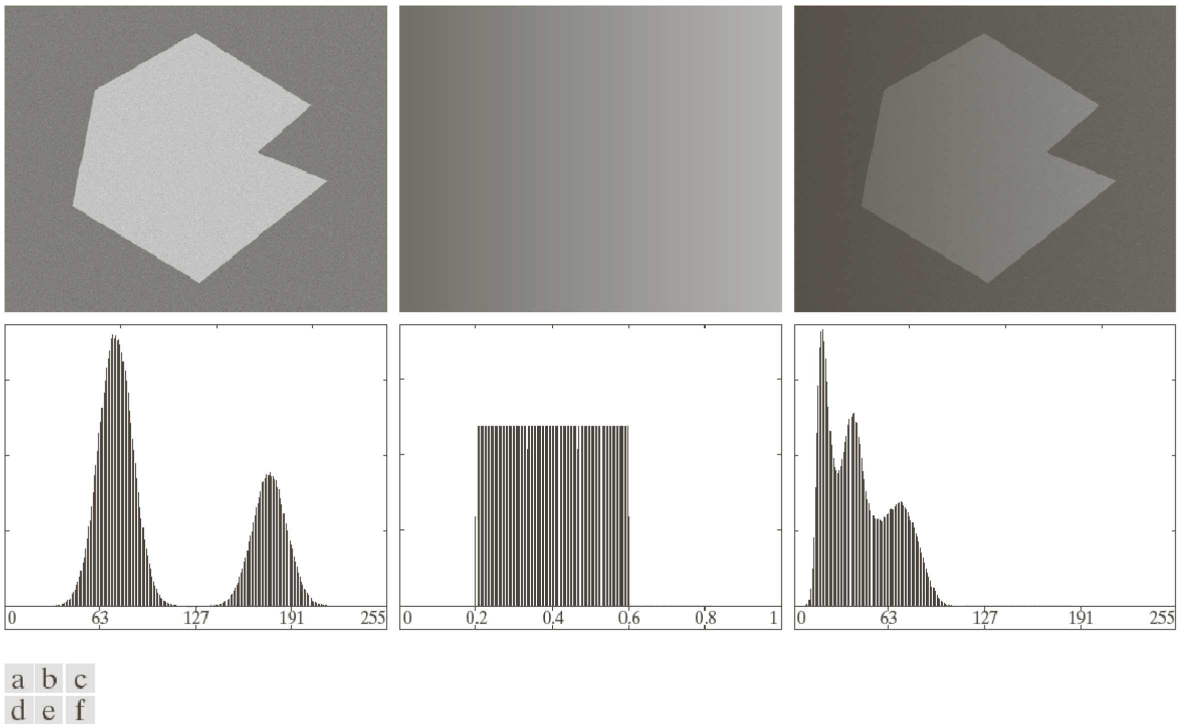


FIGURE 10.37 (a) Noisy image. (b) Intensity ramp in the range [0.2, 0.6]. (c) Product of (a) and (b). (d)–(f) Corresponding histograms.

Options for correcting non-uniform illumination: (1) Multiply with inverse of pattern by imaging flat surface with constant intensity; (2) Processing using top-hat transformation (Sec 9.6.3); (3) Variable thresholding (Sec 10.3.7)

10.3.2 Basic global thresholding

Iterative algorithm for automatic estimation of threshold T:

(1) **Select an initial estimate for T**

(2) **Segment image using $T \longrightarrow$** **Group G_1 (values $> T$)**
Group G_2 (values $\leq T$)

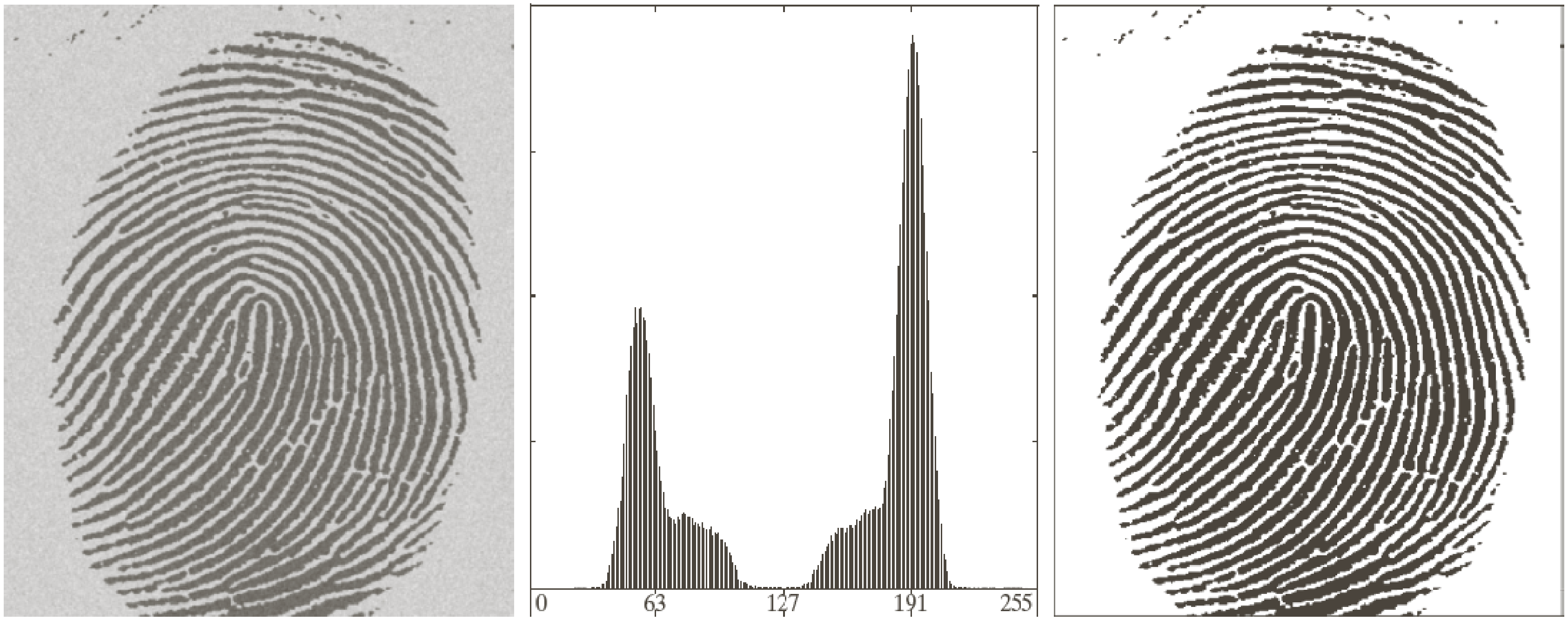
(3) **Compute average intensity values for G_1, $G_2 \longrightarrow m_1$, m_2**

(4) **Compute a new threshold value $T = \dfrac{1}{2}(m_1 + m_2)$**

(5) **Repeat (2) through (4) until the difference in T in successive iterations is smaller than ΔT**

Average intensity is good initial estimate for T

Example 10.15: Global thresholding



a b c

FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

Start with average gray level and $\Delta T = 0$

Algorithm results in $\tilde{T} = 125.4$ **after 3 iterations, so let** $T = 125$

10.3.3 Optimal global thresholding using Otsu's method

- **Otsu's method (1979) maximizes between-class variance**
- **Based entirely on computations performed on histogram (1-D) of image**
- **Normalized histogram:** $p_i = \dfrac{n_i}{MN},\ i = 0, \ldots, L-1$, **with** $\displaystyle\sum_{i=0}^{L-1} p_i = 1,\ p_i \geq 0$
- **Select threshold** $T(k)$ **to segment image** $\longrightarrow$ **Class** C_1 **(values** $[0, k]$**)**
  Class C_2 **(values** $[k+1, L-1]$**)**

$\Rightarrow$ **Prob of pixel assigned to** C_1 **(ie of** C_1 **occuring):** $P_1(k) = \displaystyle\sum_{i=0}^{k} p_i$

$\Rightarrow$ **Prob of pixel assigned to** C_2 **(ie of** C_2 **occuring):** $P_2(k) = \displaystyle\sum_{i=k+1}^{L-1} p_i = 1 - P_1(k)$

⇒ **Mean value of pixels assigned to C_1:**

$$\begin{aligned} m_1(k) &= \sum_{i=0}^{k} i\, P(i/C_1) \\ &= \sum_{i=0}^{k} i \; \overbrace{P(C_1/i)}^{=1} \; \underbrace{P(i)}_{=p_i} \;/\; \overbrace{P(C_1)}^{=P_1(k)} \quad \textbf{(Bayes' formula)} \\ &= \frac{1}{P_1(k)} \sum_{i=0}^{k} i\, p_i \end{aligned}$$

⇒ **Mean value of pixels assigned to C_2:**

$$\begin{aligned} m_2(k) &= \sum_{i=k+1}^{L-1} i\, P(i/C_2) \\ &= \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} i\, p_i \end{aligned}$$

$\Rightarrow$ **Mean intensity up to level k:** $m(k) = \sum_{i=0}^{k} i\, p_i$

- **Global mean:** $m_G = \sum_{i=0}^{L-1} i\, p_i$

$\Rightarrow$ $\boxed{P_1 m_1 + P_2 m_2 = m_G}$ **and** $\boxed{P_1 + P_2 = 1}$ **(ks temporarily omitted)**

- **"Goodness" of threshold at level k evaluated by dimensionless metric:**

$$\boxed{\eta = \frac{\sigma_B^2}{\sigma_G^2}}$$

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2\, p_i \quad \textbf{(Global variance)}$$

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 \quad \textbf{(Between-class variance)}$$

Also: $\sigma_B^2 = P_1 P_2 (m_1 - m_2)^2 = \dfrac{(m_G P_1 - m)^2}{P_1(1 - P_1)}$ $\leftarrow$ **most efficient**

Reintroduce $k \rightsquigarrow$ final results:

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

Optimum threshold is k^* that maximizes $\sigma_B^2(k)$:

$$\sigma_B^2(k^*) = \max_{k \in [0, L-1]} \sigma_B^2(k)$$

Segmentation is as follows:

$$g(x, y) = \begin{cases} 1, & \textbf{if } f(x,y) > k^* \\ 0, & \textbf{if } f(x,y) \leq k^* \end{cases},$$

The metric $\eta(k^*)$ can be used to obtain a quantative estimate of the separability of the classes and has values in the range:

$$\eta(k^*) \in [0, 1]$$

Summary of Otsu's algorithm

(1) **Compute normalized histogram of the image,** $p_i = \dfrac{n_i}{MN},\ i = 0, \ldots, L-1$

(2) **Compute cumulative sums,** $P_1(k) = \displaystyle\sum_{i=0}^{k} p_i,\ k = 0, \ldots, L-1$

(3) **Compute cumulative means,** $m(k) = \displaystyle\sum_{i=0}^{k} i\, p_i,\ k = 0, \ldots, L-1$

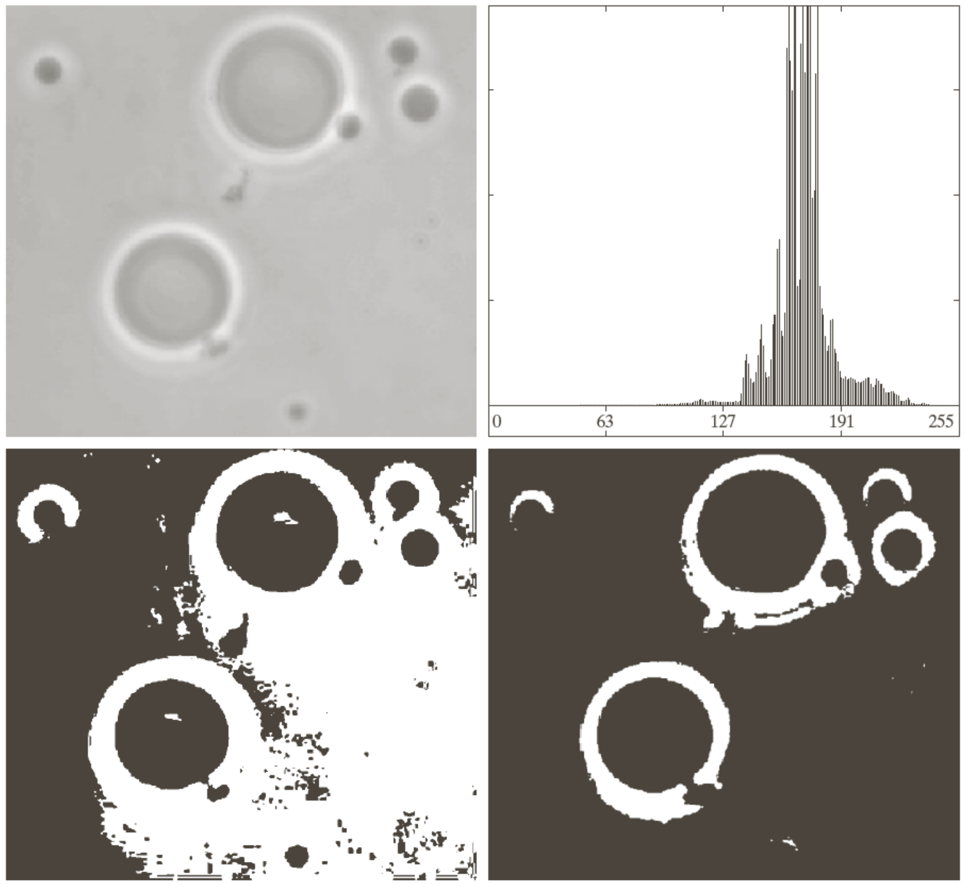
(4) **Compute global intensity mean,** $m_G = \displaystyle\sum_{i=0}^{L-1} i\, p_i$

(5) **Compute between-class variance,** $\sigma_B^2(k) = \dfrac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]},\ k = 0, ., L-1$

(6) **Obtain the Otsu threshold,** k^***, that is the value of** k **for which** $\sigma_B^2(k^*)$ **is a maximum** $-$ **if this maximum is not unique, obtain** k^* **by avaraging the values of** k **that correspond to the various maxima detected**

(7) **Obtain the separability measure** $\eta(k^*) = \dfrac{\sigma_B^2(k^*)}{\sigma_G^2}$

Example 10.16: Optimal global thresholding using Otsu's method



a b
c d

FIGURE 10.39 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

For the above image... **Threshold found by basic algorithm:** $T = 161$;

Threshold found by Otsu's algorithm: $T = 181$ **(Sep measure:** $\eta = 0.467$**)**

For fingerprint image... **basic <u>and</u> Otsu's algorithm:** $T = 125$ **(**$\eta = 0.944$**)**

10.3.4 Using image smoothing to improve global thresholding

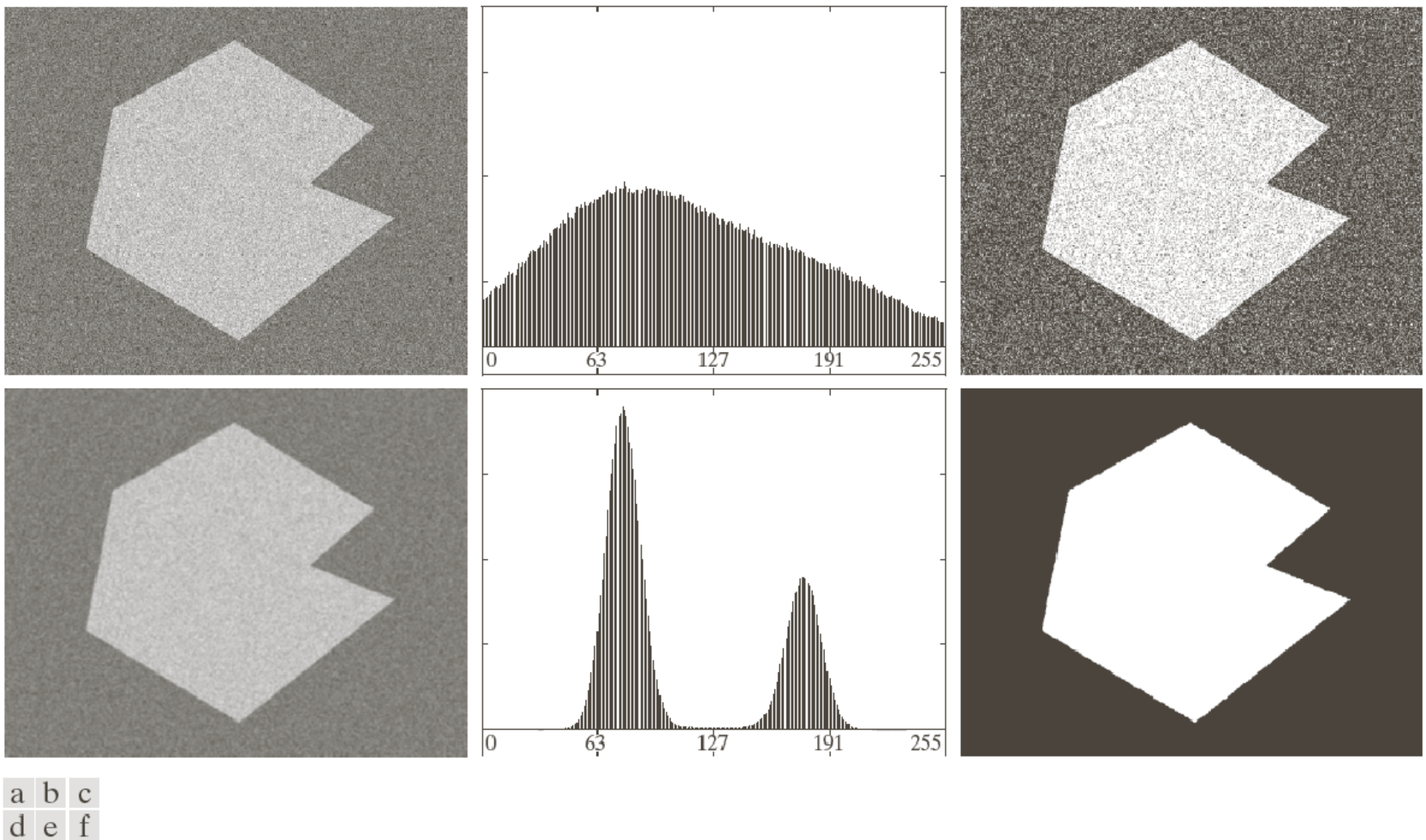


FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

Small object $\Rightarrow$ thresholding fails even after smoothing

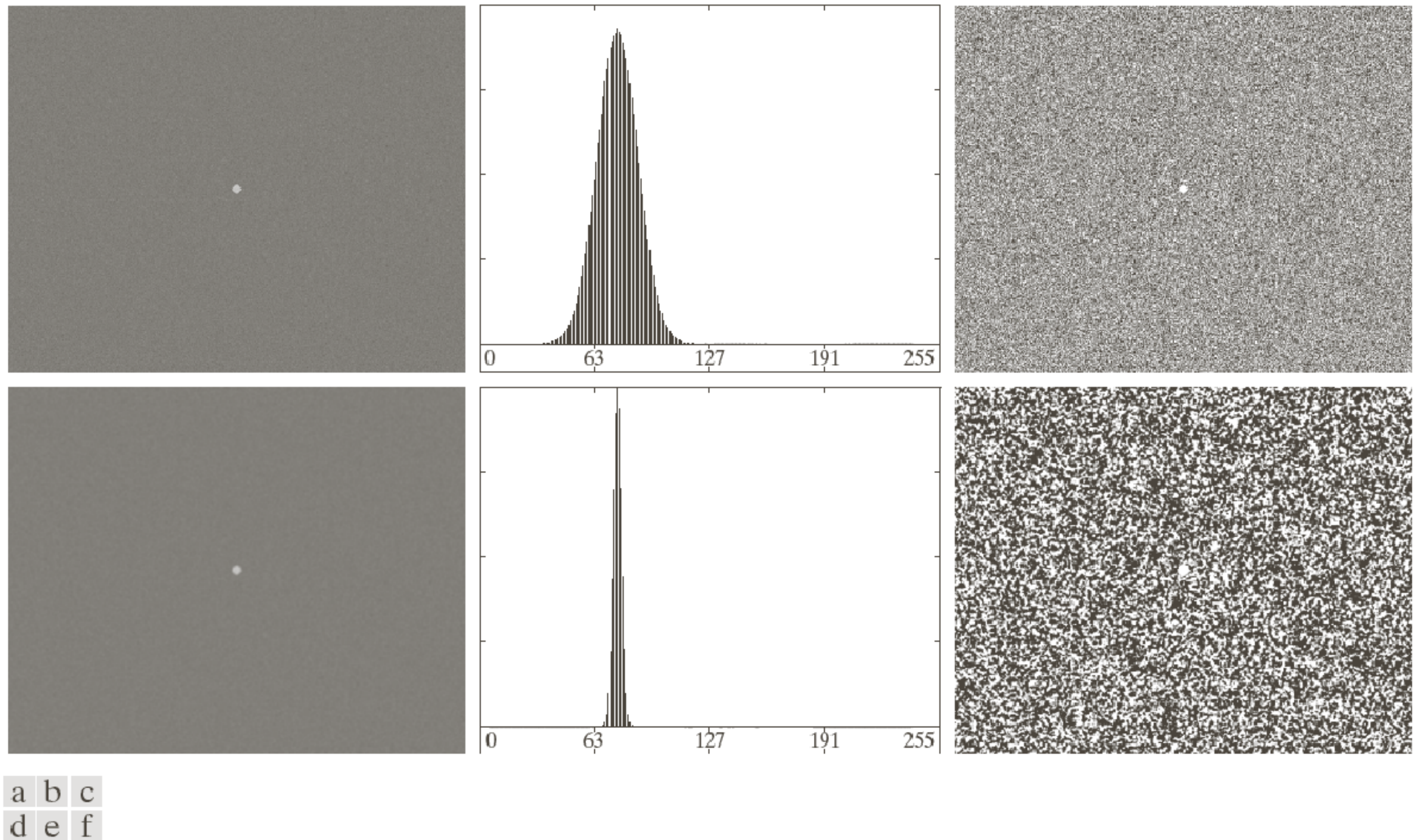


FIGURE 10.41 (a) Noisy image and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method. Thresholding failed in both cases.

10.3.5 Using edges to improve global thresholding

Strategy to obtain a histogram of which the peaks are tall, narrow, symmetric, and separated by deep valleys:
Consider only those pixels that lie on or near the edges between objects and the background

Algorithm

(1) Compute an edge image as either the magnitude of the gradient, or the absolute value of the Laplacian, of $f(x, y)$

(2) Specify a threshold value, T

(3) Threshold the image from step (1) using the threshold from step (2) to produce a binary image, $g_T(x, y)$, which is used as a mask image in the following step to select pixels from $f(x, y)$ corresponding to "strong" edge pixels

(4) Compute a histogram using only the pixels in $f(x, y)$ that correspond to the locations of the 1-valued pixels in $g_T(x, y)$

(5) Use the histogram from step (4) to segment $f(x, y)$ globally using, for example, Otsu's method

Example 10.17

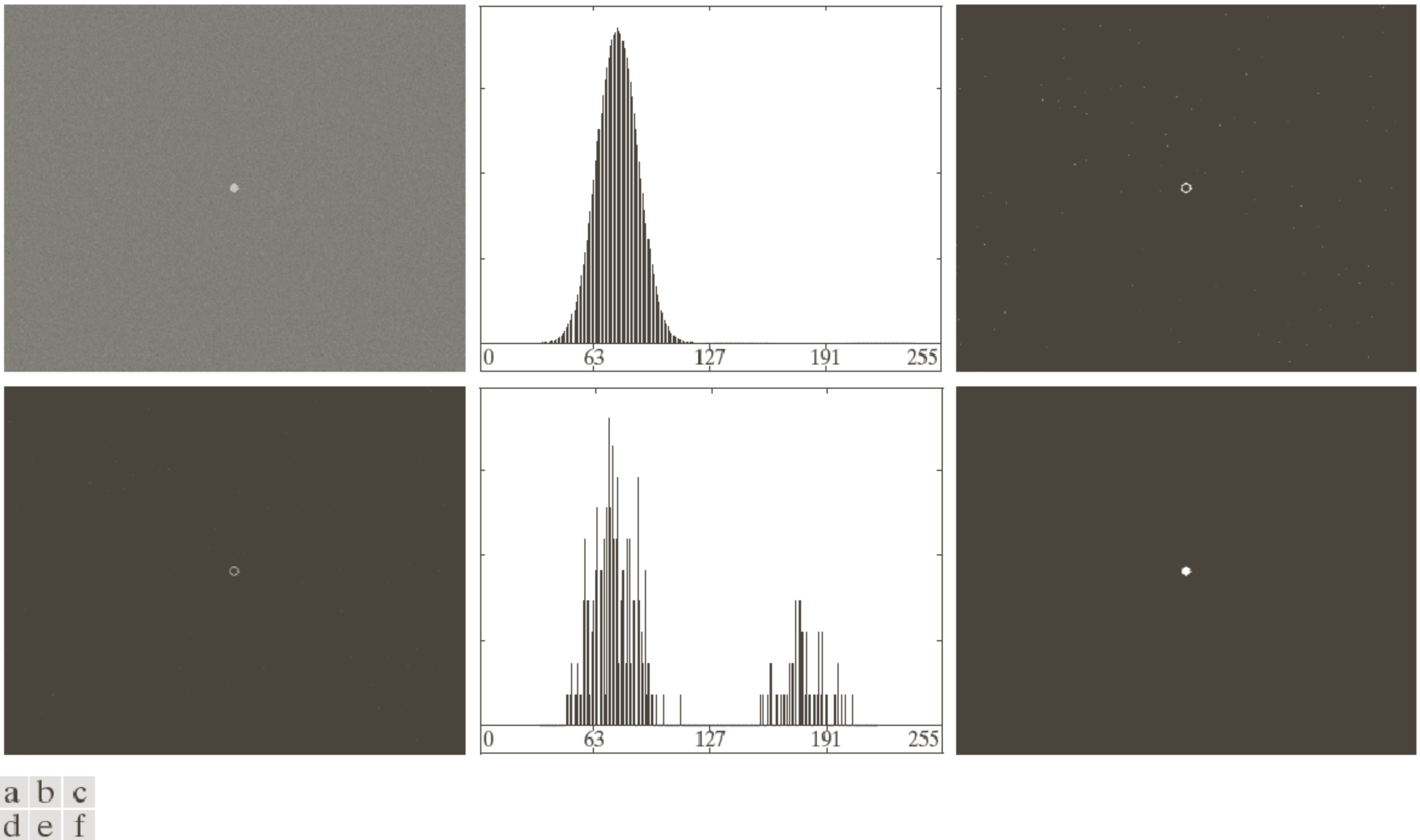
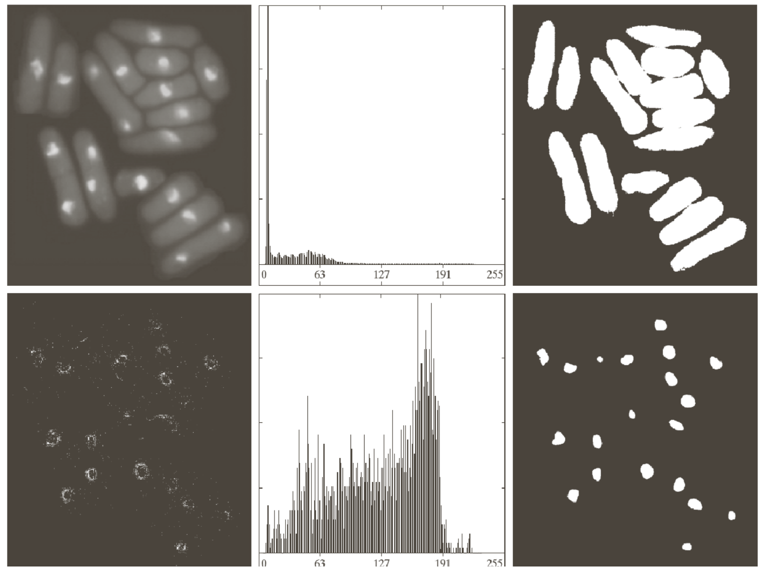


FIGURE 10.42 (a) Noisy image from Fig. 10.41(a) and (b) its histogram. (c) Gradient magnitude image thresholded at the 99.7 percentile. (d) Image formed as the product of (a) and (c). (e) Histogram of the nonzero pixels in the image in (d). (f) Result of segmenting image (a) with the Otsu threshold based on the histogram in (e). The threshold was 134, which is approximately midway between the peaks in this histogram.

Example 10.18



a b c
d e f

FIGURE 10.43 (a) Image of yeast cells. (b) Histogram of (a). (c) Segmentation of (a) with Otsu's method using the histogram in (b). (d) Thresholded absolute Laplacian. (e) Histogram of the nonzero pixels in the product of (a) and (d). (f) Original image thresholded using Otsu's method based on the histogram in (e). (Original image courtesy of Professor Susan L. Forsburg, University of Southern California.)

FIGURE 10.44 Image in Fig. 10.43(a) segmented using the same procedure as explained in Figs. 10.43(d)–(f), but using a lower value to threshold the absolute Laplacian image.