

In[40]:= **L = .**

In[41]:= **L**

Out[41]= **L**

In[42]:= **dot = Integrate[Cos[k Pi x / L] * Cos[j Pi x / L], {x, -L, L}]**

$$\text{Out[42]= } \frac{1}{\pi} L \left(\frac{\text{Sin}[(j - k) \pi]}{j - k} + \frac{\text{Sin}[(j + k) \pi]}{j + k} \right)$$

In[44]:= **Simplify[dot, {j ∈ Integers, k ∈ Integers}]**

Out[44]= **0**

In[45]:= **dot = Integrate[Sin[k Pi x / L] * Sin[j Pi x / L], {x, -L, L}]**

$$\text{Out[45]= } \frac{1}{\pi} L \left(\frac{\text{Sin}[(j - k) \pi]}{j - k} - \frac{\text{Sin}[(j + k) \pi]}{j + k} \right)$$

In[46]:= **Simplify[dot, {j ∈ Integers, k ∈ Integers}]**

Out[46]= **0**

In[47]:= **dot = Integrate[Cos[k Pi x / L] * Cos[k Pi x / L], {x, -L, L}]**

$$\text{Out[47]= } L + \frac{L \text{Sin}[2 k \pi]}{2 k \pi}$$

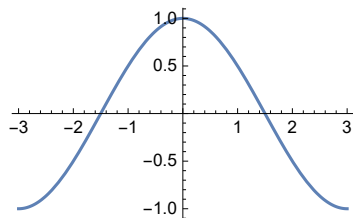
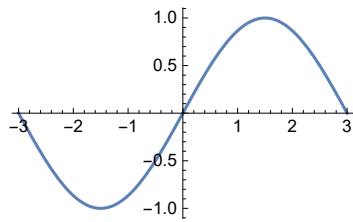
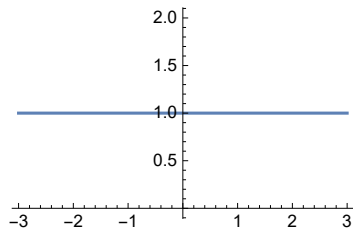
In[48]:= **Simplify[dot, {j ∈ Integers, k ∈ Integers}]**

Out[48]= **L**

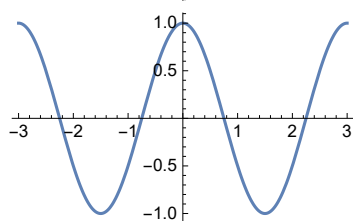
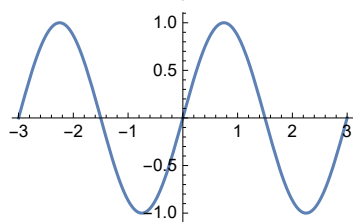
In[8]:= **L = 3**

Out[8]= **3**

```
In[20]:= Grid[{{Plot[1, {x, -L, L}]},
  {Plot[Sin[1 x Pi / L], {x, -L, L}], {Plot[Cos[1 x Pi / L], {x, -L, L}]},
  {Plot[Sin[2 x Pi / L], {x, -L, L}], {Plot[Cos[2 x Pi / L], {x, -L, L}]}}
```



```
Out[20]=
```



```
In[70]:= L = .
```

```
In[71]:= ak = 1/L * Integrate[-1 Cos[k * Pi x / L], {x, -L, -L/2}] +
  1/L * Integrate[x Cos[k Pi x / L], {x, 0, L}]
```

```
Out[71]= 
$$\frac{\sin\left[\frac{k\pi}{2}\right] - \sin[k\pi]}{k\pi} + \frac{1}{k^2\pi^2}L(-1 + \cos[k\pi] + k\pi \sin[k\pi])$$

```

```
In[72]:= ak = Simplify[ak, k ∈ Integers]
```

```
Out[72]= 
$$\frac{(-1 + (-1)^k)L + k\pi \sin\left[\frac{k\pi}{2}\right]}{k^2\pi^2}$$

```

In[73]:= **bk = 1/L * Integrate[-1 Sin[k * Pi x / L], {x, -L, -L/2}] + 1/L * Integrate[x Sin[k Pi x / L], {x, 0, L}]**

Out[73]=
$$\frac{\cos\left[\frac{k\pi}{2}\right] - \cos[k\pi]}{k\pi} + \frac{L(-k\pi\cos[k\pi] + \sin[k\pi])}{k^2\pi^2}$$

In[74]:= **bk = Simplify[bk, k ∈ Integers]**

Out[74]=
$$\frac{(-1)^{1+k}(1+L) + \cos\left[\frac{k\pi}{2}\right]}{k\pi}$$

In[75]:= **a0 = 1/L * Integrate[-1, {x, -L, -L/2}] + 1/L * Integrate[x, {x, 0, L}]**

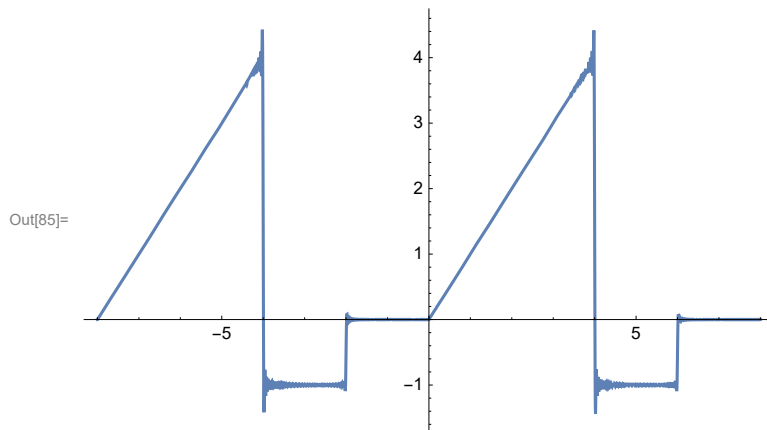
Out[75]=
$$-\frac{1}{2} + \frac{L}{2}$$

In[83]:= **ff = a0/2 + Sum[ak * Cos[k Pi x / L] + bk * Sin[k Pi x / L], {k, 1, 200}];**

In[77]:= **L = 4**

Out[77]= **4**

In[85]:= **Plot[ff, {x, -2 L, 2 L}]**

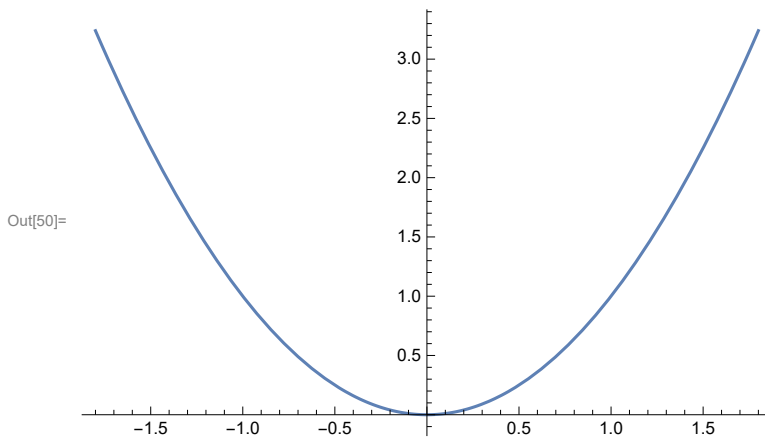


EXAMPLE

In[49]:= **f = x^2**

Out[49]= **x²**

In[50]:= **Plot[f, {x, -1.8, 1.8}]**



In[51]:= **ak = 1/L Integrate[f Cos[k Pi x / L], {x, -L, L}]**

$$\text{Out[51]} = \frac{1}{k^3 \pi^3} 2 L^2 (2 k \pi \text{Cos}[k \pi] + (-2 + k^2 \pi^2) \text{Sin}[k \pi])$$

In[52]:= **ak = Simplify[ak, k ∈ Integers]**

$$\text{Out[52]} = \frac{4 (-1)^k L^2}{k^2 \pi^2}$$

In[53]:= **bk = 1/L Integrate[f Sin[k Pi x / L], {x, -L, L}]**

Out[53]= 0

In[54]:= **a0 = 1/L Integrate[f, {x, -L, L}]**

$$\text{Out[54]} = \frac{2 L^2}{3}$$

In[64]:= **F = a0/2 + Sum[ak * Cos[k Pi x / L], {k, 1, 30}];**

In[56]:= **L = 1.8**

Out[56]= 1.8

In[68]:= **Plot[{F}, {x, -10, 12}]**

