AM 214

# TUTTEST 13 – MEMO

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$\frac{10}{16}$

(a) $A\underline{a} = \underline{a} \times \underline{a} = \underline{0}$

$$A^3 = AA^2 = A(\underline{a}\underline{a}^T - I) = (A\underline{a})\underline{a}^T - A$$

$$= 0 - A = -A \checkmark$$

(b) $$(-A^2)(-A^2) = A^2A^2 = (\underline{a}\underline{a}^T - I)(\underline{a}\underline{a}^T - I)$$

$$= \underline{a}(\underline{a}^T\underline{a})\underline{a}^T - \underline{a}\underline{a}^T - \underline{a}\underline{a}^T + I$$

$$= \underline{a}\,1\,\underline{a}^T - 2\underline{a}\underline{a}^T + I$$

$$= I - \underline{a}\underline{a}^T = -(\underline{a}\underline{a}^T - I)$$

$$= -(A^2) \checkmark\checkmark$$

$\longrightarrow$ Idempotent $\longrightarrow$ projection matrix

(c) $$Q(\underline{a}, 180^\circ) = I + s_{180^\circ}A + (1 - c_{180^\circ})A^2$$

$$= I + 0A + (1 - (-1))A^2$$

$$= I + 2A^2 = I + 2(\underline{a}\underline{a}^T - I)$$

$$= 2\underline{a}\underline{a}^T - I \checkmark\checkmark$$

H through $\underline{a}$

$$= 2P - I \quad \text{where } P = \underline{a}\underline{a}^T$$

$$= 2\underline{a}\underline{a}^T - I \checkmark$$

same

(2)

(d) $Q = I + s_\theta A + (1-c_\theta)A^2$

$$tr(Q) = tr(I) + s_\theta\, tr(A) + (1-c_\theta)\, tr(A^2)$$

but $tr(I) = 3$, $tr(A) = 0$

$$\left.\begin{aligned} tr(A^2) &= tr(\underline{a}\underline{a}^T - I) \\ &= tr(\underline{a}\underline{a}^T) - tr(I) \\ &= a_1^2 + a_2^2 + a_3^2 - 3 \\ &= 1 - 3 = -2 \end{aligned}\right\}$$

More complete than necessary.

$$tr(Q) = 3 + 0 + (1-c_\theta)(-2)$$
$$= 3 - 2 + 2c_\theta = 1 + 2c_\theta$$

$$c_\theta = \frac{tr(Q) - 1}{2} \quad ✓$$

$$Q - Q^T = I + s_\theta A + (1-c_\theta)A^2$$
$$- I - s_\theta(-A) - (1-c_\theta)A^2$$
$$= 2s_\theta A$$

$$A = \frac{Q - Q^T}{2s_\theta} \quad ✓$$

(d) $\mathrm{tr}(Q) = \frac{8}{4} = 2$

$$c_\theta = \frac{2-1}{2}, \quad \theta = 60^\circ \checkmark$$

$$\frac{Q - Q^T}{2s_\theta} = \frac{1}{4}\begin{bmatrix} 0 & -2\sqrt{6} & 0 \\ 2\sqrt{6} & 0 & -2\sqrt{6} \\ 0 & 2\sqrt{6} & 0 \end{bmatrix} / \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$\underline{a} = \frac{2}{4} \cdot \frac{1}{\sqrt{3}} \begin{bmatrix} 2\sqrt{6} \\ 0 \\ 2\sqrt{6} \end{bmatrix}$$

$$= \frac{1}{2\sqrt{3}} \begin{bmatrix} \sqrt{3}\sqrt{2} \\ 0 \\ \sqrt{3}\sqrt{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{2} \\ 0 \\ \sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \checkmark$$

Alternatively, you could write

$$\underline{a}_{\text{unnormalised}} = \begin{bmatrix} 2\sqrt{6} \\ 0 \\ 2\sqrt{6} \end{bmatrix} \overset{\text{scaled}}{=\!=} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{a}_{\text{normalised}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$